The General Theory of Relativity

Preamble: The Special Theory of Relativity

Foundations

The Principle of Relativity & the constancy of the speed of light, c.

Consequences

Coordinate transformations between inertial systems are Lorentzian

Not "space" and "time" but "spacetime"

Energy and Momentum Conservation: Defines "relativistic" mass and energy

Not "mass" and "energy" but "mass-energy"

Applications

Replaces Newton's Laws of Mechanics in inertial systems

Reduces to Newton's Mechanics when v << c

.........aside from the concept of rest energy!

Limitations

Precepts of Special Relativity apply only to inertial (unaccelerated) systems.

Spacetime geometry is presumed to be flat (Minkowsi spacetime)

The General Theory of Relativity addresses these issues by associating gravity with curvatures introduced into the very fabric of spacetime by the presence of mass-energy.

Testing the Special Theory of Relativity

What does the Special Theory Accomplish?

It explains the Results of the Michelson-Morley Experiments
It resolves the transformation problems of Maxwell's Electromagnetism

What are the testable (and surprising) predictions of Special Relativity?

Phenomenon	Special Relativity	Newtonian Mechanics
Fitzgerald-Lorentz Contraction:	$L = L_o/\gamma$	$L = L_o$
Time Dilation:	$\Delta t = \Delta t_o/\gamma$	$\Delta t = \Delta t_o$
Addition of Velocities:	$u_v = (u_v' + v)/(1 + u_v'v/c^2) \leq c$	$u_v = u_v + v$
	$u_{y,z} = (u_{y,z}'/\gamma(1 + u_v'v/c^2)$	$u_{y,z} = u_{y,z}'$
Relativistic Doppler Effect:	$\lambda = \gamma\lambda'(1 + v_{radial}/c)$	$\lambda = \lambda'(1 + v_{radial}/c)$
Total Energy:	$E = \gamma m_o c^2$	$E = m_o v^2/2$
Rest Energy:	$E_o = m_o c^2$	$E_o = 0$
Inertial Mass:	$m = \gamma m_o$	$m = m_o$
Force Law:	$\mathbf{F} = \gamma m_o \mathbf{a} + (\mathbf{F} \cdot \mathbf{u})\mathbf{u}/c^2$	$\mathbf{F} = m_o \mathbf{a}$

Note that while successes of these predictions do not prove STR (and never can) they do falsify Newtonian theory in all area where the two disagree.

Recollect Limitations:

The precepts of Special Relativity apply only to inertial (unaccelerated) systems.
The spacetime geometry must be locally flat (Euclidean or Minkowskian)

The General Theory of Relativity

Albert Einstein (1911 - 1916)

<u>Motivations & Questions</u>

- What is an "inertial" or "unaccelerated system"?
  - Are they really the same thing?
    - ... Can inertial systems be in relative acceleration?
  - Is a freely falling observer in an inertial frame?
    - ... Since such an observer feels no acceleration
- How does one handle the transformations between accelerated systems?
  - ... in a manner consistent with the Principle of Relativity.
    - (cf. Copernicus, Galileo, Newton, Einstein, ...)
- How does one deal with the special case of gravity?
  - What is the significance of the Equivalence Principle?
  - Why is gravity fundamentally different from other "forces"?)
- Dare one mention Mach's Principle?

Many revolutionary ideas in science arose because then prevailing theories were falsified by observations - or couldn't explain them. With General Relativity there was really only one <u>known</u> phenomenon - the anomalous advance of the perhelion of Mercury - that seemed do defy explanation in terms of Newtonian or Einsteinian (STR) mechanics. The main motivation for developing GTR was that the new physics (STR) provided no method for the transformation of coordinates (z,t) between frames in relative acceleration.

The Equivalence Principle

(Galileo *c.* 1610)

Experiment

The gravitational acceleration of a mass is independent of the magnitude (a.k.a. “weight) of that mass.

(Aristotle was wrong: heavier objects don’t fall faster than lighter objects!)

also

The gravitational acceleration of a mass is independent of the composition of that mass.

The Newtonian “Acceleration of Gravity” is $g = GM_{Earth}/R_{Earth}^2$

Note that $F = GMm/R_{Mm}^2$ is symmetrical in (M,nm)

The inertial mass of any object is apparently identical to its gravitational mass.

$W = gm_{gravitational}$ $\quad$ $F = m_{inertial}a_m$

with

$m_{gravitational} = m_{inertial}$

Empirically: An object’s weight is proportional to its inertial mass, the constant of proportionality being independent of the other properties of the object.

Testing the Equivalence Principle

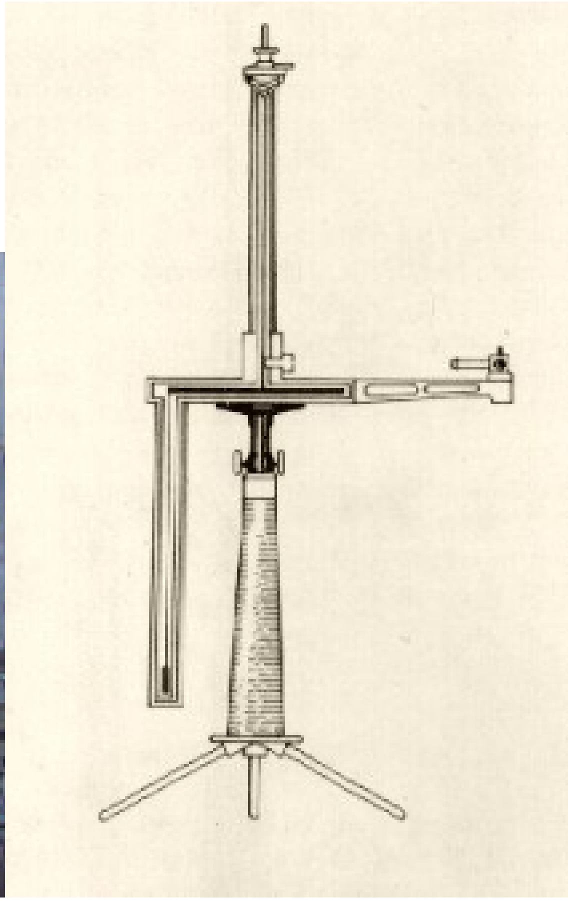
Experiment	Precision (as a limit on $\lvert\alpha - 1\rvert$)	Method
Philiponus, 500 (?)	small"	Drop Tower
Galileo, 1590 (?)	10^{-2}	Drop Tower
Newton, 1686	10^{-3}	Pendulum
Bessel, 1832	10^{-5}	Pendulum
Potter, 1923	10^{-6}	Pendulum
Eötvös, 1922.	10^{-8}	Torsion Balance
Dicke *et al.*, 1964	10^{-11}	Torsion Balance
Braginskii & Panov,1972	10^{-12}	Torsion Balance
Shapiro et. al., 1976	10^{-12}	Lunar Laser Ranging
Keiser & Faller, 1981	10^{-10}.	Fluid Support Balance
Niebauer *et al.*, 1987	10^{-10}	Drop Tower
Adelberger, 1990	10^{-12}.	Torsion Balance
APOLLO, 2006-	10^{-14}	Lunar Laser Ranging

Period of a pendulum of length L: $P = 2\pi(\alpha L/g)^{1/2}$

Free fall time from height H: $T = (2\alpha H/g)^{1/2}$

where $\alpha = m_{inertial}/m_{gravitational}$

The Leaning Tower **The Cathedral of Pisa** **Eötvös Torsion Balance**



Free-Fall Time

$T = (2H/g)^{1/2}$

Pendulum Period

$P = 2\pi(L/g)^{1/2}$

Attraction of Masses

$F = GMm/R^2$

The Equivalence Principle Revisited

Weak Equivalence Principle

The trajectory in a gravitational field of a freely falling body depends only upon its initial position and velocity - and is independent of its mass or composition.

• Inertial mass is identical to gravitational mass •

Einstein Equivalence Principle

The result of any local non-gravitational experiment conducted in an inertial frame will be independent of the location or the velocity of that frame.

• Weak Equivalence Principle plus Special Relativity: Lorentz Invariance •

Tests: Atomic and nuclear dimensionless constants

(Dimensionless physical constants are everywhere and always the same.)

Strong Equivalence Principle*

The result of any experiment, gravitational or not, conducted in an inertial frame will be independent of location, time, or the velocity of that frame.

• Application to self-gravitating objects. Gravitational self-energy •

Tests: Searches for temporal variations in G or fundamental particle masses. APOLLO.

***The only known "Theory of Gravity" which satisfies this is General Relativity.**

Application of the Equivalence Principle

<u>Einstein's *Gedankenexperiment*</u>
(Digression: Aristotle and Galileo, a half-massed thought experiment.)

So how do I know I'm in an inertial frame? Acceleration and Free-Fall.

••••••••••••••

An acceleration of kinematic origin is locally* indistinguishable from a gravitational field.

•

An inertial frame is an unaccelerated frame.

•

A freely-falling frame is an inertial frame.

•

But such inertial frames can be accelerating with respect to one another!
EXAMPLE
(So something beyond Special Relativity is needed in these circumstances.)

***Locality Qualification: Convergence and tidal effects excluded.**

The *Gedankenexperiment* Continued

Consider trajectories of light rays in accelerated and unaccelerated frames:

Starting Points

The Special Theory of Relativity
(The Principle of Relativity & the constancy of c)

The Equivalence Principle
(The indistinguishability of gravitational and inertial mass)

Scenarios

(1) A "Horizontal" Ray
The Gravitational Deflection of Light
(curvature of the observed trajectory)

(2) A "Vertical" Ray
The Gravitational Redshift or Gravitational Time Dilation
(... the same thing)

The General Theory of Relativity

Einstein (1915)
"If all accelerated systems are equivalent, then Euclidean geometry cannot hold in all of them."

Gravity is not a force in the Newtonian sense, rather:

- **The presence of matter (mass-energy) produces a curvature of spacetime**
- **Gravity is a manifestation of that curvature**
- **Inertial motion follows geodesics in that curved spacetime**

Digression: Geodesics and curved spaces

Forces are influences that cause departures from geodesic motion; gravity determines what constitutes geodesic motion.

John Archibald Wheeler : *"Spacetime grips mass, telling it how to move, and mass grips spacetime, telling it how to curve."*

...or *vice versa.*

Digression: The Einstein Field Equations

$$G_{ik} + \Lambda g_{ik} = \kappa T_{ik}$$

where:

κ and Λ are constants

G_{ik} is the Einstein Tensor

T_{ik} is the stress-energy tensor

g_{ik} is the spacetime metric tensor

$i,k = 1, 2, 3, 4$ ...corresponding to spacetime coordinates such as *(x,y,z,t)*

....which looks simple enough....

but these are, in fact, a set of ten* coupled second-order differential equations
Their solution is the metric tensor g_{ik} as a function of the coordinates

Notes

The value of κ is set by requirement that the behavior of objects will be described by Newtonian mechanics, including Newtonian gravity, in the weak field, low-velocity limit:

$$\kappa = 8\pi G/c^4$$

The term containing the Cosmological Constant Λ was later added by Einstein in order to accommodate (what he thought was) a static non-expanding universe.

* Actually 16, but with some duplicates. Conditions on g_{ik} reduce the number to six.

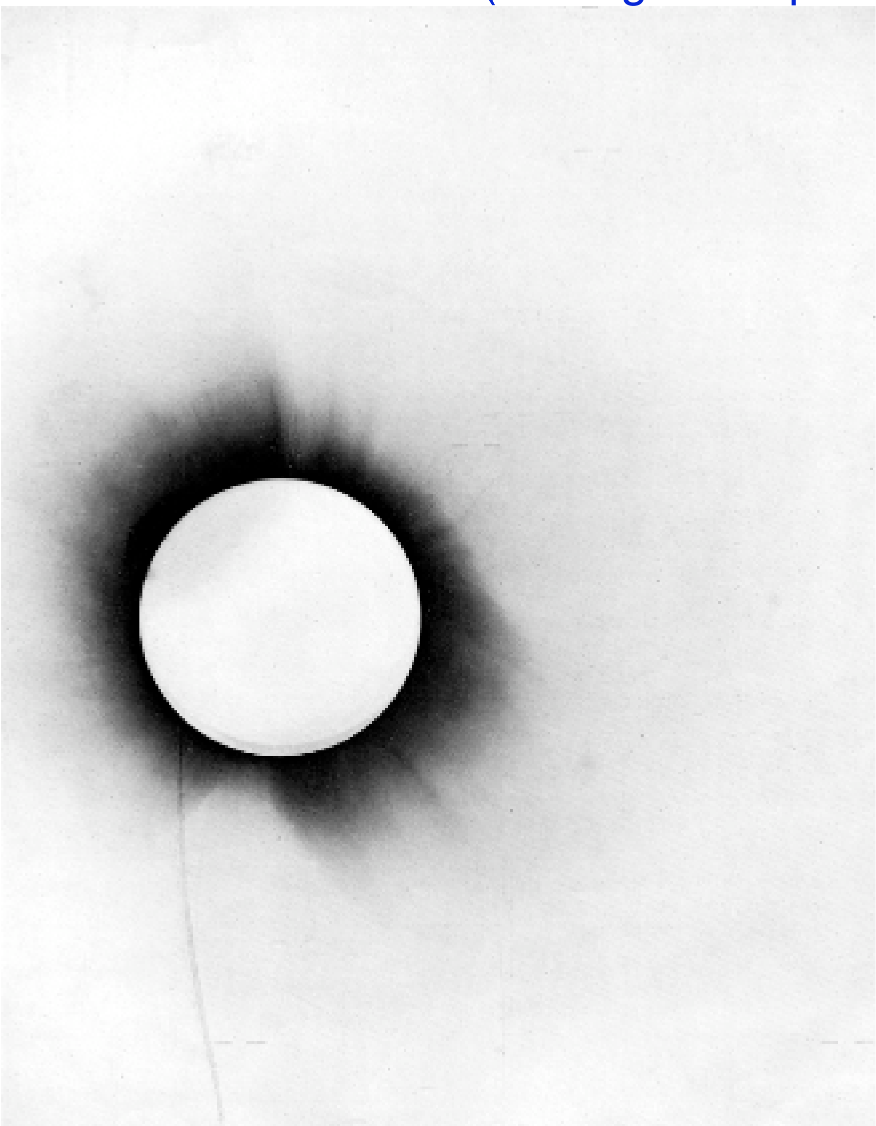
The General Theory of Relativity: Tests and Predictions

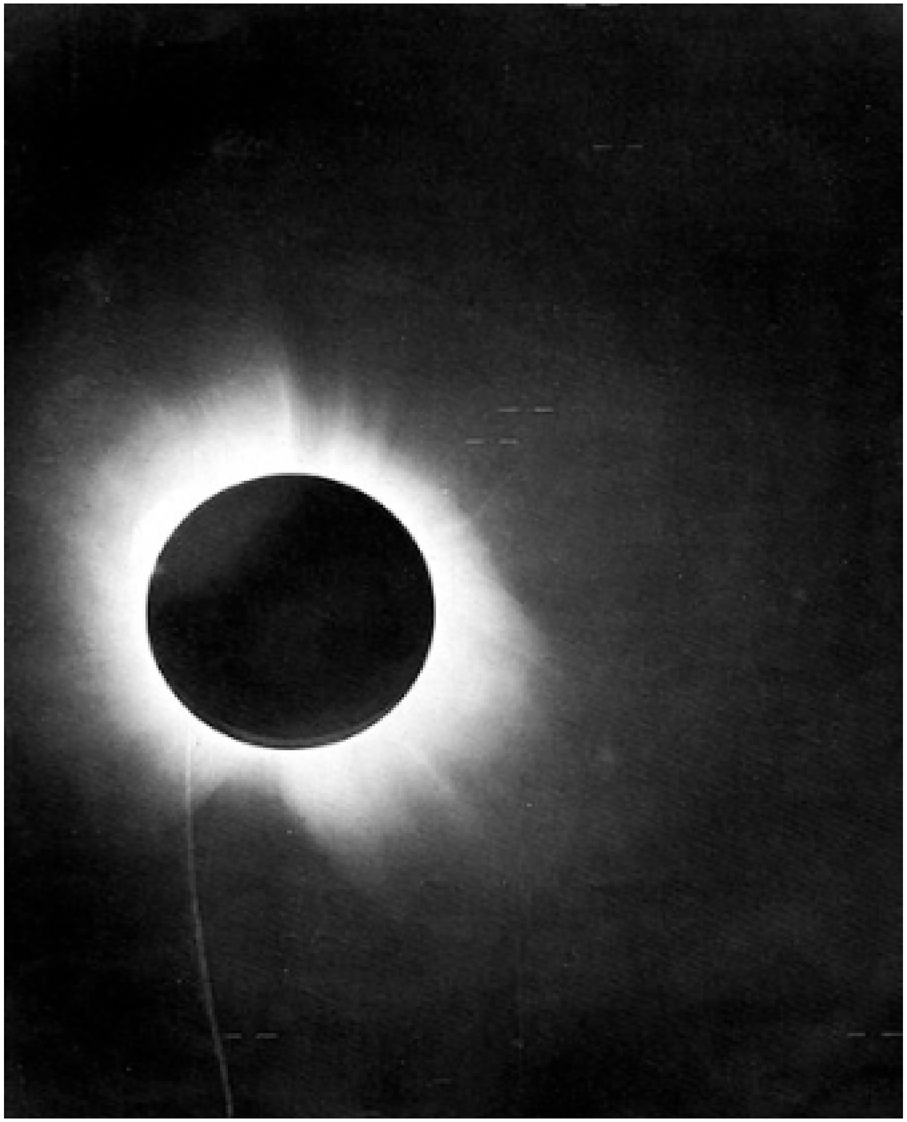
• The Advance of the Perhelion of Mercury
(Observed value is 0.4303” yr^{-1})
• The Gravitational Deflection of starlight: $\delta\theta \approx 4GM/rc^2$
(Eddington 1919; $\delta\theta_{Sun} \leq 1.75''$)
• The Gravitational Redshift: $\delta\lambda/\lambda \approx GM/rc^2$
(Hafele & Keating 1959)
• Gravitational Time Dilation: $\delta t/t \approx -GM/rc^2$
(Pound & Rebka 1959)
• Gravitational Time Delay: $\Delta t = (2GM/c^3)\ln(1- \cos\phi)$
(Shapiro 1964, $\Delta t_{Sun} \leq 190\ \mu s$)
• Gravitational Radiation & Orbital Decay
(Taylor & Hulse 1978)
• Frame Dragging and Geodetic Precession
(Lense-Thirring Effects 1918)
&
• Gravitational Lensing
• Direct Detection Gravitational Waves (LIGO)
• Additional tests of the Strong Equivalence Principle (APOLLO)
• A measurement of the speed of gravity
• Black Hole kinematics

Note: The above tests and experiments are continually being refined and improved to increasing levels of precision and subtlety.

Solar Eclipse of 29 May 1919

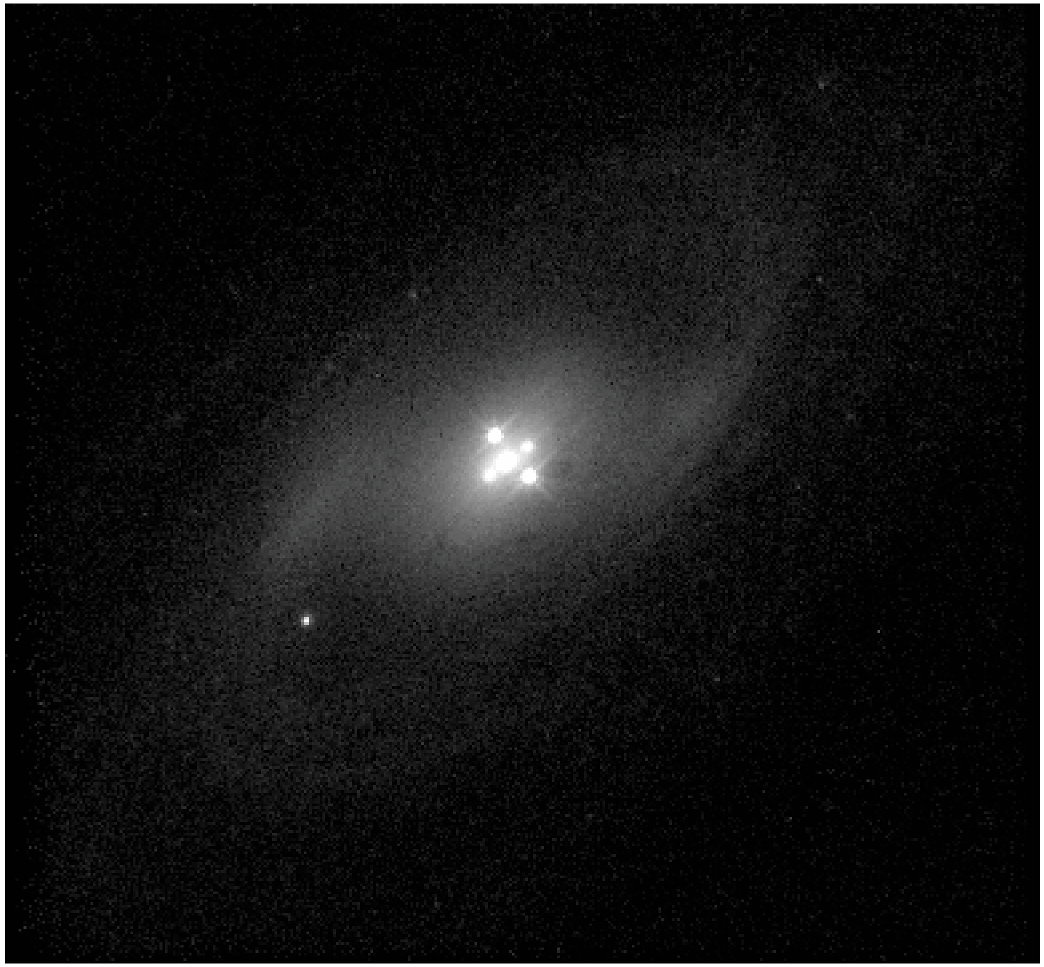
(Eddington Expedition: Note marked stars)





An Einstein Cross

ZW2237+030 = QSO2237+030 and G2237+030



There are five (5!) images of a <u>single</u> quasar and a foreground spiral galaxy in this picture

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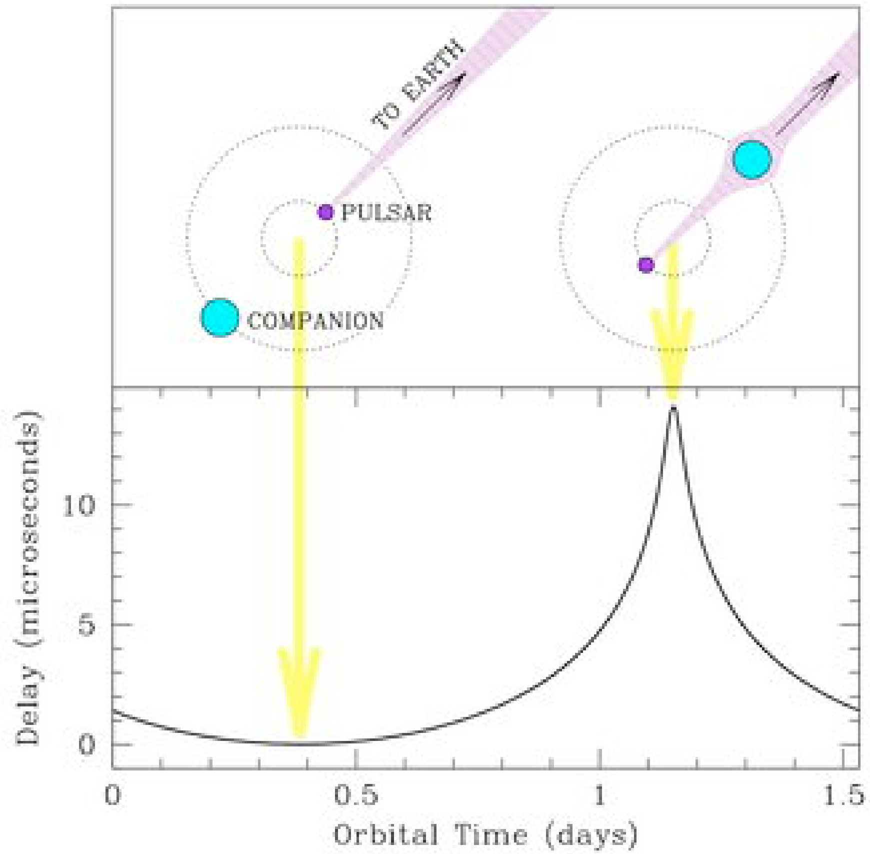
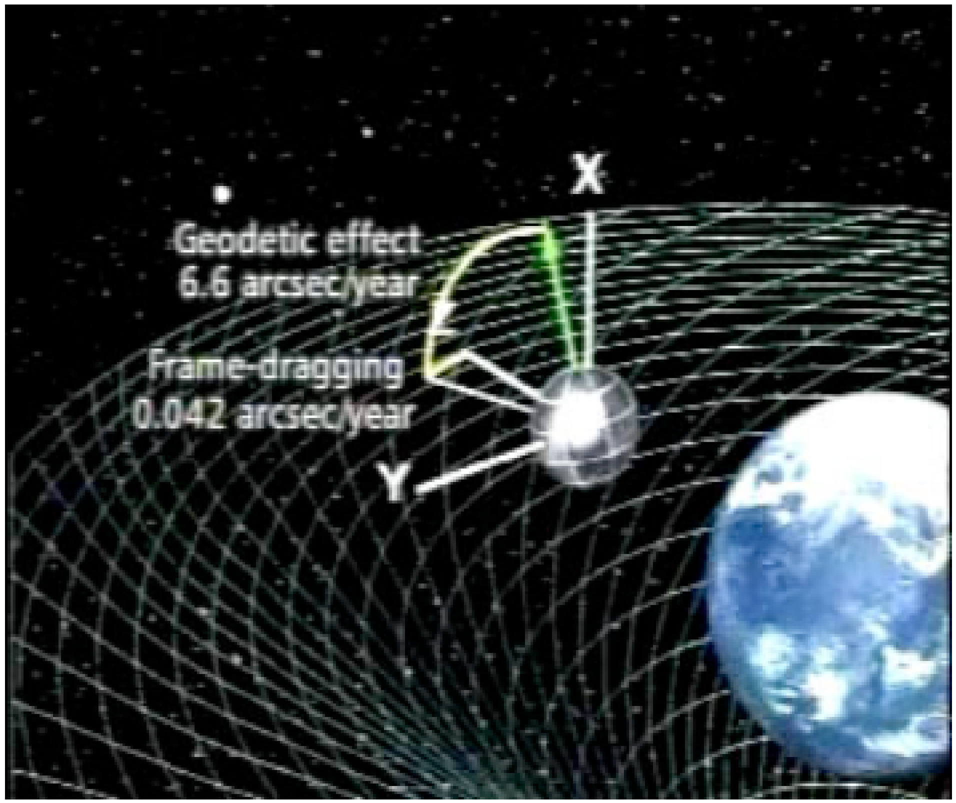
Pulsar Timing: The Shapiro Time Delay



Geodetic Effect and Frame Dragging

(Lense-Thirring Effect)



(Gravity Probe B)

APOLLO Laser in Action at Apache Point Observatory

General Relativity and Cosmology

Einstein's Original Field Equations (1915)

$$G_{ik} = (8\pi G/c^4)T_{ik}$$

Early Applications and Solutions

• Explanation: Advance of the perhelion of Mercury (1915)
• Observation:Gravitational deflection of starlight (1919)
• Theory: Schwarzschild Solution (1916)

Einstein's Cosmology

• Assumption: An isotropic universe
• Assumption: An homogeneous universe
• Prediction: An expanding (or contracting) universe!
• "Fixing" things: The Cosmological Constant, Λ (1922)

$$G_{ik} + \Lambda g_{ik} = (8\pi G/c^4)T_{ik}$$

• The Cosmological Constant, Λ: Death and Resurrection
• Hubble Expansion of the Universe (1929)
• Cosmological Acceleration (1998) & "Dark Energy"

General Relativity: Problems and Concerns

Alternative Theories of Gravity

- **Newtonian Gravity** (Falsified)
- **Modified Newtonian Dynamics (MOND)**
- **Scalar-Tensor Theories** (and time-variable "constants")
- **String Theory** (and the Strong Equivalence Principle)

Miscellaneous Current Concerns and Puzzlements

- **What is the nature of "dark matter"?**
- **What is the nature of "dark energy"?**
- **Awkward observations: The Pioneer Effect**

The Major Concern

General Relativity and Quantum Mechanics are not fully consistent

Is the problem with General Relativity, Quantum Mechanics - or both?

(Note: Special Relativity and Quantum Mechanics work together quite nicely!)

Are gravitons necessary?

Is spacetime itself quantized?

String Theory?

The Bottom Line

Einstein's General Relativity is an extremely successful theory of gravity, both in terms of its testability and its predictive power. It is also the simplest of surviving post-Newtonian theories of gravity.

Musings and Quibbles:

The exact expression for the gravitational redshift and gravitational time dilation (they are really the same thing) is

$$z = \delta\lambda/\lambda = -\delta t/t = [\ 1- (2GM/rc^2)]^{-1/2} - 1 \ \approx GM/rc^2 \approx \Phi/c^2$$

or

$$z = \delta\lambda/\lambda = -\delta t/t = [\ 1- (R_S/r)]^{-1/2} - 1 \ \approx R_S/2r^2$$

Φ = **GM/r** is the Newtonian Gravitational Potential at **r**
The quantity $R_S = 2GM/c^2$ is the Schwarzschild Radius.

Note that this can give a <u>blueshift</u> if the observer is sitting in the deeper potential well.
One should really write: $z \approx \Delta\Phi/c^2$ **where** $\Delta\Phi = \Phi_{source} - \Phi_{observer}$

Object	Escape Velocity (km/s)	Radius (km)	Schwarzschild Radius (km)
Earth	11	6,738	0.00001 (1 cm)
Sun	620	700,000	2.95
White Dwarf	6,200	7,000	3
Neutron Star	280,000	7	6

Numbers: At the surface of the Earth $r \approx 7 \times 10^8 R_S$ so $z \approx 7 \times 10^{-10}$
A one second "tick" of a clock at the surface would appear to a distant free-floating observer to take 1.0000000007 seconds.