Nuclear Reactions

Nomenclature
Atomic Number (Z) → 6C\textsuperscript{12} ← Atomic Weight (A)

Chemical Symbol ↑ (C for Carbon)

Atomic Number = Number of Protons (+)
(Identifies the element: 6 → “Carbon”)

Atomic Weight = Number of Protons (+) plus Neutrons (0)
(Identifies the isotope: “Carbon-12”)

Nuclear Reaction Examples

Nuclear Fusion:
\[ ^{92}\text{U}\textsuperscript{235} + {}_0\text{n}\textsuperscript{1} \rightarrow ^{92}\text{U}\textsuperscript{23} \]
neutron ↑

Nuclear Fission:
\[ ^{92}\text{U}\textsuperscript{236} \rightarrow ^{54}\text{Xe}\textsuperscript{140} + ^{38}\text{Sr}\textsuperscript{94} + {}_0\text{n}\textsuperscript{1} + {}_0\text{n}\textsuperscript{1} + \gamma \]
gamma ray ↑

The Simplest Fusion Reaction:
\[ ^1\text{H}\textsuperscript{1} + ^1\text{H}\textsuperscript{1} \rightarrow ^1\text{H}\textsuperscript{2} + \text{e}^+ + \nu \]
“deuterium” ↑ ↑ positron

also Matter-Antimatter Annihilation:
\[ \text{e}^- + \text{e}^+ \rightarrow \gamma \]
electron ↑ ↑ positron
Thermonuclear Fusion in Stars

The Proton Proton Reactions

\[ _1^1H + _1^1H \rightarrow _1^2H + e^+ + \nu \]

followed by

\[ _1^1H + _1^2H \rightarrow _2^3He + \gamma \]

The positron \( e^+ \) just gets converted to a gamma ray \( \gamma \) by

\[ e^+ + e^- \rightarrow \gamma \]

The light isotope of helium \( _2^3He \) finds another of its kind

whereupon

\[ _2^3He + _2^3He \rightarrow _2^4He + _1^1H + _1^1H \]

and we get back two of the six hydrogen nuclei (protons) we started with.

The net result is

\[ _1^1H + _1^1H + _1^1H + _1^1H \rightarrow _2^4He + 4\gamma 's + 2\nu 's \]

So how much energy is released in this process?
Energy Production by Hydrogen Fusion

The basic process is

\[ {}_1\text{H} + {}_1\text{H} + {}_1\text{H} + {}_1\text{H} \rightarrow {}_2\text{He}^4 + 4\gamma' \text{s} + 2\nu' \text{s} \]

as four hydrogen nuclei “fuse” to make one helium nucleus.

In the process, some mass “disappears”

The mass of four hydrogen nuclei is \(6.694 \times 10^{-27} \text{ kg}\)
The mass of one helium nucleus is \(6.644 \times 10^{-27} \text{ kg}\)
The difference is only \(0.050 \times 10^{-27} \text{ kg}\)

.... or 0.007 of the mass of hydrogen we started with.

SO WHERE DOES IT GO?

This 0.007 or 0.7% of the mass is converted to energy.
This appears mostly in the form of gamma radiation, \(\gamma\)
(some produced by the annihilation of positrons \(e^+\))
A very small amount is effectively “wasted” in the neutrino \(\nu\)

But just how much energy is that?
Energy Yield in Hydrogen Fusion

When 1 kg of hydrogen is converted to 0.993 kg of helium
0.007 kg of mass is converted to energy.

\[ E = mc^2 \]

\[ E = (0.007 \text{ kg}) \times (3 \times 10^8 \text{ m/s})^2 = 6.3 \times 10^{14} \text{ joules} \]

Recollect that

\[ M_{\text{Sun}} = 2 \times 10^{30} \text{ kilograms} \]
\[ L_{\text{Sun}} = 3.8 \times 10^{26} \text{ watts} \]

If the Sun was all hydrogen which was all converted to helium the energy released would be

\[ 0.007 \times M_{\text{Sun}} c^2 = 1.26 \times 10^{45} \text{ joules} \]

The Sun could then shine at its present luminosity for

\[ 110 \times 10^9 \text{ years} \]

But the Sun isn’t all hydrogen and not all of the Sun’s hydrogen can be fused to make helium........
The Lifetime of the Sun

Why the “Thermonuclear” in “Thermonuclear Fusion”? 

High temperatures are required to overcome the electrostatic repulsion between (positively charged) nuclei. (An element’s atomic number, Z, equals its nuclear charge.)

Temperatures in excess of $10,000,000 \ °K$ are usually needed.

These temperatures are attained only in the deep interior.

So not all of the Sun’s “fuel” is the “kitchen”

A more careful calculation reduces the Sun’s lifetime from the above crude estimate by about a factor of ten to

$$\tau_{\text{Sun}} = 11 \times 10^9 \ \text{years}$$

But there was, historically, another possibility considered ....
The CNO Reactions

The first reaction sequence proposed to power the Sun was the six-step “catalytic” Carbon-Nitrogen-Oxygen (CNO) cycle

1. \( ^1H + ^6C \rightarrow ^7N + \gamma \)
2. \( ^7N \rightarrow ^6C + e^+ + \nu \)
3. \( ^6C + ^1H \rightarrow ^7N + \gamma \)
4. \( ^7N + ^1H \rightarrow ^8O + \gamma \)
5. \( ^8O \rightarrow ^7N + e^+ + \nu \)
6. \( ^7N + ^1H \rightarrow ^6C + 2^{He}_4 \)

Again, the positrons get annihilated via \( e^+ + e^- \rightarrow \gamma \)

The Net Result is

\( ^1H + ^1H + ^1H + ^1H \rightarrow 2^{He}_4 + 5\gamma's + 2\nu's \)

which is almost identical to the Proton-Proton result:

\( ^1H + ^1H + ^1H + ^1H \rightarrow 2^{He}_4 + 4\gamma's + 2\nu's \)

The energy yield is the same!

......but there was a problem with this “catalytic” process......
The CNO Problem

Interior temperatures of the Sun don’t get high enough to overcome the electrical repulsion between protons $^1H$ and the nuclei of $^6C$ and $^7N$ (in steps 1, 3, 4, and 6)

Note: The electric force between charges $Q$ and $q$ separated by distance $r$ is

$$ F_{\text{electric}} = \frac{+Qq}{r^2} $$

But

The net result and energy yield is the same as the proton-proton reactions:

$$ 4 \times ^1H^1 \rightarrow ^2He^4 + \text{energy} $$

And

This happens to be the process which powers more massive ($>1.5$ MSun) main sequence stars (which do get hot enough!)
Main Sequence Stars are powered by thermonuclear fusion

- Hydrogen is fused to helium.
  Basically: \( _1^1\text{H} + _1^1\text{H} + _1^1\text{H} + _1^1\text{H} \rightarrow _2^4\text{He} \)
- Approximately 0.7% of the mass “disappears”
  1.000 kg of Hydrogen makes 0.993 kg of Helium and yields \( 6.3 \times 10^{14} \) joules of energy \((E = mc^2)\)
- There are two routes for this process
  The Proton-Proton Reaction dominates below 1.5 \( M_{\text{Sun}} \)
  The CNO reaction dominates above 1.5 \( M_{\text{Sun}} \)
  (They contribute equally to the energy at 1.5 \( M_{\text{Sun}} \))

This “Hydrogen Burning” continues until fuel exhaustion
....when all core hydrogen has been converted to helium

With hydrogen exhaustion the main sequence stage ends as the star begins to die ............