Matter and Energy: Special Relativity

Albert Einstein (1879 - 1955)

(Also: Brownian Motion,The Photoelectric Effect, General Theory of Relativity)

The Special Theory of Relativity

"On the Electrodynamics of Moving Bodies" (1905)

$$E = mc^2$$

Conclusions and clarifications regarding Mass and Energy (Einstein)

- The total energy of a moving mass is $E = mc^2$ or $E = \gamma m_o c^2$

where m_o is the rest mass,
$m = \gamma m_o$ is the inertial mass and
$\gamma = [1 - (v/c)^2]^{-1}$ is the Lorentz Factor
$E_o = m_o c^2$ is the rest energy (the energy if $v = 0$), and
$(E - E_o) = (\gamma - 1)m_o c^2$ is the kinetic energy associated with the motion
... and the speed of light becomes a fundamental "cosmic speed limit"

Newton: The only energy is the Kinetic Energy associated with the motion
i.e., $E = (1/2)mv^2$ ··· and $E = 0$ for $v = 0$

Special Relativity

Some Other (testable and much tested) Consequences*:

Inertial Mass: $m = \gamma\, m_o$

Newton: $m = m_o$

Fitzgerald-Lorentz Contraction: $L = L_o/\gamma$

Newton: $L = L_o$

Time Dilatation: $\Delta t = \gamma\, \Delta t_o$

Newton: $\Delta t = \Delta t_o$

Addition of Velocities: $v_{rel} = (u + v)/(1 + u\cdot v/c^2) \leq c$

Newton: $v_{rel} = u + v$

Transverse Doppler Effect: $\nu/\nu_o = \gamma\,(1 - v_{radial}/c)$

Classical: $\nu/\nu_o = (1 - v_{radial}/c)$*

...as well as observing the conversion of mass to energy and vice versa.

Note that the precepts of Special Relativity apply only to motions observed within inertial (unaccelerated) systems. Accelerated frames, including gravitational " accelerations" were addressed in Einstein's General Relativity (1915)

(Newtonian Mechanics was thought applicable to both inertial and accelerated systems)

Digression: Newton's Second Law of Mechanics

Einstein adopts the same operational definition of force as Newton
It is the rate at which momentum **p** changes in response to an applied force **F**

$$\mathbf{F} = d\mathbf{p}/dt \quad \text{where} \quad \mathbf{p} = m\mathbf{v}$$

However:

The more familiar form **F** = m**a** where **a** = d**v**/dt is only true if m is a constant!

- This is not the case in Newtonian mechanics when $m = m(t)$
- It is never the case in Relativistic mechanics where $m = m(v) = \gamma m_o$

Moreover:

- The momentum is not simply m**v** but is related to the (total) energy and the rest mass according to the quadratic expression: $p^2c^2 = E^2 - m_o^2c^4$

(Compare this to the Newtonian equivalent: $p^2 = 2m_oE$)

The Special Relativistic expression corresponding to Newton's "F = ma" is

$$\mathbf{F} = m[\mathbf{a} + (\gamma/c)^2(\mathbf{v}\cdot\mathbf{a})\mathbf{v}\ ]$$

...and the resulting acceleration is generally not parallel to the applied force!
(This would occur only if the force **F** was parallel to the velocity **v**.)

Again, the precepts of Special Relativity apply to observations or measurements made in inertial (unaccelerated) systems.

Energy

The Special Theory of Relativity: $E = mc^2$

The State of Matter in 1905

- Matter is composed of atoms.
- Elements are defined and distinguished by their chemical properties
- The atom is the smallest unit of an element
- Atoms of different elements have different weights
- Chemical properties vary in a regular way with atomic weight (or atomic number)
- Molecules are combinations of two or more atoms
- Atoms have spatial extent and (charge) structure.

Some reference numbers:

The mass of the hydrogen atom is $m_H = 1.67 \times 10^{-27}$ kg

(The energy equivalent is about 1.50×10^{-10} joule)

The “radius” of a hydrogen atom is $a_o = 5.29 \times 10^{-11}$ m

Avogadro’s Number is $N = 6.022 \times 10^{+23}$ mole^{-1}

But What was the State of Energy in 1905?

Energy in History

(Note historical distinctions between "light," "heat","energy" and "matter")

The Classical View

- **Empedocles** (484 - 424 BCE)**:** Earth, Water, Air, **Fire** (with *philia* and *neikos*.)
- **Anaxagoras** (500 - 428 BCE)**:** Continuity of Matter *versus*
- **Democritus** (460 - 370 BCE)**:** Atomism
- **Aristotle** (384 - 322 BCE)**:** Light as substance possessing **energy**; photons

Etymology: εργον = "work", ενεργεια = "vigor" (Aristotle)

Heat, Light, and Energy

- **Francis Bacon** (1521 - 1626): Suggests **heat** is related to **motion**
- **Gottfried Liebnitz** (1676 - 1689): The observed conservation of ***vis viva*** (= mv^2)
- **Guillaume Amontons** (1702): Gases and the concept of absolute zero
- **Daniel Bernoulli** (1700 - 1782) : **Kinetic Theory** (*Hydrodynamics*)
- **Antoine Lavoisier** (1743 - 1794): Combustion and energy as substance: **caloric**.
- **Benjamin Thompson** (1798): Frictional heat from kinetic energy. Death of *caloric.*

Some More History & Etymology

Thomas Young (1807): "Energy" for energy of motion (*cf. vis viva)*
Gustave-Gaspard Coriolis (1829): "Kinetic Energy" (= $(1/2)mv^2$)
William Rankine (1853): "Potential Energy"

The Nature of Energy

Definitions

Energy is the ability to do work.

Work is required to change the energy of a system:

$W = |\Delta E|$

Work is expended when a system is displaced by a **Force**:

$\mathbf{W = F \cdot d}$

(Note that a displacement* must be accomplished for work to be done.)

Power is the rate at which work is done or energy expended:

$\Delta W = P\ \Delta t$

MKS Units of Force, Energy, and Power (MKS = Meter-Kilogram-Second)

Force: **1 newton = 1 kg m s^{-2}**

Energy & Work: **1 joule = 1 newton meter = 1 watt s = 1 kg m^2 s^{-2}**

Power: **1 watt = 1 joule s^{-1} = 1 kg m^2 s^{-3}**

* ...hence work is done (energy is expended) in compressing a gas:

$W = |\Delta E| = p\Delta V$

Forms of Energy

Kinetic Energy

...associated with the motion of mass

$K = Mv^2/2$

Radiative Energy

...carried by electromagnetic radiation

$E = h\nu$

Potential Energy

... “stored” energy potentially available to do work

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Heat

Note that “heat” has sometimes been used as a synonym for “energy”.
Heat is kinetic energy associated with microscopic motions within a system

$E = (3/2)NkT$ (... in an ideal monatomic gas; $k = k_B$)

Energy in waves carried by matter (*e.g.*, sound & surf) is also kinetic energy.

More Forms of Energy: Twentieth Century Additions

- **Rest Mass Energy (Einstein’s $E = m_o c^2$)**
- **Vacuum Energy (Quantum Fluctuations)**
- **Zero Point EM energy (Casimir Effect 1948)**

.... and maybe “Dark Energy”?

The Transport of Energy

How does energy move from place to place?

RADIATION

Energy carried by radiation
Electromagnetic Waves and Photons

CONVECTION

Energy carried by large scale motions of matter
Convective Flows of Matter

CONDUCTION

Energy carried by small scale motions of matter
Conduction of heat, electricity

WAVE PROPAGATION

Energy carried by oscillatory motions of matter
Longitudinal & Transverse Waves - at all scales

and finally

GRAVITATIONAL WAVES

Energy carried by fluctuations in spacetime (Einstein 1916)
Generated by and dissipated by mass oscillations

Thermodynamics & Statistical Mechanics

Experimental Foundations of Thermodynamics

Otto von Guericke (1650): Vacuum pump and the “Magdeburg Hemispheres”
(Aristotle: “Nature abhors a vacuum.”)
Robert Boyle & Robert Hooke (1656): Air pump and gas (P,V,T) relations
Denis Papin & Thomas Savery (1697): Safety Valves and the Steam Engine

Efficiency and Power of Engines

Sadi Carnot (1824): *Reflections on the Motive Power of Fire*
...useful “caloric” lost in any real process: Efficiency is ≤ 100%

William Thompson (Lord Kelvin, 1849): “Thermodynamics”
...inevitable heat lost in any real process

Theoretical Thermodynamics & Kinetic Theory

Rudolf Clausius (1850): Heat content and (macroscopic) entropy

James Maxwell and Boltzmann (1871): Statistical Thermodynamics

Ludwig Boltzmann (1875): **$S = + k \log W$**

where **W** is the number of possible states of the system and **k** is Boltzmann’s constant (sometimes k_B), one of the fundamental constants of nature.

Heat as Motion: Kinetic Theory

Newton (1701): Heat as substance; pressure as repulsion between molecules

versus

Bernoulli (1738): Heat as kinetic energy; pressure as a collisional process.

The Ideal Gas

A gas consist of numerous particles, each of which has a finite mass.
(atoms, molecules, ..)

The particles are very small in size compared to their average separations.
(atoms, molecules, ..)

The particles are in motion with an isotropic distribution of velocities.
(the system is relaxed)

The particles do not interact with each other except by (elastic) collisions.
(interparticle forces are short range forces)

and

Collision physics is Newtonian, with energy and momentum being conserved.
(non-relativistic velocities; elastic collisions)

Kinetic Theory and Statistical Mechanics

Predictions & Consequences

For a volume **V** containing a mass **M** in the form of **N** particles, each of mass **m**:

$$M = Nm$$

The <u>pressure</u> **P** exerted on the container walls by collisions is:

$$P = M\langle v^2\rangle/3V$$

where $\langle v^2\rangle$ is the <u>mean</u> <u>square</u> velocity of the particles.

The average <u>kinetic energy</u> of a particle is:

$E = m\langle v^2\rangle/2$ or $E = 3PV/2N$

But the Perfect Gas Law is:

$$PV = NkT$$

where **T** is the temperature and **k** is Boltzmann's Constant. Thus

$$E = 3kT/2$$

so the temperature of a gas gives the average kinetic energy of its particles.

Heat (as parametrized by temperature) is kinetic energy!

…but what about light?

A Thermodynamics Primer

Digression on Nomenclature

"System" and "Surroundings"
Boundaries
Averages
Macroscopic Properties: M, V, N; E, U, P, T, S,....
Microscopic Properties: m_k, $\mathbf{v}_k$, $\mathbf{s}_k$ for k = 1, 2, ...N

System Types and Boundaries

Isolated: No matter or energy crossing the boundary
Closed: No matter crossing the boundary
Adiabatic: No heat crossing the boundary (otherwise Diathermic)
Open: Heat, matter, and work can all cross the boundary

Notes

Systems can do work on their surroundings - or have work done on them.
Example: $\Delta W = P\Delta V$

A system is said to be in a state of Thermodynamic Equilibrium when its (macroscopic) properties are constant in time. An isolated system will approach this state.

The Concept of Entropy

Boltzmann (1868)

- A system can exist in many equally probable microscopic states.
- Many microscopic states can give the same macroscopic observables.
- The probability of a given macroscopic state is proportional to the number, W, of microscopic states which give the same macroscopic observables.
- The entropy S associated with a macroscopic state is then defined by:

$$S = +k \log_e W$$

where W is the number of contributing microscopic states and k is a constant.

The Boltzmann Constant is one of the fundamental constants of nature.

$$k = 1.3806 \times 10^{-23} \text{ joule } {}^\circ\text{K}^{-1}$$

- A system's entropy increases with the addition of energy to the system according to

$$\Delta S = \Delta Q/T$$

...where T is the temperature of the system and ΔQ is the added energy.

Entropy is an independent macroscopic property of a system, just like mass, volume, number of particles, or internal energy; it cannot be expressed in terms of those other quantities. The entropy of a system depends upon its history.

The Laws of Thermodynamics

First Law: Conservation of Energy

The increase in energy of a closed system is equal to the energy added in the form of heat minus the work done by the system on its surroundings:

$$\Delta U = \Delta Q - \Delta W$$

Second Law: Entropy Always Increases

The entropy of an isolated system will only increase with time. Generally

$$\Delta S \geq 0$$

The entropy will approach a maximum value as the system evolves in time.

Third Law: Absolute Zero

As a system’s temperature approaches absolute zero all processes cease, and the system entropy approaches its minimum value.

also

Zeroth Law: Thermodynamic Equilibrium

A system in thermodynamic equilibrium is by definition its most probable state; all processes have essentially gone to completion and its macroscopic variables are unchanging. Also, if two systems are in thermodynamic equilibrium with a third they are in thermodynamic equilibrium with each other.

Thermodynamic Humor

Zeroth Law

“You must play the game.”

First Law

“You can’t win the game.”

Second Law

“You can’t even tie the game.”

Third Law

“You can’t get out of the game.”

... attributed to C. P. Snow

Thermodynamics: Fragments

Work done on a system increases its internal energy

First Law: $\Delta U = \Delta Q - \Delta W$ (Usually $\Delta U = \Delta Q - P\Delta V$)

Second Law: $\Delta Q = T\Delta S$ (For a reversible process)

$\Delta U = T\Delta S - \Delta W$ (Usually $\Delta U = \Delta Q - P\Delta V$)

But S, T, U are state functions so the above holds for both reversible and irreversible changes.

If the system has additional external variables other than the volume, which change or if the number of particles in the system N changes, one can write

$$\Delta W = P\Delta V + F_k\Delta x_k - \mu_n\Delta N_n \qquad \text{(summation convention)}$$

where the are F_k generalized forces corresponding to external variables x_k and the μ_k are the chemical potentials associated with particle of type n.

Energy Flow

- **Energy ΔQ can only flow from the “hotter” system to a “cooler” system.**

ΔT will be positive if ΔU is positive

- **This energy flow can (usually) be used to do work.**
- **Some of the energy is dissipated (as heat) during any such process. This energy loss ΔE is related to the change in entropy of the overall system**

$\Delta E = S\Delta T$ (always negative)

Available Energy

- The energy available to do work in a system is the stored energy minus the energy that would be lost as dissipated heat if the energy is extracted. The stored energy consists of the internal energy U and the the energy that might be available as the system does work on its surroundings by changing its volume. The available energy for two special cases is

$A = U - TS$ if T and V are held constant (Helmholtz Free Energy)

or

$G = U + PV - TS$ if T and P are held constant (Gibbs Free Energy)

Again, the available (“useful”) energy is always less than the stored energy.