The forward and inverse problems in local helioseismology

Jason Jackiewicz[1], Laurent Gizon[1], Aaron Birch[2]

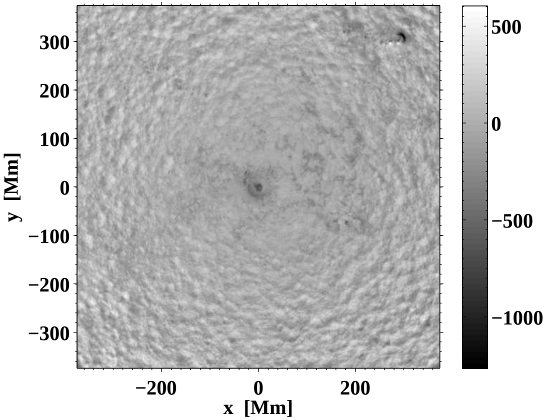
[1]Max Planck Institute for Solar System Research
Katlenburg-Lindau, Germany
[2]NWRA CoRA, Boulder, Colorado, USA

23 August 2007

Outline

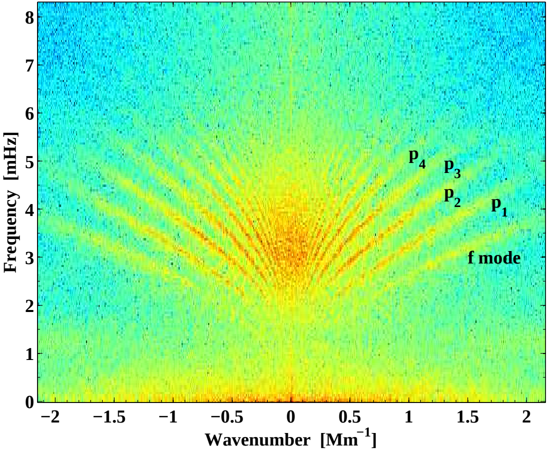
1. Helioseismic data and measurements
2. The forward problem
3. 2+1 dimensional OLA inversion
4. Inversion results for flows
5. Conclusions

Doppler velocity map

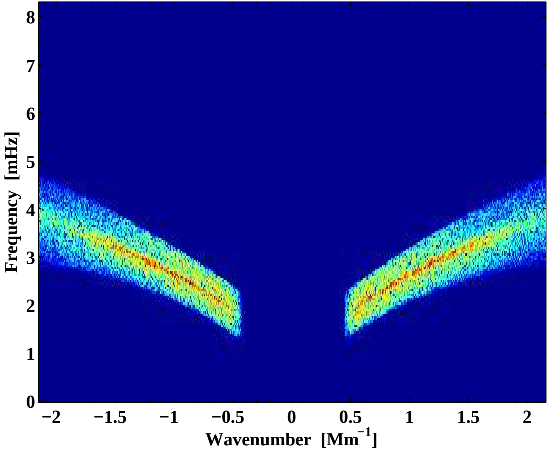


- line-of-sight velocity
- 6 hour-average of SOHO/MDI data
- pixel size 0.12° (∼ 1.46 Mm)
- AR 9787

Power spectrum of solar oscillations

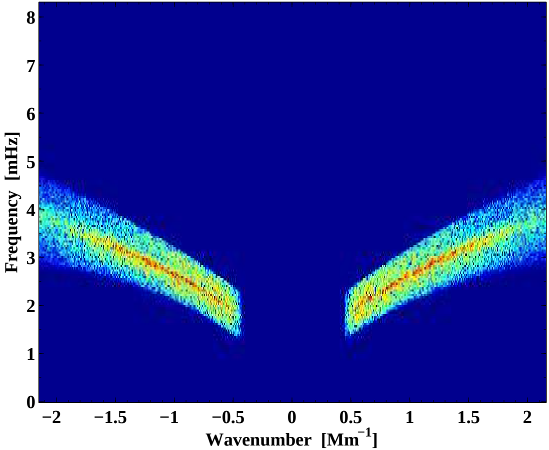


Filtered f-mode power spectrum



- we filter each individual ridge (not phase-speed filtering)

Filtered f-mode power spectrum



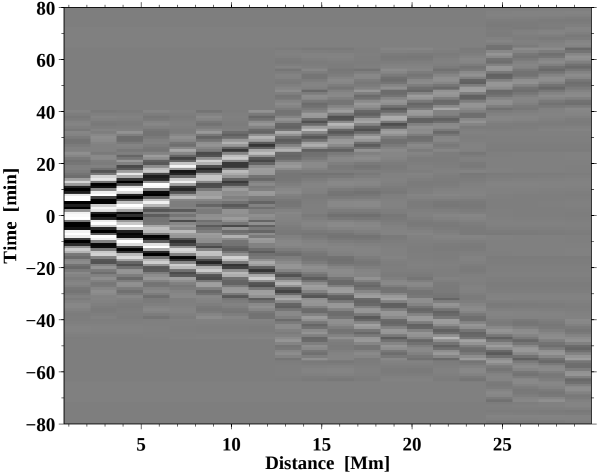
- we filter each individual ridge (not phase-speed filtering)
- we do this for f, p_1, p_2, p_3, and p_4

Cross-covariance function

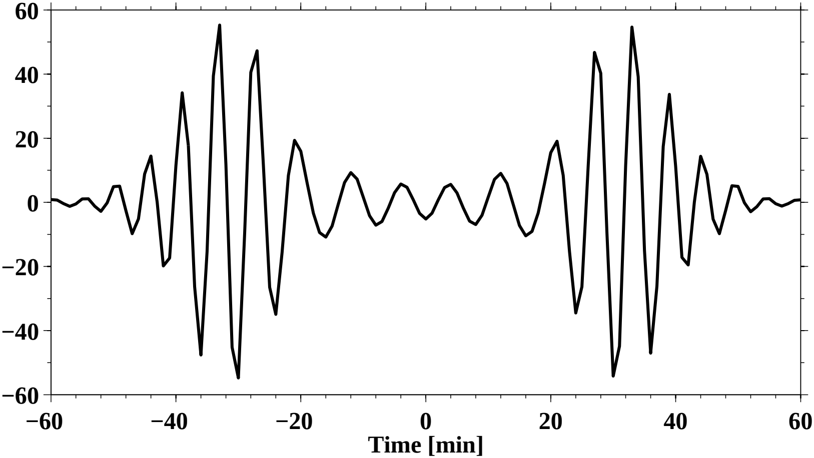
$$C(\boldsymbol{x_1}, \boldsymbol{x_2}, t) \sim \sum_{t'} \psi(\boldsymbol{x_1}, t')\psi(\boldsymbol{x_2}, t' + t)$$

- $\psi(\boldsymbol{x}, t)$ is the filtered line-of-sight velocity at point $\boldsymbol{x}$ on the solar surface
- $t > 0$ gives information about waves going from $\boldsymbol{x_1}$ to $\boldsymbol{x_2}$
- $t < 0$ gives information about waves going from $\boldsymbol{x_2}$ to $\boldsymbol{x_1}$
- in practice we average the cross covariance over points of an annulus of radius Δ: $C_\alpha(\boldsymbol{x}, t; \Delta)$

F-mode average cross covariance 'oi'



F-mode average cross covariance $\Delta = 14.6$ Mm

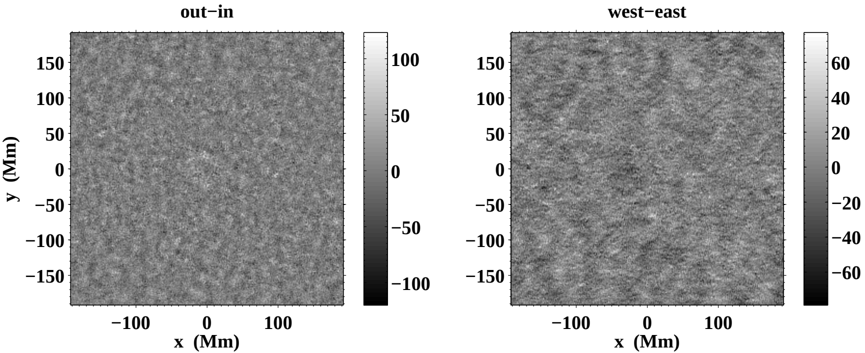


Travel-time measurements

$$\tau_\alpha(\boldsymbol{x};\Delta) = \sum_t W_\alpha(\Delta,t)\left[C(\boldsymbol{x},t;\Delta) - C^0(\Delta,t)\right]$$

- C^0 is the background unperturbed cross covariance (model)
- travel-time differences are sensitive to flows, and we use 3 type of measurements (point-to-annulus): out-in, west-east, north-south
- we measure $\tau_\alpha(\Delta;\boldsymbol{x})$ for $\Delta = 1.5 - 30$ Mm

Travel-time measurements, 6 hrs, $\Delta = 8.7$ Mm



Linear sensitivity of travel times to flows

- Relate the measurements (travel-time differences) to the perturbation we wish to study (flows):

$$\tau(\boldsymbol{x_1}, \boldsymbol{x_2}) = \iiint \mathrm{d}^3 r \, \mathbf{K}(\boldsymbol{x_1}, \boldsymbol{x_2}; \boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r})$$

Linear sensitivity of travel times to flows

- Relate the measurements (travel-time differences) to the perturbation we wish to study (flows):

$$\tau(\boldsymbol{x_1}, \boldsymbol{x_2}) = \iiint \mathrm{d}^3 r \, \mathbf{K}(\boldsymbol{x_1}, \boldsymbol{x_2}; \boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r})$$

- The kernel **K** is computed from a background solar model. Perturbations to the model due to flows are treated in the first Born approximation.

Linear sensitivity of travel times to flows

- Relate the measurements (travel-time differences) to the perturbation we wish to study (flows):

$$\tau(\boldsymbol{x_1}, \boldsymbol{x_2}) = \iiint \mathrm{d}^3 r \; \mathbf{K}(\boldsymbol{x_1}, \boldsymbol{x_2}; \boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r})$$

- The kernel **K** is computed from a background solar model. Perturbations to the model due to flows are treated in the first Born approximation.
- The input power spectrum of the model should match as closely as possible the observed power spectrum

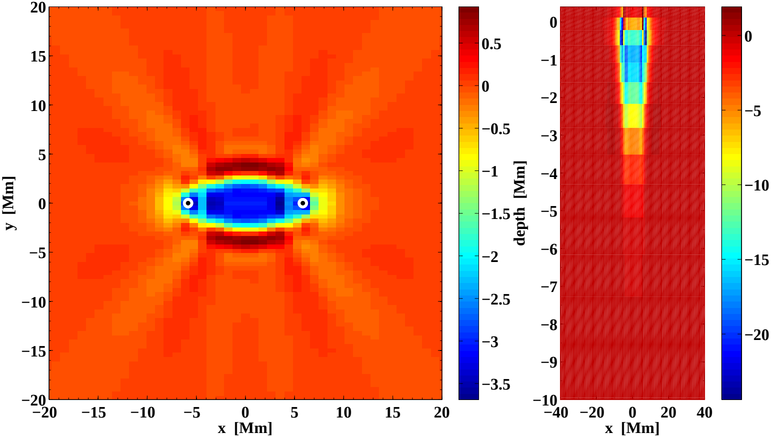
Linear sensitivity of travel times to flows

- Relate the measurements (travel-time differences) to the perturbation we wish to study (flows):

$$\tau(\boldsymbol{x_1}, \boldsymbol{x_2}) = \iiint \mathrm{d}^3 r \; \mathbf{K}(\boldsymbol{x_1}, \boldsymbol{x_2}; \boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r})$$

- The kernel **K** is computed from a background solar model. Perturbations to the model due to flows are treated in the first Born approximation.
- The input power spectrum of the model should match as closely as possible the observed power spectrum
- The kernels are computed for each travel-time measurement type (all available ridges, geometries, and distances)

An example f-mode kernel (K_x), $\Delta = 11.6$ Mm



- see poster PH.36 (Birch and Gizon) for more

The problem to solve

$$\tau(\boldsymbol{x_1}, \boldsymbol{x_2}) = \iiint \mathrm{d}^3 r \; \mathbf{K}(\boldsymbol{x_1}, \boldsymbol{x_2}; \boldsymbol{r}) \cdot \boldsymbol{u}(\boldsymbol{r}) + \text{noise}$$

- Given τ and $\mathbf{K}$ and noise, find $\boldsymbol{u}$.

Strategy of a 2D OLA inversion

1. Search for a linear combination of the sensitivity kernels so that the result (averaging kernel) resembles some localized 'target function'. The coefficients (weights) of the linear combination are w.

Strategy of a 2D OLA inversion

1. Search for a linear combination of the sensitivity kernels so that the result (averaging kernel) resembles some localized 'target function'. The coefficients (weights) of the linear combination are w.
2. To invert for the x component of the flow:

$$\begin{aligned}
\mathcal{K}_x &= \sum_\alpha w^\alpha \mathsf{K}_x^\alpha = \text{skinny gaussian} \\
\mathcal{K}_y &= \sum_\alpha w^\alpha \mathsf{K}_y^\alpha = 0 \\
\mathcal{K}_z &= \sum_\alpha w^\alpha \mathsf{K}_z^\alpha = 0
\end{aligned}$$

Strategy of a 2D OLA inversion

1. Search for a linear combination of the sensitivity kernels so that the result (averaging kernel) resembles some localized 'target function'. The coefficients (weights) of the linear combination are w.
2. To invert for the x component of the flow:

$$\begin{aligned}\mathcal{K}_x &= \sum_\alpha w^\alpha \mathsf{K}_x^\alpha = \text{skinny gaussian} \\ \mathcal{K}_y &= \sum_\alpha w^\alpha \mathsf{K}_y^\alpha = 0 \\ \mathcal{K}_z &= \sum_\alpha w^\alpha \mathsf{K}_z^\alpha = 0\end{aligned}$$

3. If successful, the weights then average the travel times to give an estimate of the local flow:

$$\bar{u}_x^i(\boldsymbol{x}) = \sum_\alpha w_i^\alpha \tau_i^\alpha$$

1D OLA depth inversion

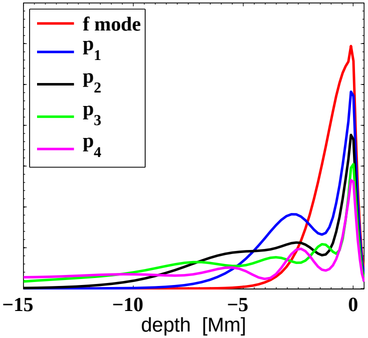
- Try to match some target function at a particular depth z_0 with the individual 1D ridge kernels

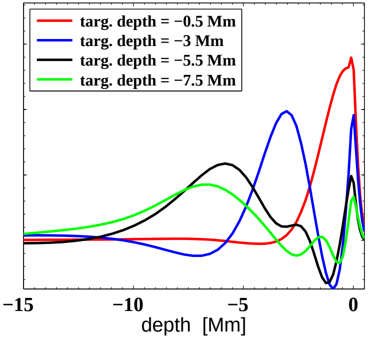
1D OLA depth inversion

- Try to match some target function at a particular depth z_0 with the individual 1D ridge kernels
- Use these new weights to combine the 2D maps to give an estimate of the flow around z_0:

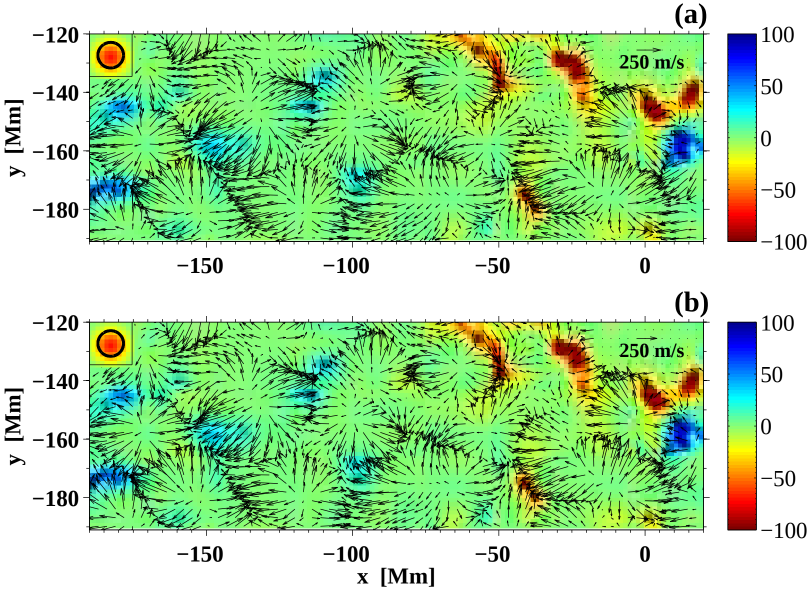
$$\tilde{u}_x(\boldsymbol{x}, z_0) = \sum_i c_i(z_0)\bar{u}_x^i(\boldsymbol{x})$$

1D kernels and averaging kernels

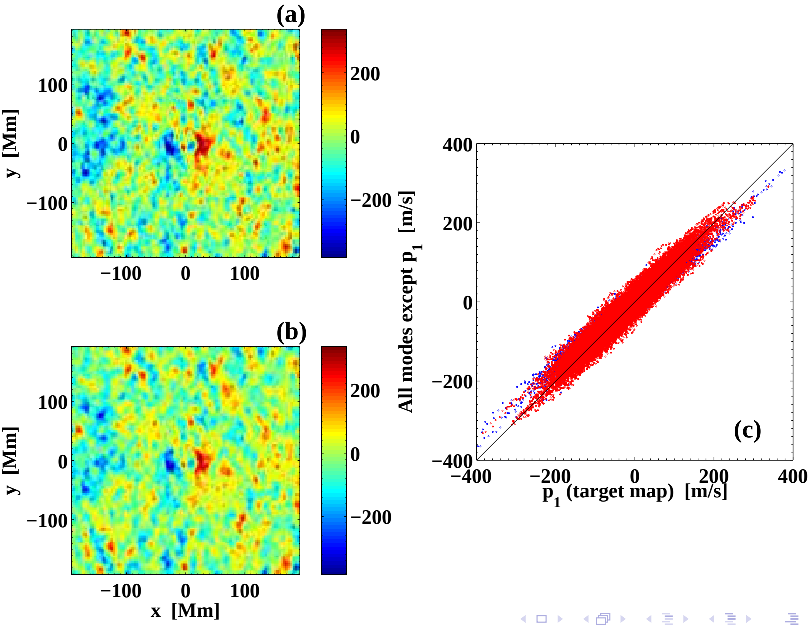




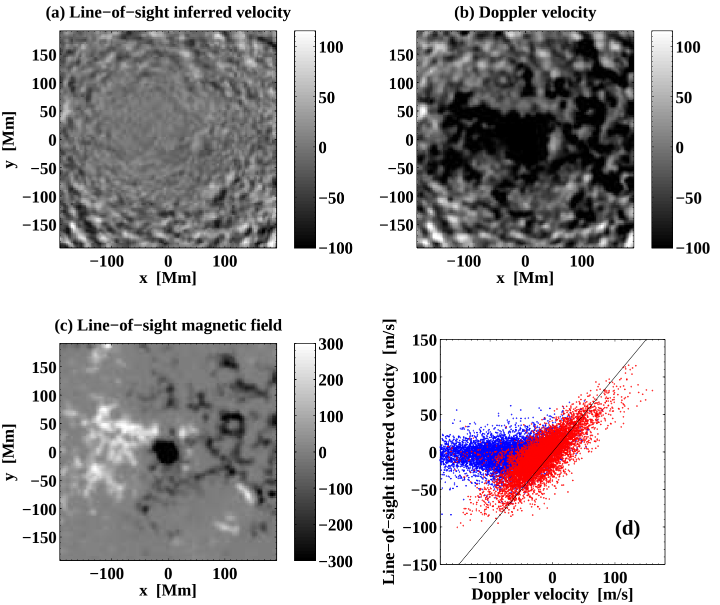
Test 1: f-ridge inversion



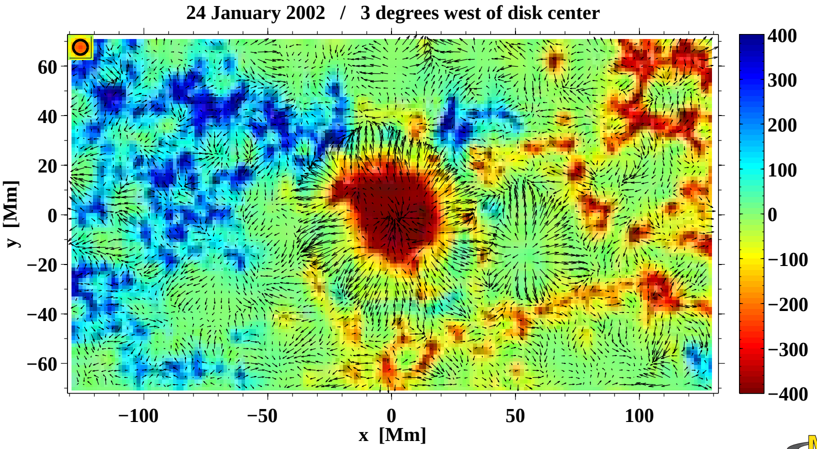
Test 2: p_1-ridge inversion



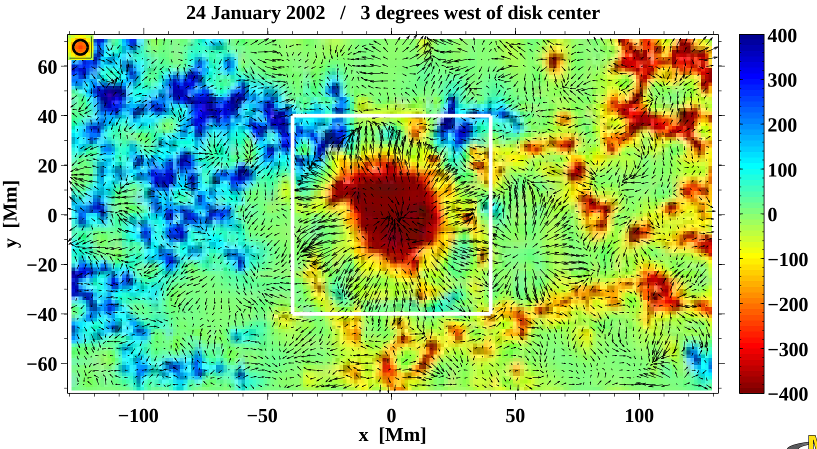
Test 3: Comparison with Doppler velocity



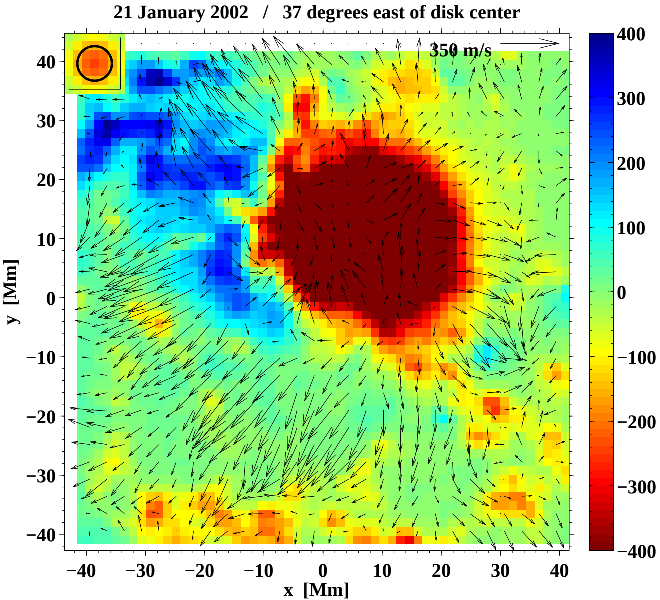
Active region 9787



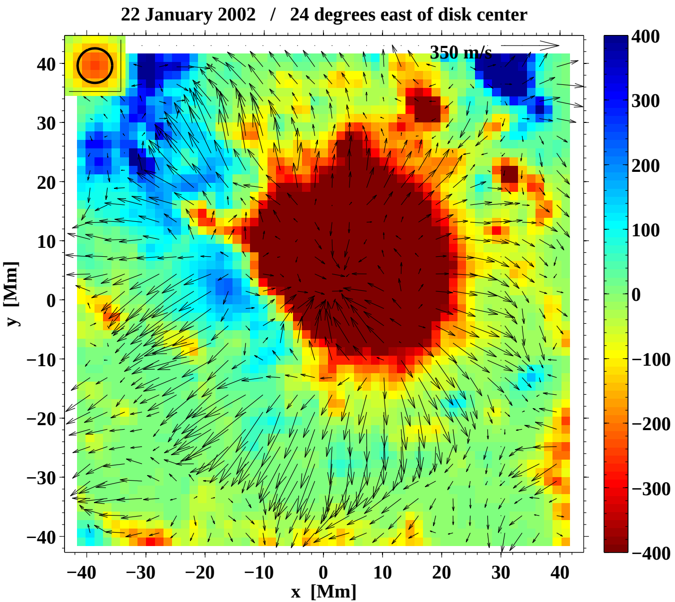
Active region 9787



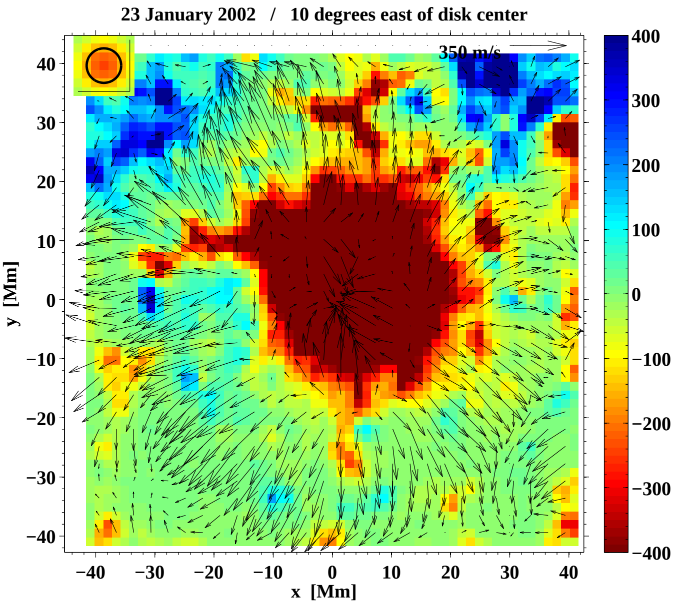
Evolution of flows around a sunspot



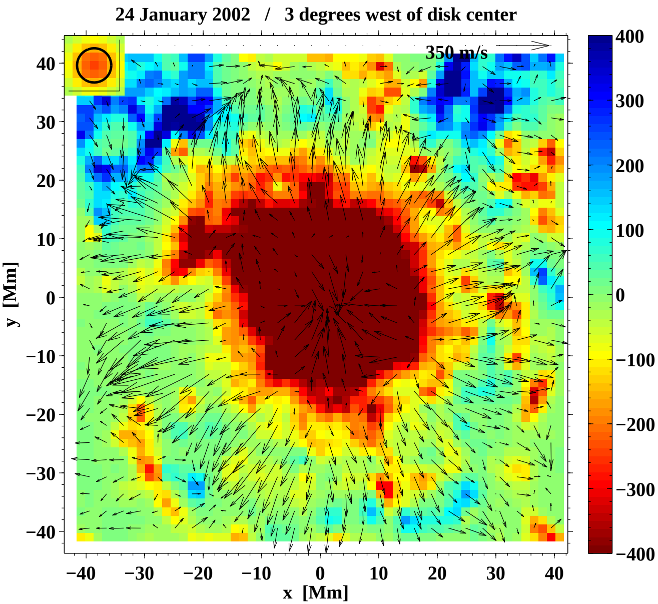
Evolution of flows around a sunspot



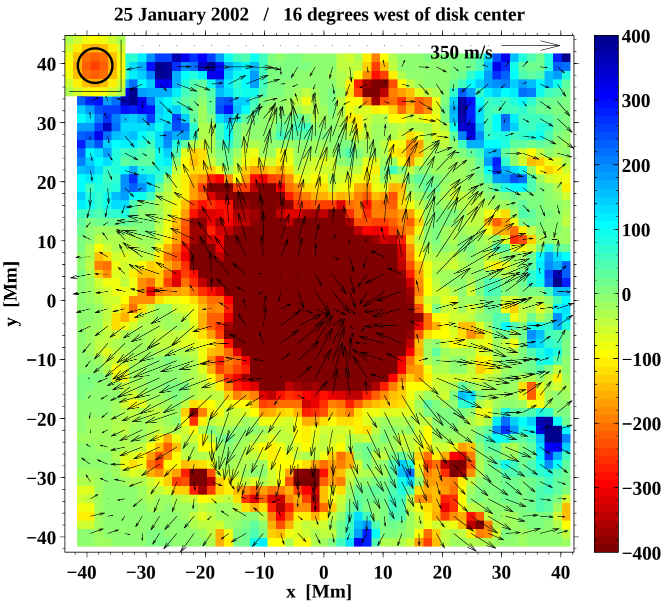
Evolution of flows around a sunspot



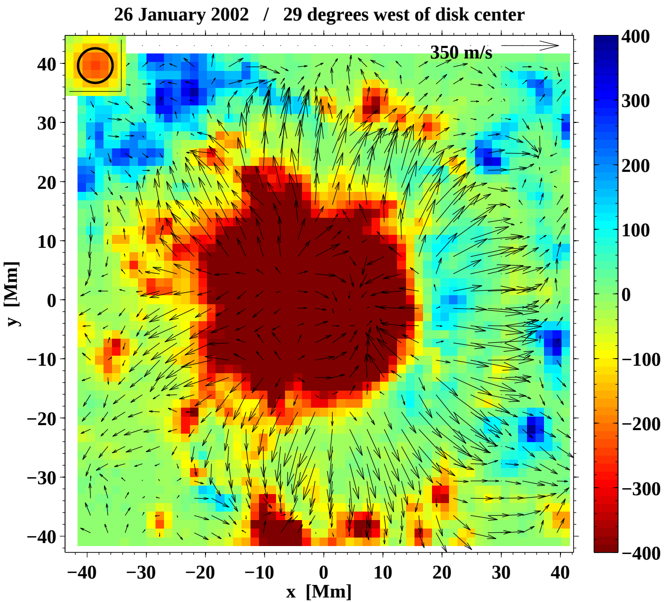
Evolution of flows around a sunspot



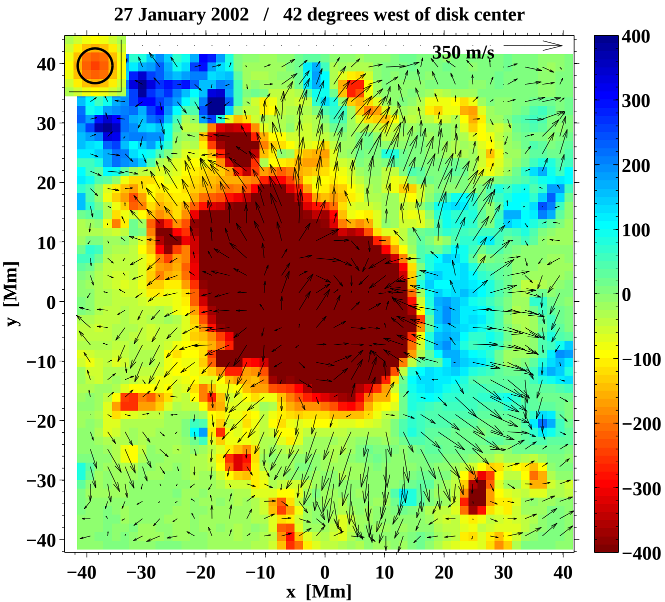
Evolution of flows around a sunspot



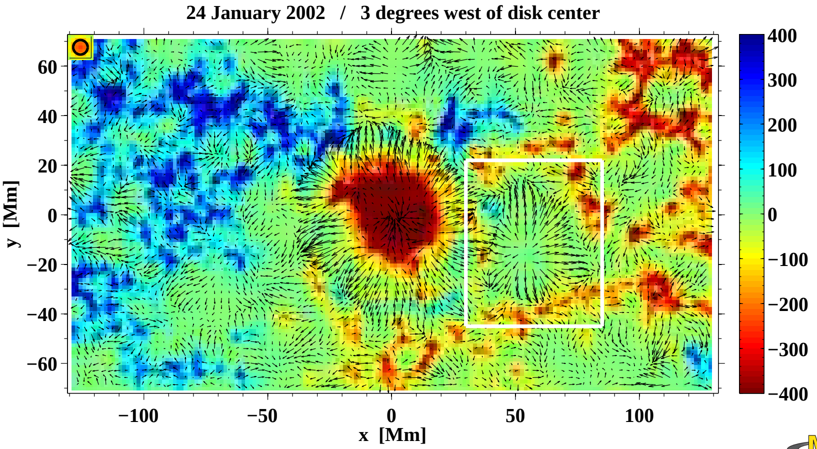
Evolution of flows around a sunspot



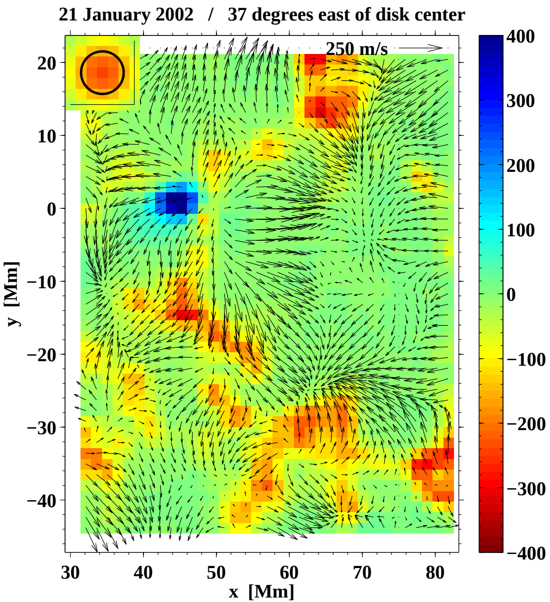
Evolution of flows around a sunspot



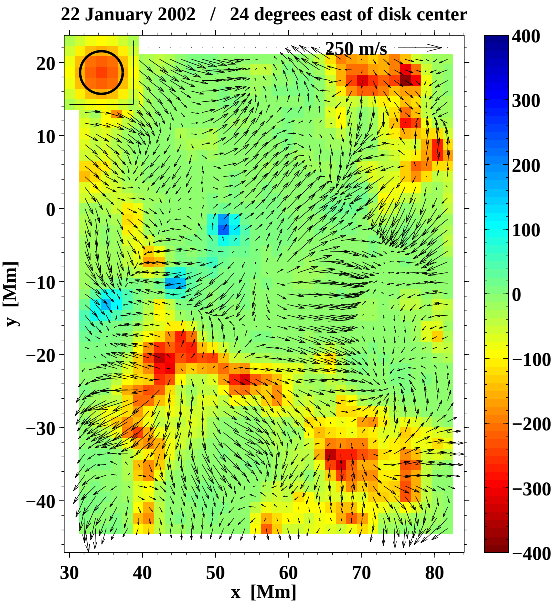
Active region 9787



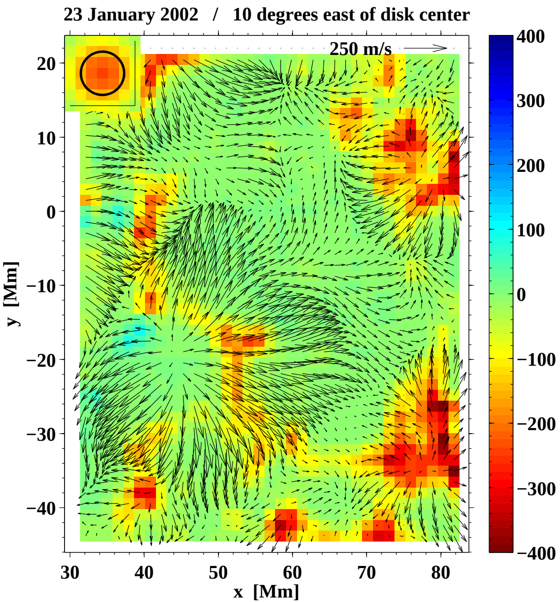
Evolution of a supergranule



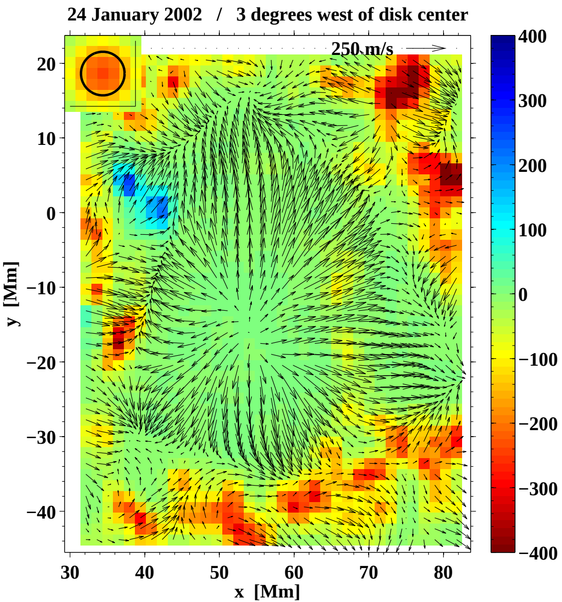
Evolution of a supergranule



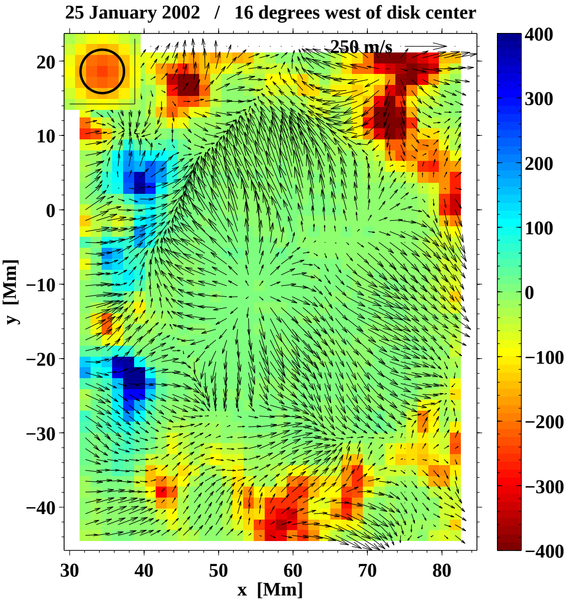
Evolution of a supergranule



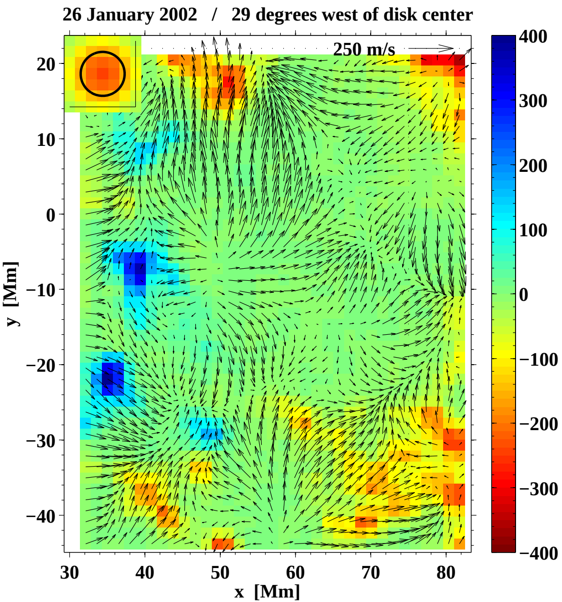
Evolution of a supergranule



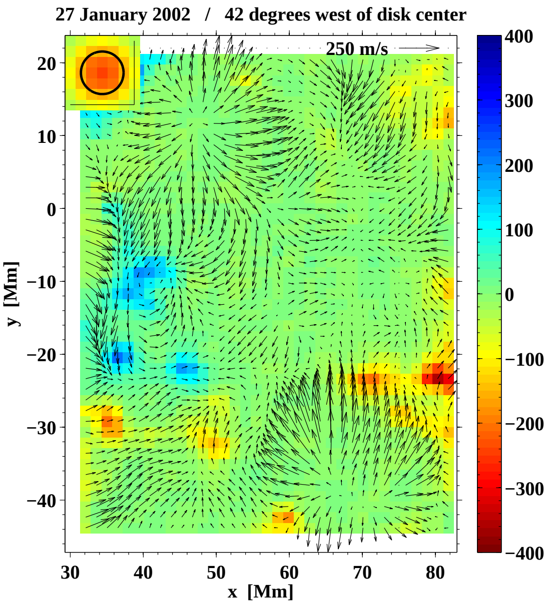
Evolution of a supergranule



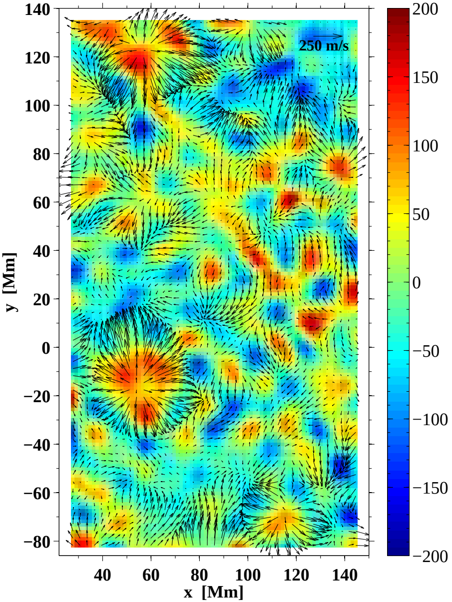
Evolution of a supergranule



Evolution of a supergranule



Vertical flows using only f modes



Conclusions

1. We obtain results similar to earlier time-distance inversions for flows, but with a more consistent procedure
2. Able to invert for the vertical component of the flow, since we minimize contamination by the horizontal component
3. We take into account the full noise covariance matrix
4. Sub-wavelength resolution is possible