

# Completing the Copernican Revolution

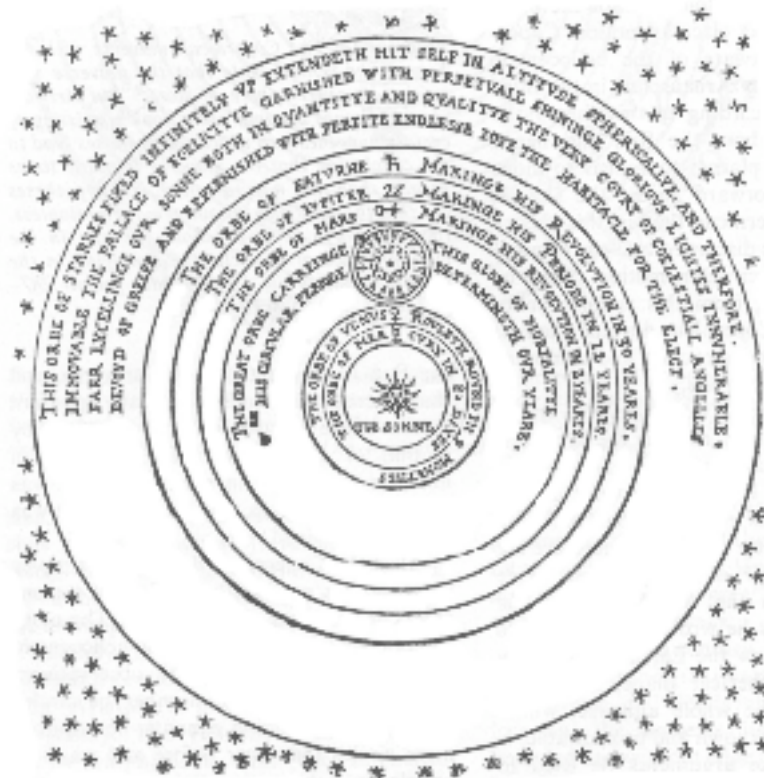
Thomas Digges (1546-1595)

*A Perfit Description of the Caelestiall Orbes...* (1576)

A Heliocentric System - but not a Heliocentric Universe

(cf. Democritus & Aristarchus)

Largely based upon Tycho's attempts to measure distances and parallaxes.



# Completing the Copernican Revolution

## Johannes Kepler (1571-1630)

*Astronomia Nova (1609) & Harmonice Mundi (1619)*

*also, The Optical Part of Astronomy (1604)*

Kepler and Tycho: Observations of Planetary Motions

Kepler's Determination of the Martian Orbit: Methodology

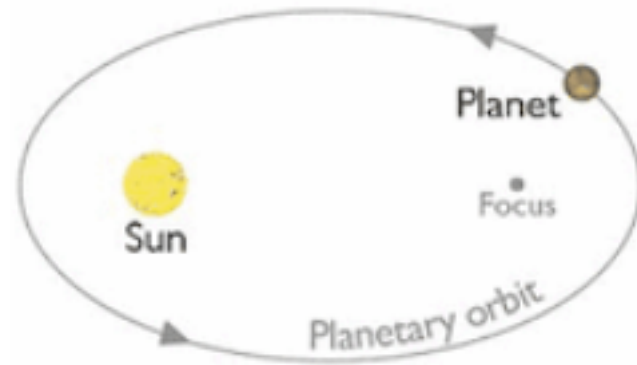
### Kepler's Empirical Laws of Planetary Motion

- I. The orbits of the planets are ellipses with the Sun at a focus of the ellipse.  
(Motions are neither circular nor combinations thereof. Note strictly heliocentric.)
- II. The Planet - Sun line sweeps out equal areas in equal intervals of time.  
("Law of Areas" - Motions are nonuniform.)
- III. The square of the orbital period is proportional to the cube of the length of the orbit's semi-major axis.  
("Harmonic Law" - The constant of proportionality is the same for all planets.)  
....again, a "not quite" heliocentric system.

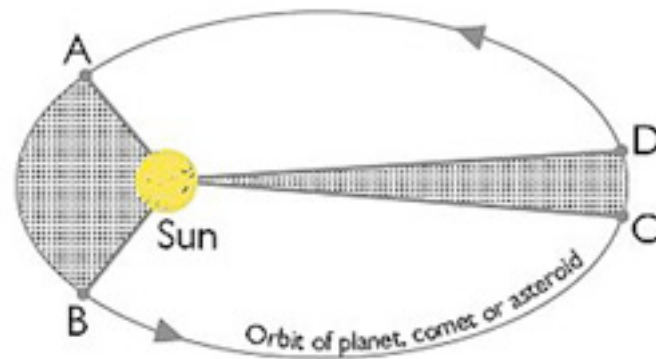
**Kepler's Laws are empirical laws providing a mathematical description of observed planetary motions.**

# Kepler's Laws of Planetary Motion

**Kepler I:**



**Kepler II:**



**Kepler III:**

$$P^2 = Ka^3 \quad \text{where } K \text{ is a constant.}$$

# Testing the Copernican Model (and the falsification of the Ptolemaic Model!)

## Galileo Galilei (1564 - 1642)

*Sidereus Nuncius* (1610)

*Dialogue Concerning the Two Chief World Systems* (1632)

### Aristotelian Physics Falsified

Observational and experimental verification of assumptions

### Experiments with falling objects

The acceleration of falling objects:  $v = gt$  independent of  $m$ !  
(also  $d = (v/2)t$  implying constant acceleration,  $g$ .)

### The Law of Inertia

The ideas of inertia, mass, and momentum

Abolishing the Prime Mover.

“Gravitational Mass” and “Inertial Mass”

**The Leaning Tower**



**The Cathedral of Pisa**



# Galileo's Astronomical Observations

**New Technology: The Telescope** (....and the microscope)

*Sidereus Nuncius* (1610) - “The Sidereal Messenger”

- **Lunar topography** (Mountains, valleys, craters, and “seas”)
- **The Milky Way - stars!** (*cf.* Democritus)
- **Sunspots** (and Solar rotation)
- **Planetary markings** (and rotation)
- **Saturn's rings** (a puzzlement!)
- **Spherical planets shining by reflected sunlight** (*cf.* the brightness problem)
- **The Galilean Satellites of Jupiter** (“Medicean Stars”) and **Jovicentric motion.**  
**and especially**

## **The gibbous phases of Venus!**

A clear falsification of the Ptolemaic Model - (but not of Tycho's Construction)

## **History: Galileo & The Inquisition**

Orders from the Inquisition (Decree of 1616)

The *Dialogue*.. (1632) and the Index of Proscribed Works

The Trial and Conviction of Galileo (1633)

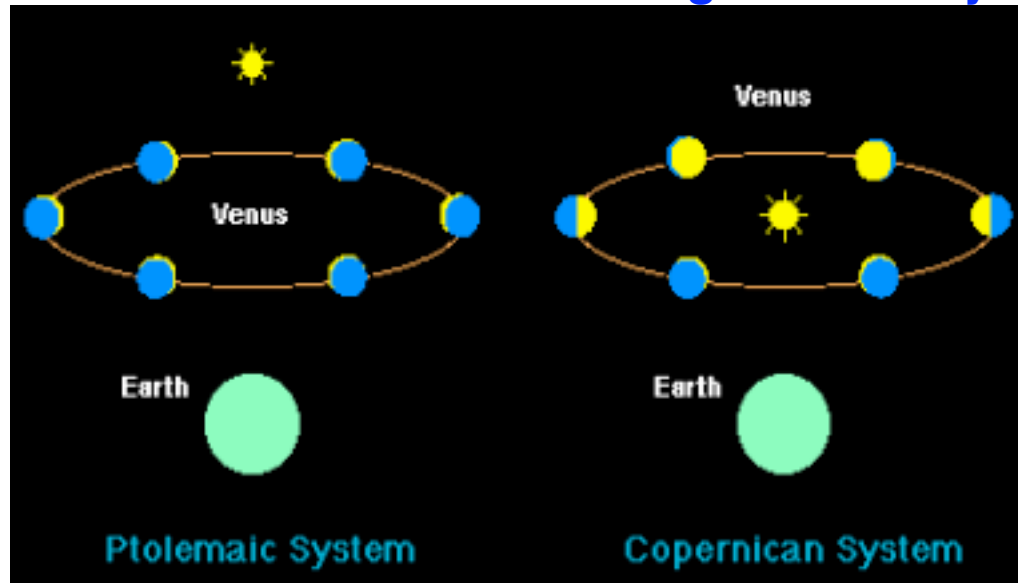
*“Epur si muove”*

Rehabilitation of the *Dialogue* and Galileo (1741, Benedict XIV)

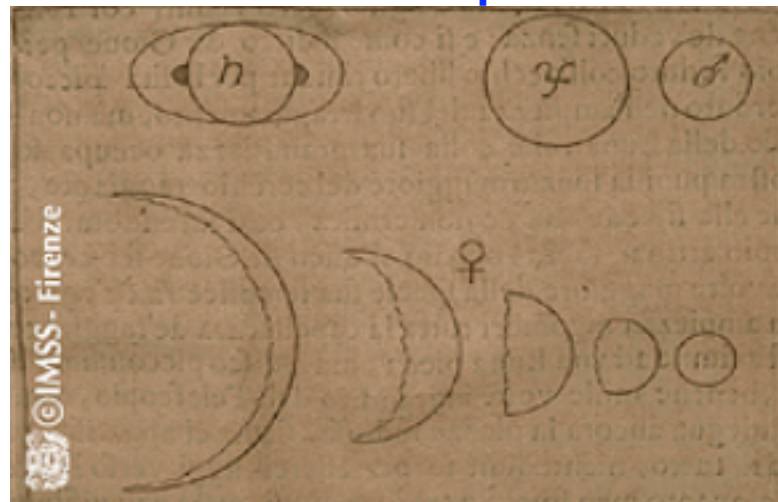
Heliocentrism officially accepted by the church (1748)

Galileo exonerated (1992, John Paul II)

## Predictions: The Phases of Venus according to Ptolemy and Copernicus



## Observations: Galileo's observations of planets - and the phases of Venus



# Celestial Mechanics: The Why of Planetary Motions

## Descriptions of How Celestial Objects Move

Aristotle, Hipparchus, and Ptolemy: **The Ptolemaic System**  
Aristarchus, Copernicus, and Kepler: **The Copernican System**  
(and the **Tychonic System** as a persistent variant of the latter)

## Digression

Reductionism and Natural Law: **Kepler's "Laws"**

## Explaining the "Why" of Planetary Motion

- **Newton's Mechanics**
- **Newton's Law of Gravity**
- **Newton's "Method of Fluxions"**  
(or Leibnitz' "Calculus")

## The Birth of Mathematical Physics

Newton showed that **Scientific Theories** (rational explanations of natural phenomena possessing predictive power) could be expressed in the universal language of mathematics. Indeed, the most fundamental "Laws of Nature" could be expressed in these terms.

**This might reasonably be considered the most "revolutionary" idea of post-Aristotelean science.**

# Sir Isaac Newton (1643-1727)

Newtonian Mechanics  
Newton's Law of Gravity  
Fluid Mechanics

Optics

The Calculus ("The Method of Fluxions")

(Also Economics & Finance)

## Newton's Laws of Mechanics

1. An object in any state of motion will remain in that state of motion unless acted upon by an external force. (Galileo's experiments)  
(If the net force is zero, the momentum  $mv$  is a constant.)
2. A mass,  $m$ , responds to a net force  $F$ , with an acceleration  $a$ , in the direction of the force according to  $F = ma$ . (An "operational" definition of force)  
(i.e., the rate of change of the momentum,  $mv$ , is equal to the applied force,  $F$ .)
3. For every action there is a reaction, equal in magnitude and opposite in direction. (Newton: An astute observation)  
(All forces occur as equal but oppositely directed pairs.)

.... these Laws provide a means of explaining or predicting the motion of any object of given mass in response to a known force. Alternatively, the properties of the force can be inferred from the observed motions of the mass.

# Newton's Law of Gravity

Newton deduced the Law of Gravity by using Newton's Laws of Mechanics and observations of the motions of objects moving under the influence of gravity. (i.e., the Moon, planets, and the apocryphal apple).

Two (point) objects of masses  $M$  and  $m$  experience a mutual force of attraction. The force is along the line joining the two masses, is proportional to the product of the masses, and is inversely proportional to the square of the distance,  $r$ , between them:

$$\mathbf{F} = [GMm/r^2]\hat{\mathbf{e}}_r$$

where  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$  is the Newtonian Gravitational Constant.

Newton's "Law of Gravity" together with Newton's Laws of Mechanics provided explanations for a variety of previously not-understood phenomena, including:

- The behavior of falling objects and trajectories of projectiles
- The near-sphericity of massive objects and the oblateness of rotating objects
- Orbital motions of the planets, their moons, and other astronomical objects
  - Tidal phenomena - on the Earth and elsewhere
  - Precession of the Earth - and of orbits
  - ..... and many others

# Kepler's Laws Explained, Corrected, and Generalized

Kepler's Laws of “planetary motion” actually hold in any two body gravitating system - almost.

**Kepler I:** Elliptical motions are a consequence of the inverse-square nature of the gravitational force. (cf. Coulomb's Law) Kepler's First Law is also generalized to include open (parabolic and hyperbolic) orbits as possibilities.

**Kepler II:** The Law of Areas follows from the central nature of the gravitational force. (Conservation of angular momentum holds for any torque-free or “central” force.)

**Kepler III:** Kepler's expression  $a^3 = KP^2$  is a good approximation for describing planetary motions about the Sun. However

$$a^3 = (G/4\pi^2)(M + m)P^2$$

is the more exact expression\* and, moreover, applies to any two-body system involving masses **M** and **m** moving under the influence of their mutual gravitational attraction.

**G is the Newtonian Constant of Gravitation.**

\*The change is from Kepler's  $K = 1$  to Newton's  $K = (M + m)$  if the units of **a**, **P**, and mass are astronomical units, years, and solar masses. In these “solar units” we have:

$$a^3 / P^2 = (M + m)$$

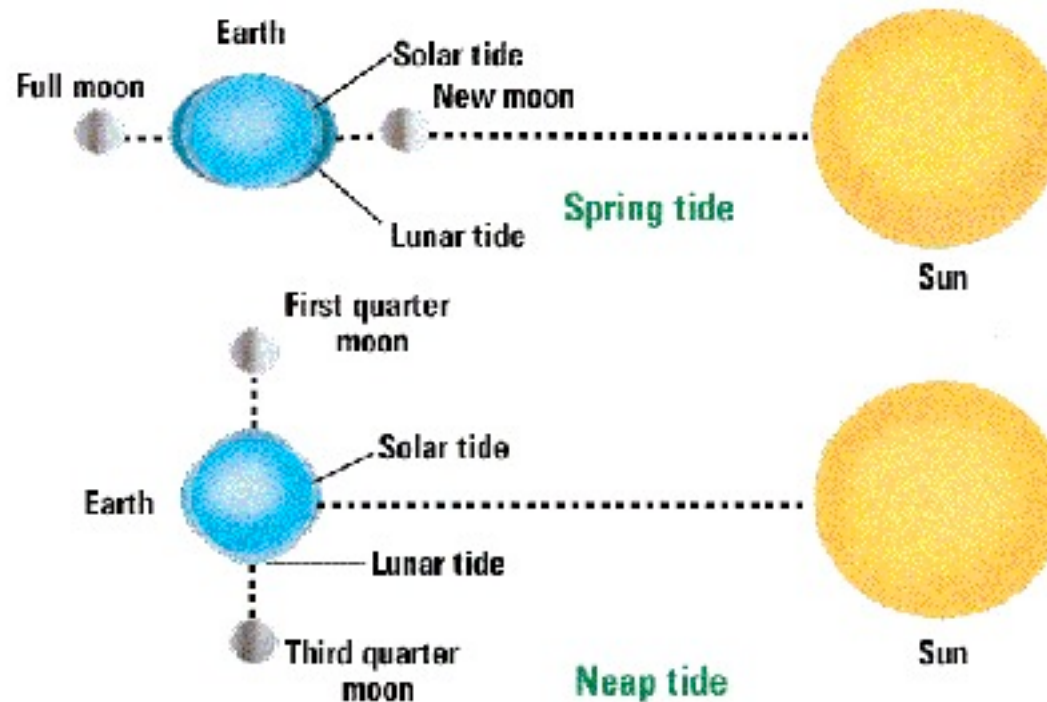
Note: If **a** and **P** can be determined then the mass of the system (**M+m**) is determined.

# Other Matters of Gravity

## Tides on the Earth and elsewhere

Tidal friction and orbital changes

Tidal couplings & Tidal Heating

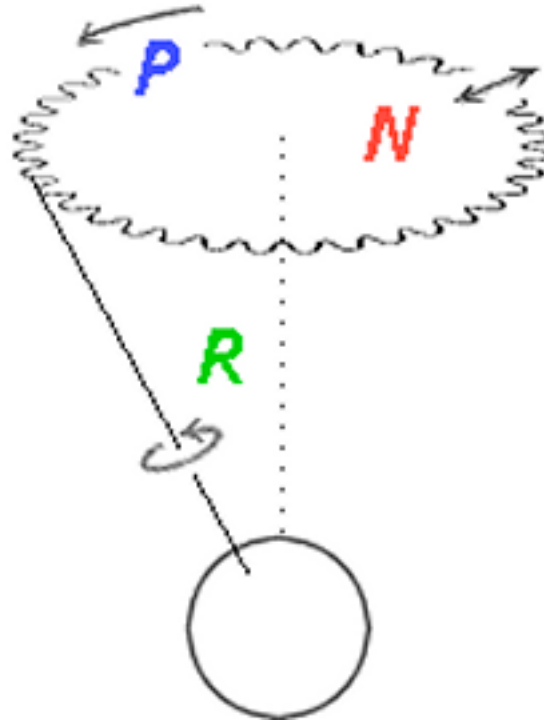


# Precession & Nutation of spinning bodies and their orbits

Precession of the Equinoxes:  $\theta = 23.5^\circ$  with  $P = 25,700$  years

Nutation of the Pole:  $\Delta\theta = \pm 9''$  with  $P = 18.61$  years

(Period of Moon's node: Retrograde  $P = 18.61$  years)



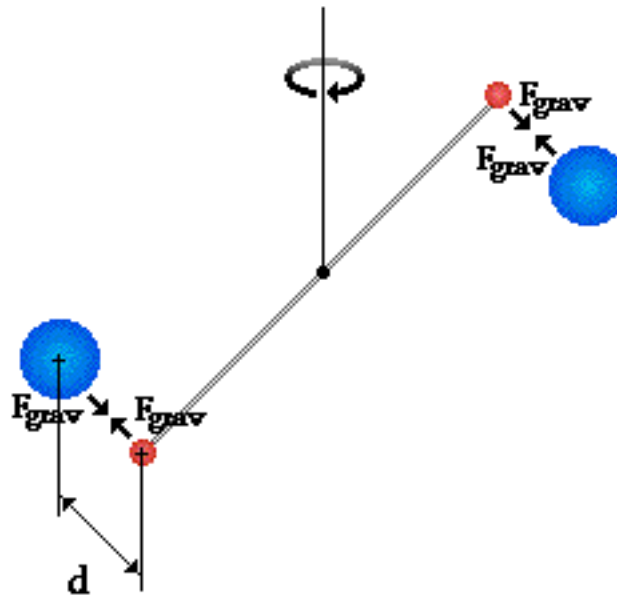
Figures of equilibrium for rotating bodies

e.g., the oblateness of the Earth

# Digressions

Inertial and Gravitational Mass: The Eötvös experiments.  
The Fundamental Forces of Nature and Gravity: Is Gravity “different”?  
Testing the Inverse-Square Law of Gravity  
Quantum gravity?

## Measurement of $G$ Cavendish (1798) Cavendish's Torsion Balance



Upcoming: General Relativity, Gravity, and Cosmology

## Digression: Planetary Orbits and Motions

- Planetary motion is adequately described by Kepler's Laws
- The motion of the planet in that orbit is specified by seven orbital elements:

Size and Shape:  $a$ ,  $e$

Orientation in space:  $i$ ,  $\omega$ ,  $\Omega$

Position in the orbit:  $P$ ,  $T$

where:

$a$  is the length of the semimajor axis of the orbital ellipse

$e$  is the eccentricity of the ellipse

$i$  is the inclination of the orbital plane to the ecliptic (the Earth's orbital plane)

$\omega$  is the argument of perhelion (locates perhelion point)

$\Omega$  is the longitude of the ascending node (locates intersection of plane with the Earth's)

$P$  is the orbital period

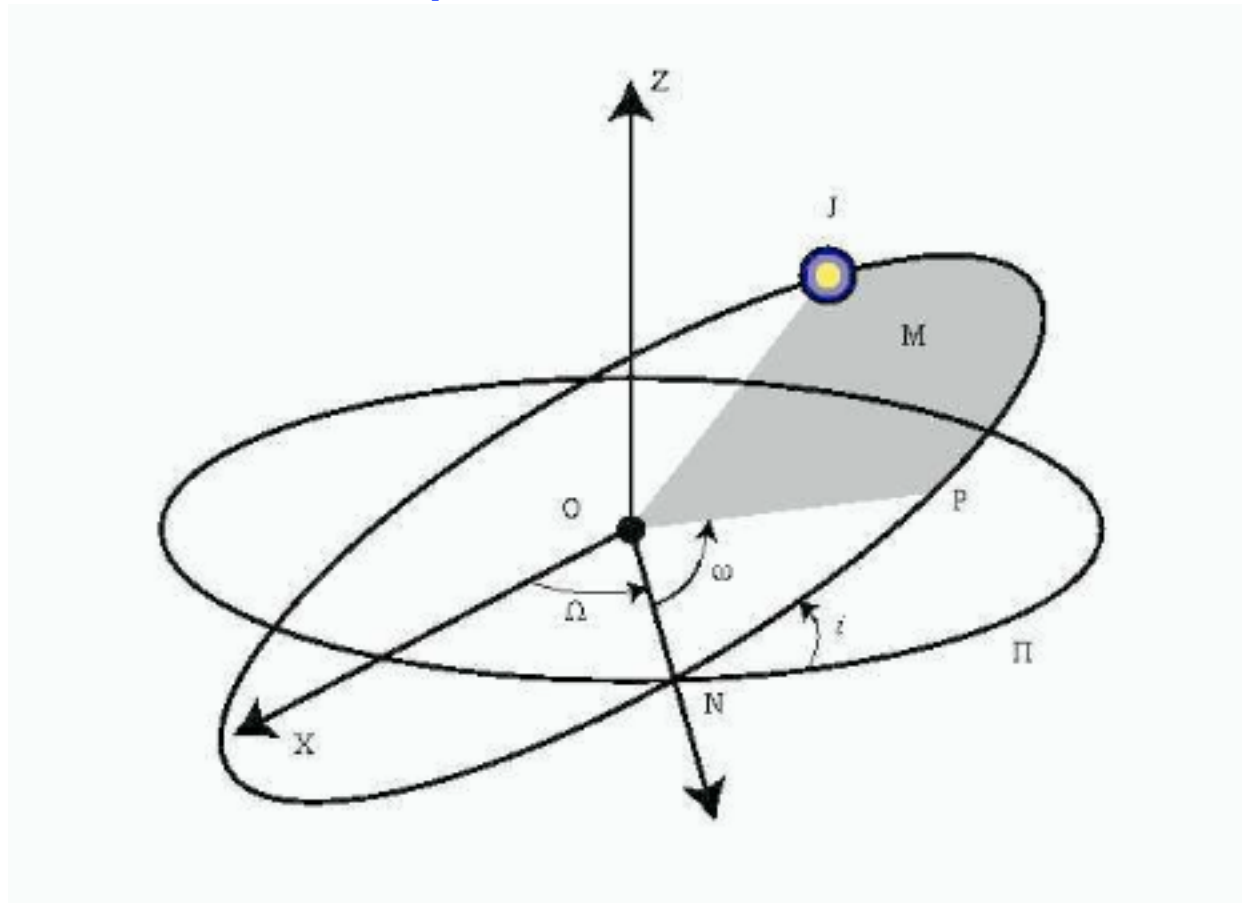
$T$  is a time of perhelion passage (*modulo*  $P$ )

Note that orientation angles are referred to the Earth's orbit

The location of a planet in its orbit at any time,  $t$ , is sometimes specified by its Mean Anomaly,  $M(t)$ , which can be obtained from the orbital elements using Kepler's law of Areas. See the figure below.

(Note: There are some alternative conventions for defining the orbital elements.)

## The Keplerian Orbital Elements



$\Pi$  is the Earth's orbital plane and  $X$  is the direction of the Vernal Equinox

$Z$  is the direction of the North Ecliptic Pole

$P$  is the perihelion point of the orbit

$O$  is the location of the Sun;  $J$  is the location of the planet

$ON$  is the line of nodes; the angle  $POJ$  is the Mean Anomaly