Celestial Mechanics: The Why of Planetary Motions

Attempts to Describe How Celestial Objects Move

Aristotle, Hipparchus, and Ptolemy: The Ptolemaic System
Aristarchus, Copernicus, and Kepler: The Copernican System
(and the **Tychonic System** as a persistent variant of the latter)

Summary: The Universe in 1650

- **The Ptolemaic Cosmology is Disproved (Galileo)**
- **A Heliocentric Solar System (Copernicus)**
- **A non-Heliocentric Universe (Digges)**
- **Planetary Motions Accurately Described (Kepler)**

Explaining why Planets Move the way they do

The Classical View: “Crystalline Spheres” and “Prime Movers”
Galileo & Kepler: Speculations about “planetary magnetism”

Digression:

The Scientific Method, Reductionism, and the Laws of Nature

Sir Isaac Newton (1643-1727)

Newtonian Mechanics (and Mathematical Physics)
Newton's Law of Gravity (and the Laws of Nature)
Fluid Mechanics (fluid flows, viscosity, turbulence, ...)
Optics (lenses, prisms, mirrors; composition of light ....)
The Calculus ("The Method of Fluxions". Liebnitz.))
Also Economics & Finance(Chancellor of the Exchequer), **Theology**

Newton's Laws of Mechanics

1. An object in any state of motion will remain in that state of motion unless acted upon by an external force. (Galileo's experiments)
(If the net force is zero, the momentum mv is a constant.)

2. A mass, m, responds to a net force F, with an acceleration a , in the direction of the force according to F = ma. (An "operational" definition of force)
(*i.e.*, the rate of change of the momentum, mv, is equal to the applied force, F.)

3. For every action there is a reaction, equal in magnitude and opposite in direction. (Newton: An astute observation)
(All forces occur as equal but oppositely directed pairs.)

.... these Laws provide a means of explaining or predicting the motion of any object of given mass in response to a known force. Alternatively, the properties of the force can be inferred from the observed motions of the mass.

Newton's "Law of Gravity"

Motivations: Observations of falling objects ("The Apple") and the motions of the Moon and Planets. Application of Laws of Mechanics to Astronomy.

GRAVITY DESCRIBED

Two (point) masses M and m experience a mutual force of attraction.
The force is along the line joining the two masses
The force is proportional to the product of the masses (*i.e,* to Mxm)
The force is inversely proportional to the square of the distance, r, between the masses (*i.e,* to $1/r^2$)

****

Mathematically:

$$\mathbf{F} = -[GMm/r^2]\,\hat{\mathbf{e}}_r$$

where G = 6.67 x 10^{-11} N m^2 kg^{-2} is the Newtonian Gravitational Constant

Digression: Mass and Weight

At the surface of the Earth the weight, W, of a mass, m, is:

$$\mathbf{W} = [GM_{Earth}/R^2_{Earth}]\,m = g_{Earth}m$$

where g_{Earth} is called "the acceleration of gravity"

Also: Units of mass and weight (a force)

Kepler's Laws Explained and Generalized

Newton: Kepler's Laws hold in any two body gravitating system - almost.

Kepler I: Elliptical motions are a consequence of the inverse-square nature of the gravitational force. Also, Kepler's First Law is generalized to include open (parabolic and hyperbolic) orbits as possibilities. (*cf.* Coulomb's Law).

Kepler II: The Law of Areas follows from the central nature of the gravitational force. (Conservation of angular momentum holds for any torque-free or "central" force.)

Kepler III: Kepler's expression $\mathbf{a^3 = KP^2}$ is a good approximation for describing planetary motions about the Sun. However,

$$\mathbf{a^3 = (G/4\pi^2)(M + m)P^2}$$

is the more exact expression* and, moreover, applies to **any** two-body with masses **M** and **m** moving under the influence of their mutual gravitational attraction.

G is the Newtonian Constant of gravitation.

Notes

* If the units of **a** (length) , **P**(time) , and **M** or **m** (mass) are astronomical units, years, and solar masses, respectively, then $(G/4\pi^2) = 1$ and

$$\mathbf{a^3/ P^2 = (M + m)}$$

Note that in the Solar System $M_{sun} = 1$ and $m_{planets} \leq 0.0007$ (Jupiter)

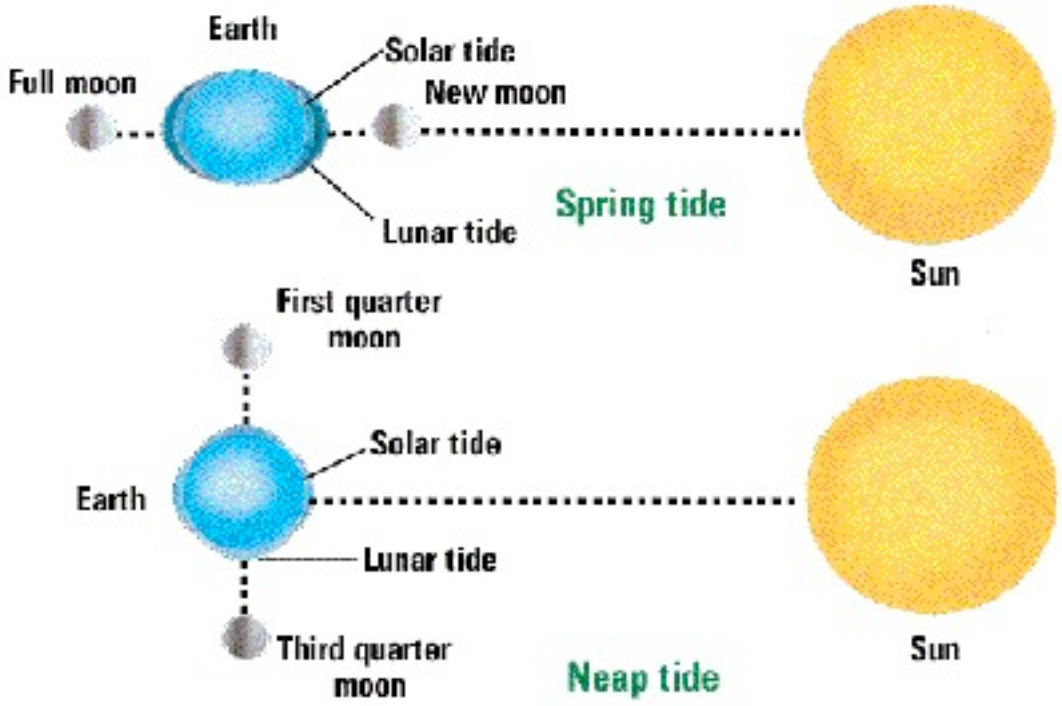
• If **a** and **P** can be determined then the mass of the system **(M+m)** is determined.
(The principal method for determining masses for astronomical objects)

Other Matters of Gravity

Tides on the Earth and elsewhere

Tidal friction and orbital changes

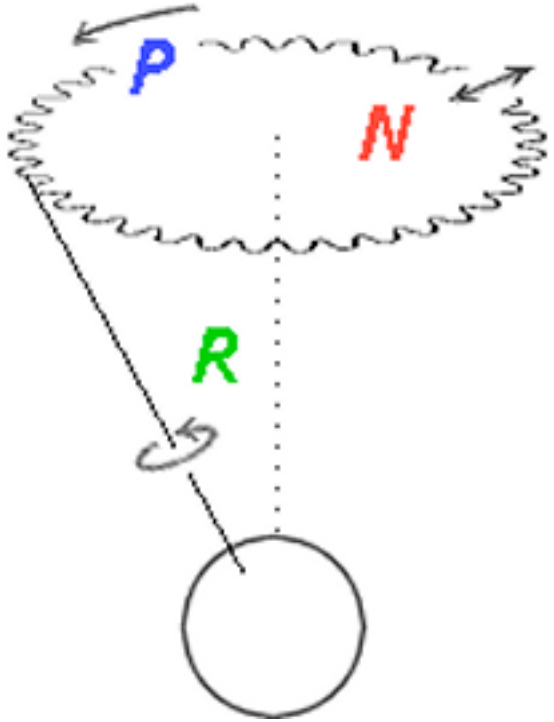
Tidal couplings & Tidal Heating



Also: Figures of equilibrium for rotating bodies

Precession & Nutation of spinning bodies and their orbits

Precession of the Equinoxes: $\theta = 23.5°$ with P = 25, 700 years
Nutation of the Pole: $\Delta\theta = \pm 9''$ with P = 18.61 years
(Period of Moon's node: Retrograde P = 18.61 years)



Digressions

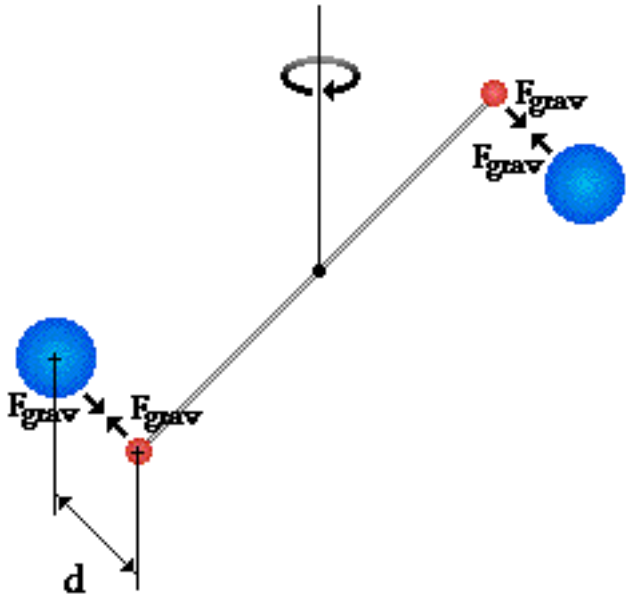
Inertial and Gravitational Mass: The Eötvös experiments.
The Fundamental Forces of Nature and Gravity: Is Gravity "different"?
Testing the Inverse-Square Law of Gravity

Measurement of G

Digression: Physical Constants of Nature (G, c, h, k; e, m_e, ...)

Cavendish (1798)

Cavendish's Torsion Balance



Upcoming: General Relativity, Gravity, and Cosmology