The Masses of Stars

Masses of stars can sometimes be determined from the orbits of the bodies about them.

Recollect Kepler's Third Law as revised by Newton:

$$M + m = (4\pi^2/G)\, a^3/P^2$$

where

M and m are the masses of the two bodies
a is the semimajor axis of the orbital ellipse
P is the orbital period

In "solar units" this is

$$M + m = a^3/P^2$$

The Sun's Mass

Using the Earth's orbit about the Sun gives the Sun's mass:

$M_{\text{Sun}} = 1.99 \times 10^{30}$ kilograms (kg)

(about 333,000 times the mass of the Earth)

Stellar Masses & Binary Stars

In "solar units" we have

$$M + m = a^3/P^2$$

where M and m are measured in units of the Sun's mass, the semimajor axis a is measured in astronomical units, and the orbital period P is measured in years

Now:

Roughly half of all stars are found in binary star systems

or (same statement)

About a third of all star systems are binary systems

If we can measure a and P for a binary system we can compute the total mass $M + m$ of the system.

...and how do we do that?

Binary Stars

A **Binary Star System** consists of two stars in orbit about one another.

Binaries are classified on the basis of what can be observed:
This depends on the brightnesses of the stars, the orbital parameters, the viewing angle, the system’s distance, and the available observing technology.
e.g., Visual Binaries, Astrometric Binaries, Single- and Double Lined Spectroscopic Binaries, Spectrum Binaries, Eclipsing Binaries, ...
A system may fit into more than one class - and its “type” may change with improvements in technology or better observations.

Note that there are also triple- and quadruple- star systems. Some consist of a binary pair in orbit about another star - or another binary pair. These systems are much rarer than binaries or single star systems.

Types of Binary Stars

Visual Binary

A resolved pair of stars seen to be in orbit about each other.

Astrometric Binary

A single star observed to be in orbit about an "invisible" companion.

Spectroscopic Binary

A system in which the orbital motion is seen in the periodic shifting of the stellar spectral lines. These include:

Double-Lined Systems: Spectral lines of both stars are seen
Single-Lined Systems: Spectral lines of only one star is seen

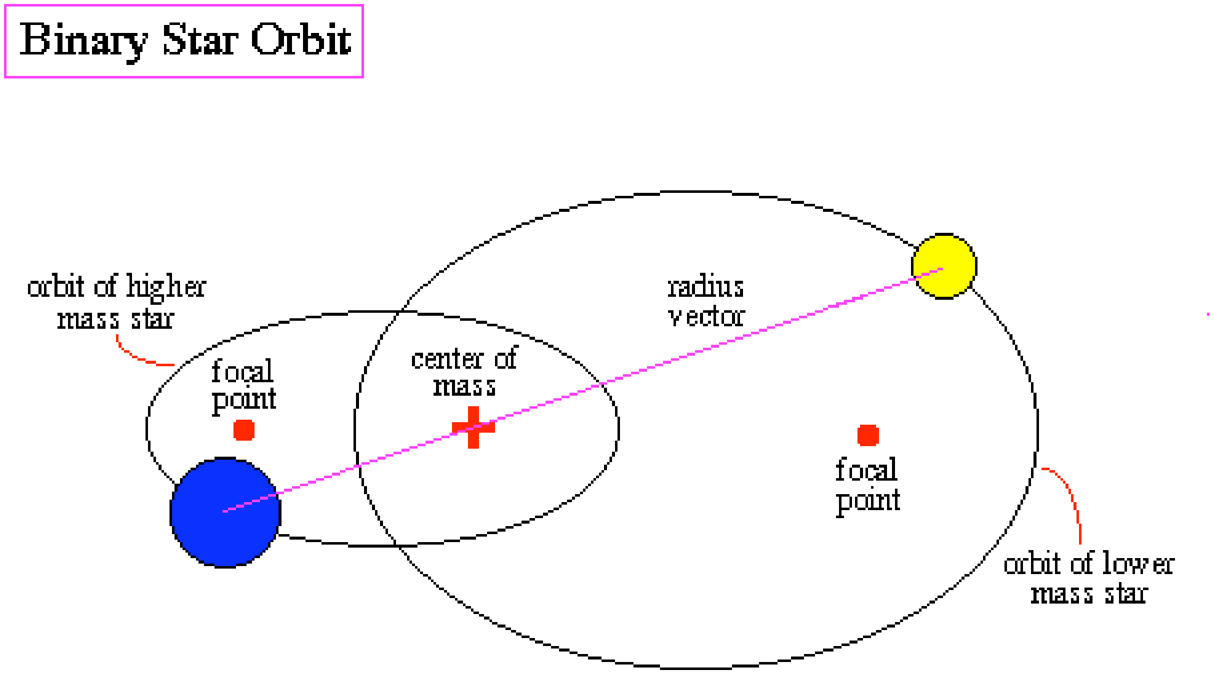
Special Cases

Eclipsing Binaries: Edge-on systems
..... producing periodic eclipses

Spectrum Binaries: Face-on spectroscopic binaries
..... giving "composite" spectra & colors

Visual Binaries

- **Two stars are observed to orbit about a fixed point between them with an orbital period P.** (Example: 6 yr)
- **This fixed point is called the center of mass or barycenter.**



- **The ratio of the apparent distances of the two stars from the system barycenter is always equal to the inverse mass ratio:**

$$M_{StarA}/M_{StarB} = r_{StarB}/r_{Star\ A}$$ (Example 2:1)

Visual Binaries

- The apparent (angular) distance $r_{StarB} + r_{Star\ A}$ between the two stars depends upon:
  - Where the stars are in their orbital motion
  - The actual size (a) and shape (ε) of the orbit

    and
  - The inclination angle (i) at which the orbit is viewed

    (....by using Kepler's First Law)
  - The distance to the system (d)

- If the last two quantities can be determined the orbital size, a, can be determined. (Example: 6 au)

- With the orbital period P known, we know the total mass:

  $M_{StarA} + M_{StarB} = a^3/P^2$ (Example: 6 M_{sun})

- But we also know the mass ratio:

  $M_{StarA}/M_{StarB} = r_{StarB}/r_{Star\ A}$ (Example 2:1)

- So we can figure out the individual masses (4 M_{sun} and 2 M_{sun})

Spectroscopic Binaries

Assume the orbits are circular and viewed edge-on.
(Note that this is almost never the case! see below.)

The measured radial velocity of Star A is observed to vary between $+v_A$ and $-v_A$ with period P. Its orbital speed is therefore v_A , its orbit circumference is v_AP, and its orbital radius is $r_A = v_AP/2\pi$. We know r_A

Similarly for Star B: $r_B = v_BP/2\pi$. We know r_B
The separation of the stars is $a = r_B + r_B$. We know a
The total mass is $M_A + M_B = a^3/P^2$ We know a and P
The mass ratio is $M_A/M_B = r_B/r_A = v_B/v_A$ We know M_A/M_B

So we can calculate the individual masses M_A and M_B

Alas, unless the inclination angle can be determined only a lower limit on the masses can be obtained. Fortunately, a few of these systems are also visual or eclipsing binaries.

Summary: Stellar Masses

Masses can be determined for the stars in visual binary systems if the distance to the system can be measured.

Masses can be determined for the members of double-lined spectroscopic binary systems if the inclination of the system can be established.

Masses cannot generally be determined for other types of binary star system, where only one star is detected directly.

Results

For reference: M_{Sun} = 1.99 x 10^{30} kilograms (kg)

- **Observed masses lie in the range ≈ 0.1 M_{Sun} to ≈ 30 M_{Sun}.**

- **The more massive stars are generally the most luminous.**

For main sequence stars stars, approximately

$$L_{Star}/L_{Sun} \approx [M_{Star}/M_{Sun}]^{3.5}$$

Note that white dwarfs, giants, and supergiants do not obey this main sequence "Mass-Luminosity Relation".

Summary: Physical Properties of Stars

Photospheric ("Surface") Temperatures

3,000°K (Red M stars) **to 40,000°K** (Blue-White O stars)

Luminosities

Main Sequence: $L_{M5V} \approx 10^{-2} L_{Sun}$ **to** $L_{O5V} \approx 10^{+6} L_{Sun}$

Also red supergiants at $10^{+5} L_{Sun}$ to white dwarfs at $10^{-4} L_{Sun}$.

Stellar Radii

Main Sequence: $R \approx 0.3$ to $18\ R_{Sun}$ (M5V to O5V)

Giants & Supergiants: $R \approx 1{,}000$ to $12\ R_{Sun}$ (M5V to O5V)

White Dwarfs, Neutron Stars: $R_{wd} \approx 0.01\ R_{Sun}$ & $R_{ns} \approx 10^{-5}\ R_{Sun}$

Stellar Masses

$M \approx 0.1\ M_{Sun}$ **to** $M \approx 30\ M_{Sun}$

Also white dwarfs at 0.6 to 1.2 M_{Sun}

Added Information: Photospheric Compositions