Atomic Fermion Condensates: Using 'Old' Physics to Solve a New Problem

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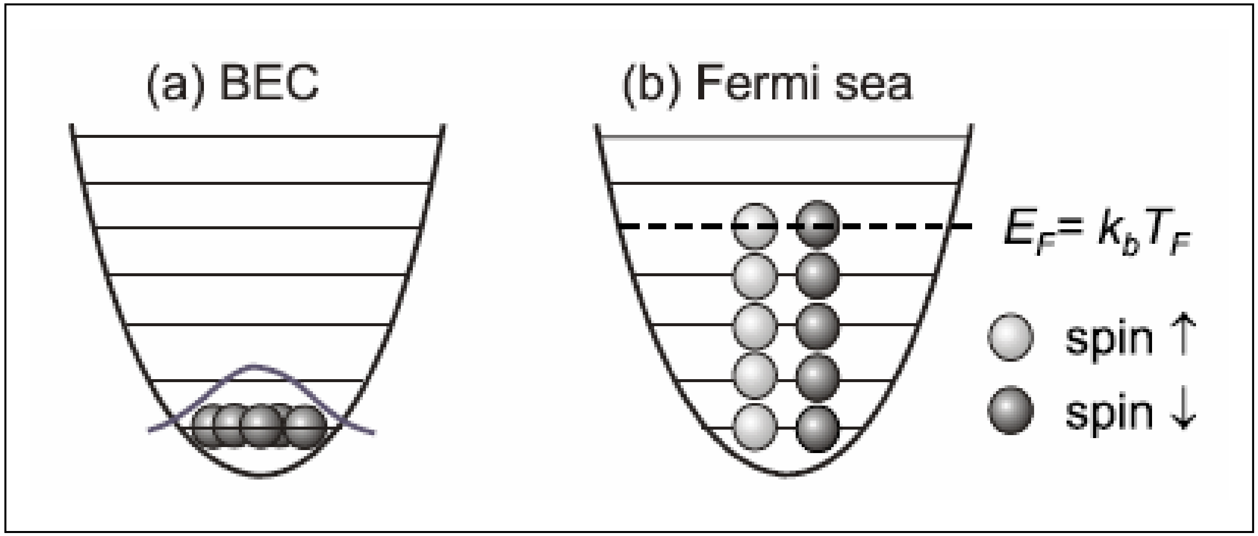
Outline

1. **Introduction to atomic BEC of fermions**
   a. How it happens
   b. Experimental work confirming BEC and methods
   c. Why study this?
2. **General Fermi liquid theory**
   a. Scattering amplitudes, Fermi liquid parameters
   b. Measurable verifications
3. **Induced interaction model and local Fermi liquid theory**
   a. Feynman diagrams
   b. Microscopic model parameter
4. **Results of calculation and comparison with experiment**
5. **BCS superconductivity/superfluidity**
   a. Pairing interactions
   b. Transition temperatures.
6. **Future (current) calculations**

Bose Einstein Condensation (BEC)

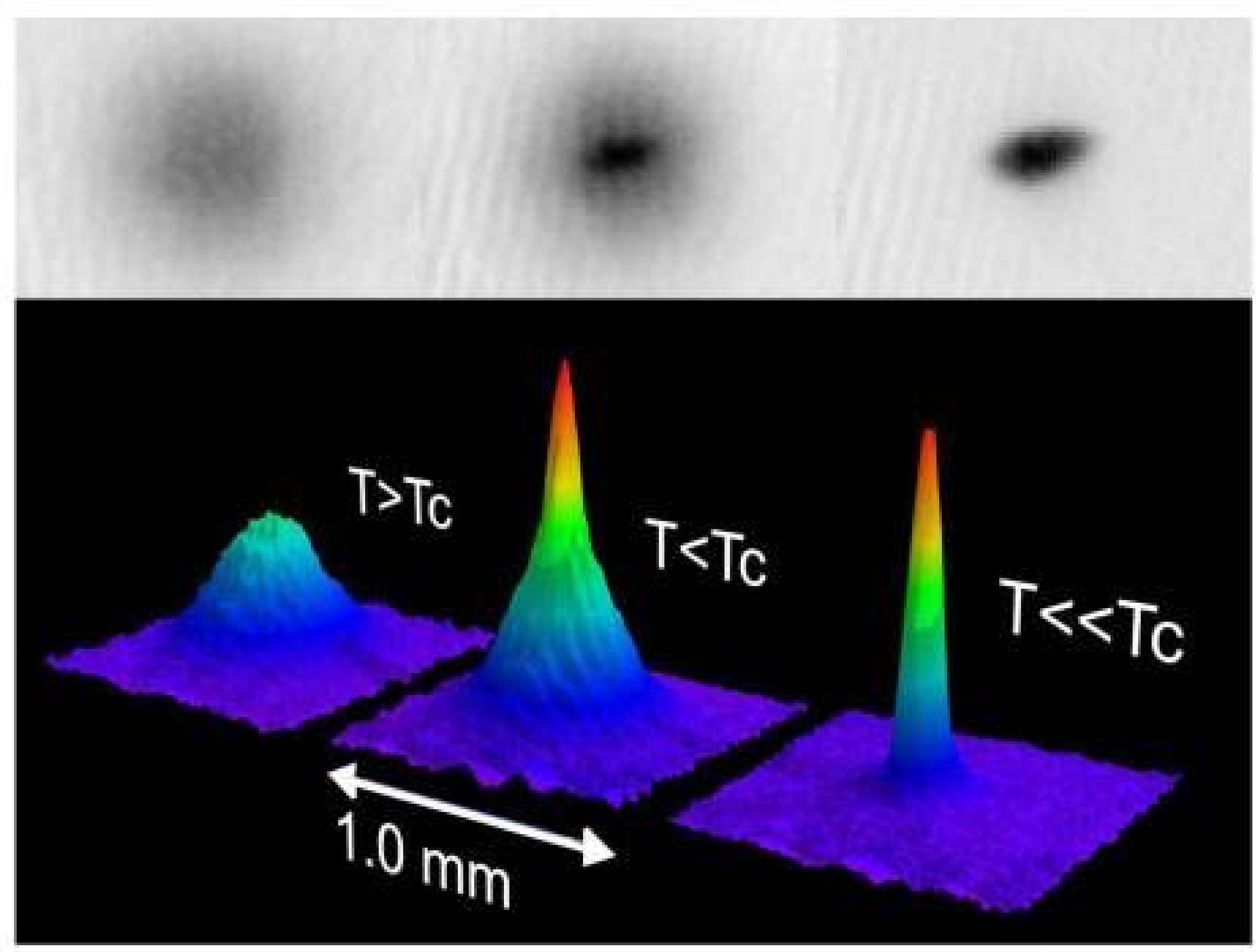
- Not an uncommon phenomenon
- Involves a collapse of all particles in the system into the lowest energy state - degeneracy
- Examples all throughout physics such as:
  - Cooper pairs in superconductors
  - ^{4}He atoms in superfluid liquid Helium
  - Proton or neutron pairs in nuclei or neutron stars
  - Alkali atoms in atomic gases

BEC of … fermions ??



Think of Cooper pairs, made up of electrons, which of course are fermions. Then the pairs are composite bosons and may Bose condense.

BEC of fermions !!



^{40}K atoms

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What is required ??

- extremely low temperatures (nanokelvin)
  - evaporative cooling
  - laser cooling **(see http://www.colorado.edu/physics/2000/bec/temperature.html)**
- magnetic trap to keep atoms confined
- fermions (odd number):
  - $^{40}\mathrm{K}$: $19p^{+} + 19e^{-} + 21n = 59$ fermions
  - $^{6}\mathrm{Li}$: $3p^{+} + 3e^{-} + 3n = 9$ fermions
- different spin states of fermionic atoms to allow for collisions to help the system cool down - this subdues the Pauili principle

What is interesting ?

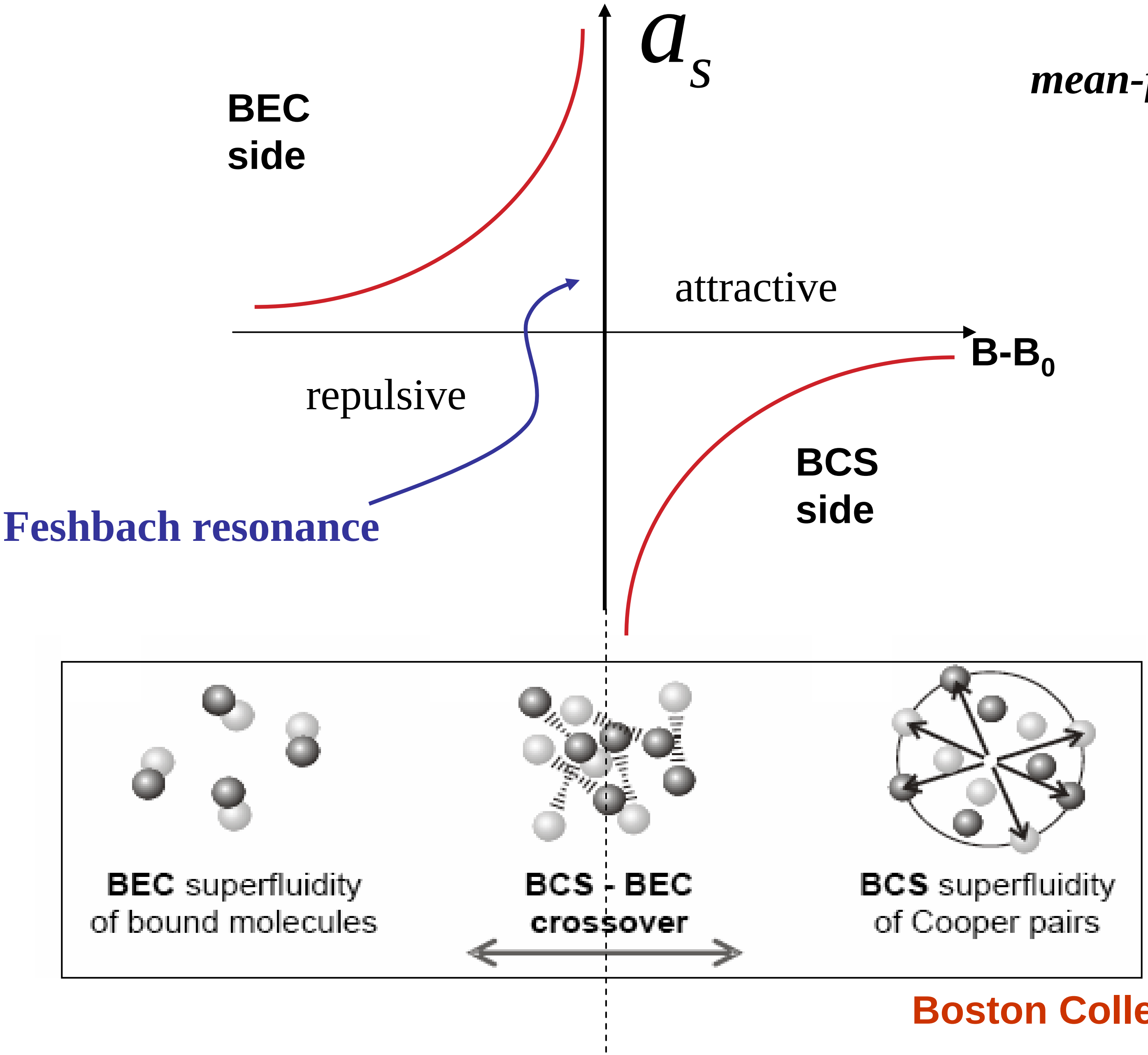
Fundamental questions:

1. How do the bosonic degrees of freedom emerge from the underlying fermionic ones?
2. What is the connection between fermion superfluidity and bose condensation?
3. Can the atomic interactions be tuned to favor one type of pairing over another, and what is the best tuner?
4. How do we detect superfluidity, what are the signatures?

Basic Concepts

1. s-wave scattering length
2. Molecular binding energy
3. Superfluidity

s-wave scattering length



a_s

mean-field

BEC side

attractive

repulsive

B-B$_0$

BCS side

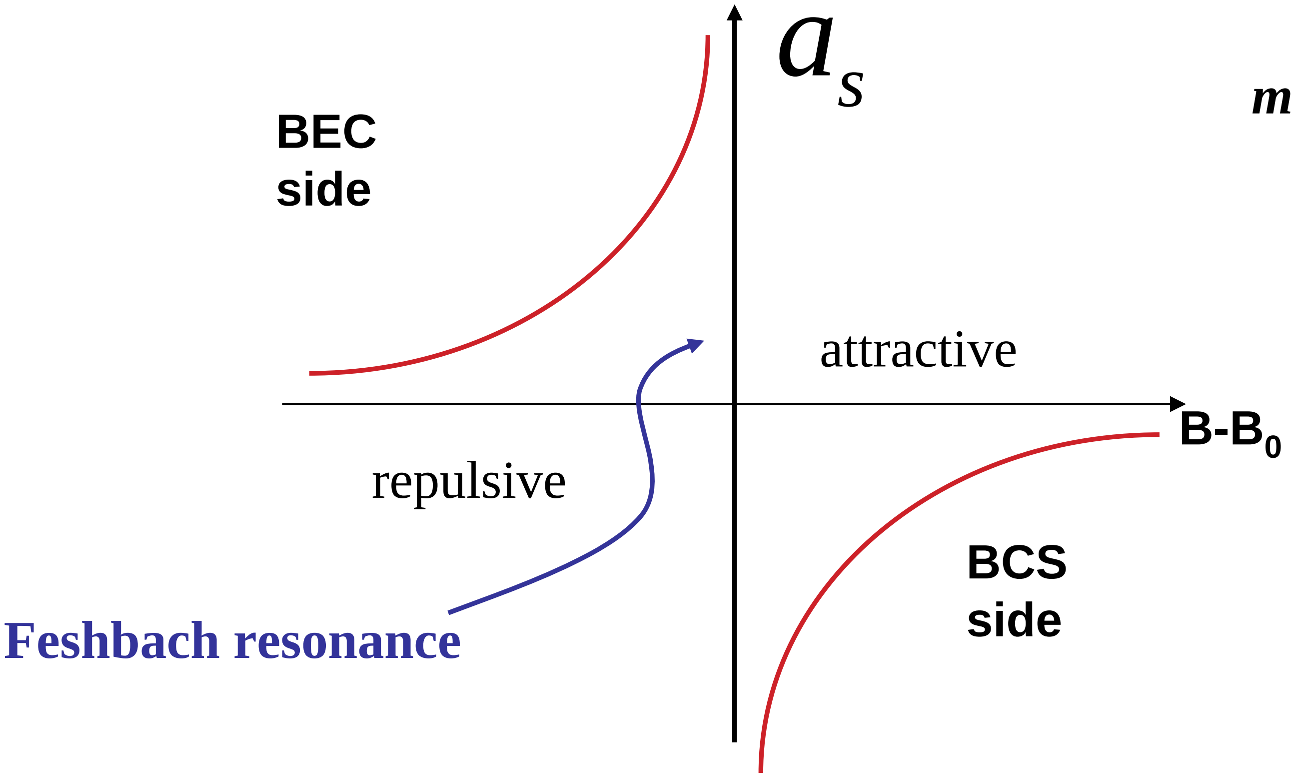
Feshbach resonance

BEC superfluidity of bound molecules

BCS - BEC crossover

BCS superfluidity of Cooper pairs

s-wave scattering length



a_s

BEC side

attractive

B-B$_0$

repulsive

BCS side

Feshbach resonance

mean-field

$$a_s = 174\, a_{Bohr} \left(1 - \frac{\Delta B}{B - B_0} \right)$$

Interactions are tunable!

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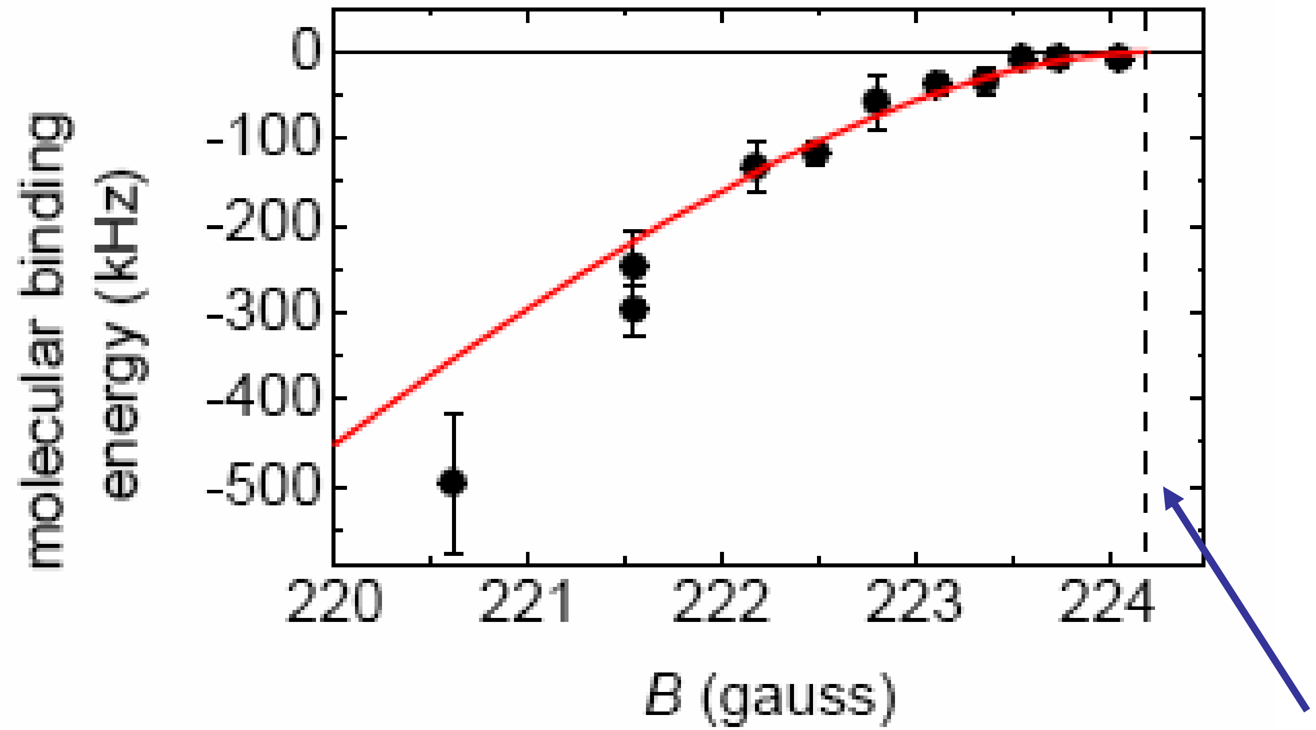
Binding energy



Feshbach

$$E_b = -\frac{h^2}{m a_s^2}$$

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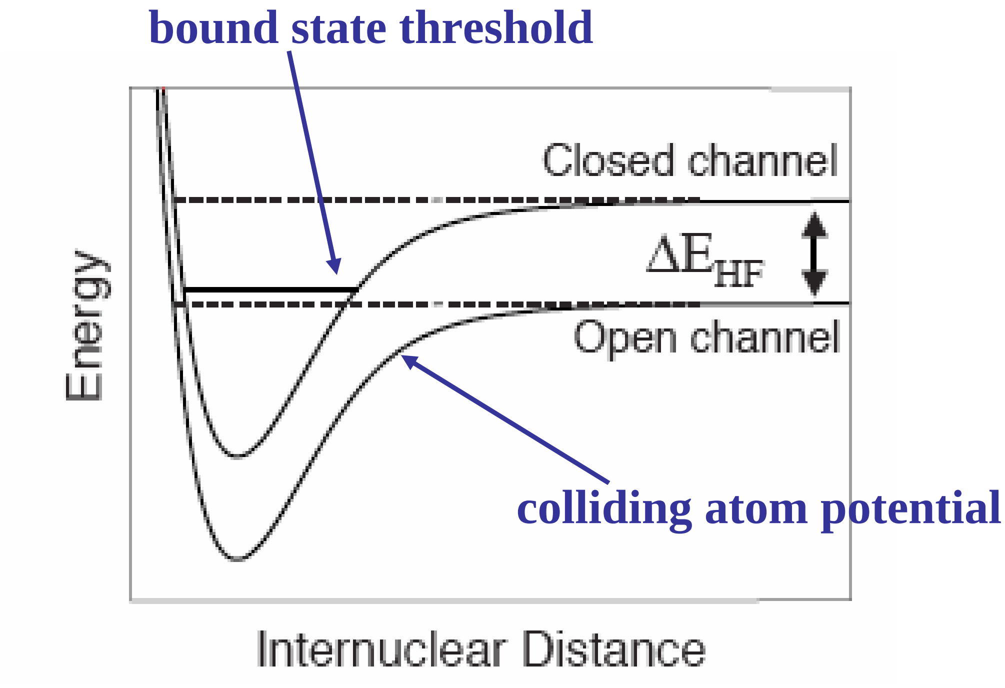
Feshbach resonances

- occurs when the energy of a bound state of 2 atoms equals the kinetic energy of a colliding pair of atoms
- different hyperfine states of atoms can be shifted with a magnetic field



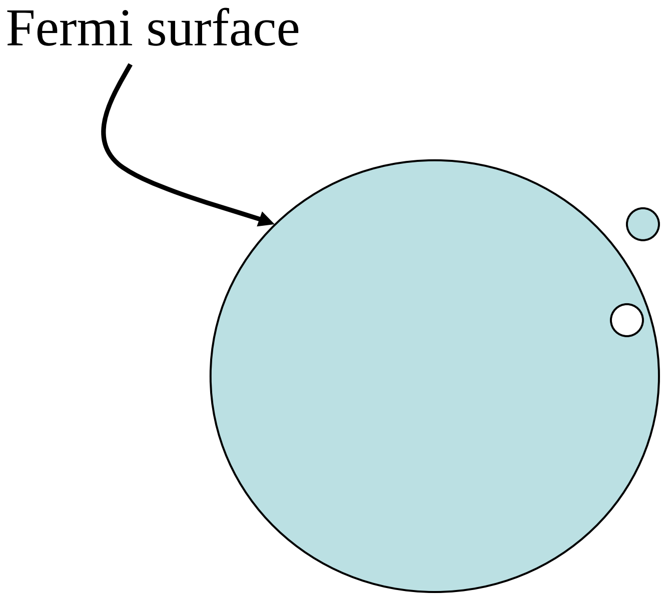
- coulomb interaction couples together different hyperfine states at small distances, allowing spin flips to occur
- **many-body effects become extremely important near the resonance**

The ‘old’ physics:

Fermi liquid theory

Phenomenological way of looking at the interactions between 2 (quasi)particles due to changes in the distribution of all of the other particles.

The Fermi Liquid



- One-to-one correspondence
- Pauli exclusion principle
- Filled to Fermi level

- Adiabatically turn on interactions or temperature, most states unaffected: Dilute gas of excitations - quasiparticles (holes)

Fermi gas + interactions ⟶ Fermi liquid

Quasiparticles ⟶ Dressed quasiparticles

Landau Fermi liquid theory

- Energy of a system of quasiparticles (qp):

$$E = E_0 + \sum_{p\sigma} \varepsilon_{p\sigma}\delta n_{p\sigma} + \frac{1}{2}\sum_{pp',\sigma\sigma'} f_{pp'}^{\sigma\sigma'}\delta n_{p\sigma}\delta n_{p'\sigma'}$$

single qp energy

qp interaction function

change in qp distribution

- Landau interaction function:

$$f_{pp'}^{\sigma\sigma'} = f_{pp'}^{s} + \sigma\cdot\sigma'\, f_{pp'}^{a} \longleftrightarrow N(0)\, f_{pp'}^{s,a} = \sum_{l} F_l^{s,a} P_l(\hat{p}\cdot\hat{p}')$$

Landau Fermi liquid parameters

$$N(0) = \frac{m^* p_F}{\pi^2}$$

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Physical quantities of Landau parameters

- effective mass

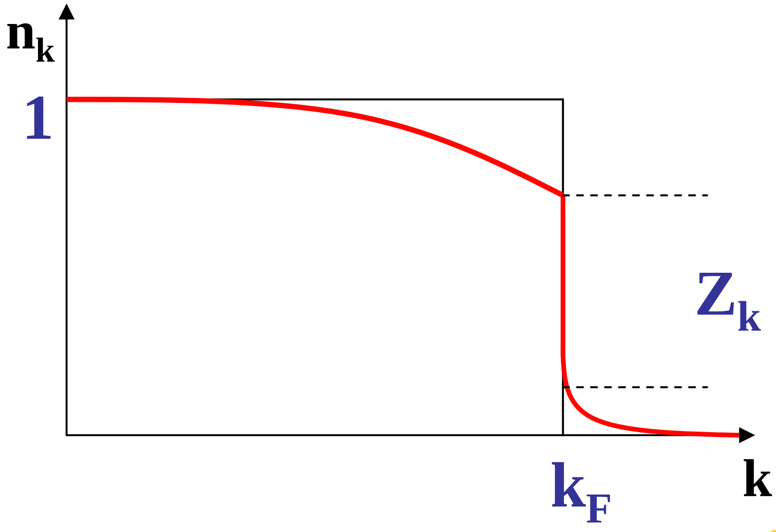
$$\frac{m^*}{m} = 1 + \frac{F_1^s}{3}$$

F_0^a: spin fluctuation

F_1^s: current fluctuation

- specific heat

$$C_V \propto N(0)T \propto m^* T$$



- spin susceptibility

$$\chi \propto \frac{N(0)}{1+F_0^a}$$

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Interactions: ‘Two-body’ interactions with ‘two-body’ wavefunctions

$$\Psi(r\sigma, r'\sigma') \rightarrow \Psi(p\sigma, p'\sigma') \rightarrow A_{pp'}^{\sigma\sigma'}\Phi(p\sigma, p'\sigma')$$

scattering amplitudes

$$A_{pp'}^{\sigma\sigma'} = A_{pp'}^{s} + \sigma\cdot\sigma' A_{pp'}^{a}$$

- as before: $A_{pp'}^{s,a} = \sum_{l=0}^{\infty} A_l^{s,a} P_l(\hat{p}\cdot\hat{p}')$

$m_0 \neq 0$

- related to Fermi liquid parameters $A_l^{s,a} = \dfrac{F_l^{s,a}}{1+\dfrac{F_l^{s,a}}{2l+1}}$

Induced Interactions

- A way to introduce a microscopic parameter into Fermi liquid theory to model fermionic systems
- The Pauli principle is ‘built in’ to the equations so that the full antisymmetry of the wave functions is preserved
- It is a full many-body procedure that takes into account ‘diagrams’ absent in Fermi liquid theory (RPA) and is fully self-consistent

Approximation: The scattering processes are all on the Fermi surface, so large momentum transfers need to be treated differently.

Induced interaction model have been applied to:

- **liquid ^{3}He**
- **superfluid ^{4}He**
- **heavy fermion superconductors**
- **high Tc superconductors**
- **superconducting ferromagnets**
- **neutron stars**
- **fermion condensates**

Basic scheme

Interaction strength = **direct term** + **induced interactions**

Fermi liquid parameters

local, on-site term, contains microscopic parameter

- purely quantum fluctuations due to antisymmetry
- includes Fermi liquid parameters

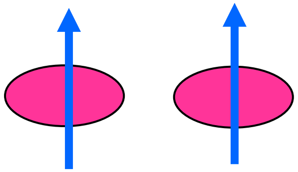
Direct term

$$D^{\sigma\sigma'}_{pp'} = D^{s}_{pp'} + \sigma\cdot\sigma' D^{a}_{pp'} \qquad D^{s,a}_{pp'} = \sum_{l} D^{s,a}_{l} P_{l}(\cos\theta)$$

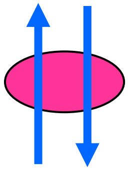
- take local (l=0) moment
- $D^s_0 = -D^a_0 = \frac{U}{2}$ **(U - Hubbard model)**

$$D^{\uparrow\uparrow}_0 = D^s_0 + D^a_0 = 0$$

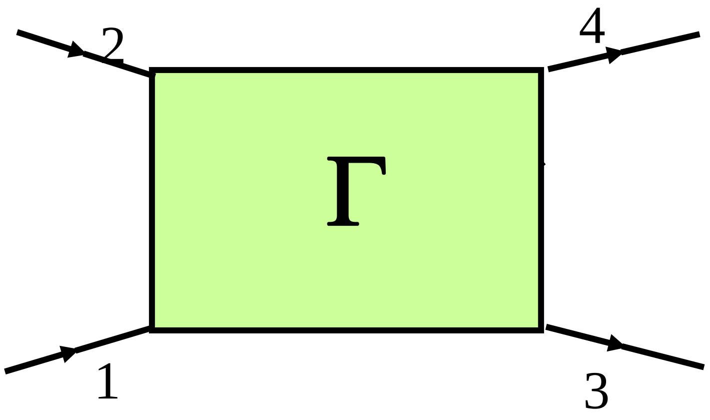
Pauli!



$$D^{\uparrow\downarrow}_0 = D^s_0 - D^a_0 = U$$



Induced interaction term



2-body reducible scattering vertex

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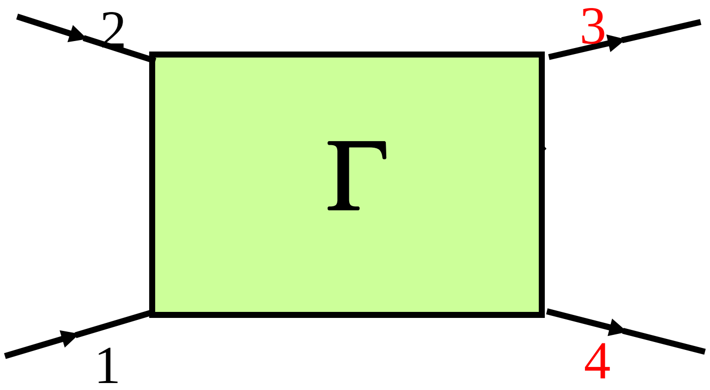
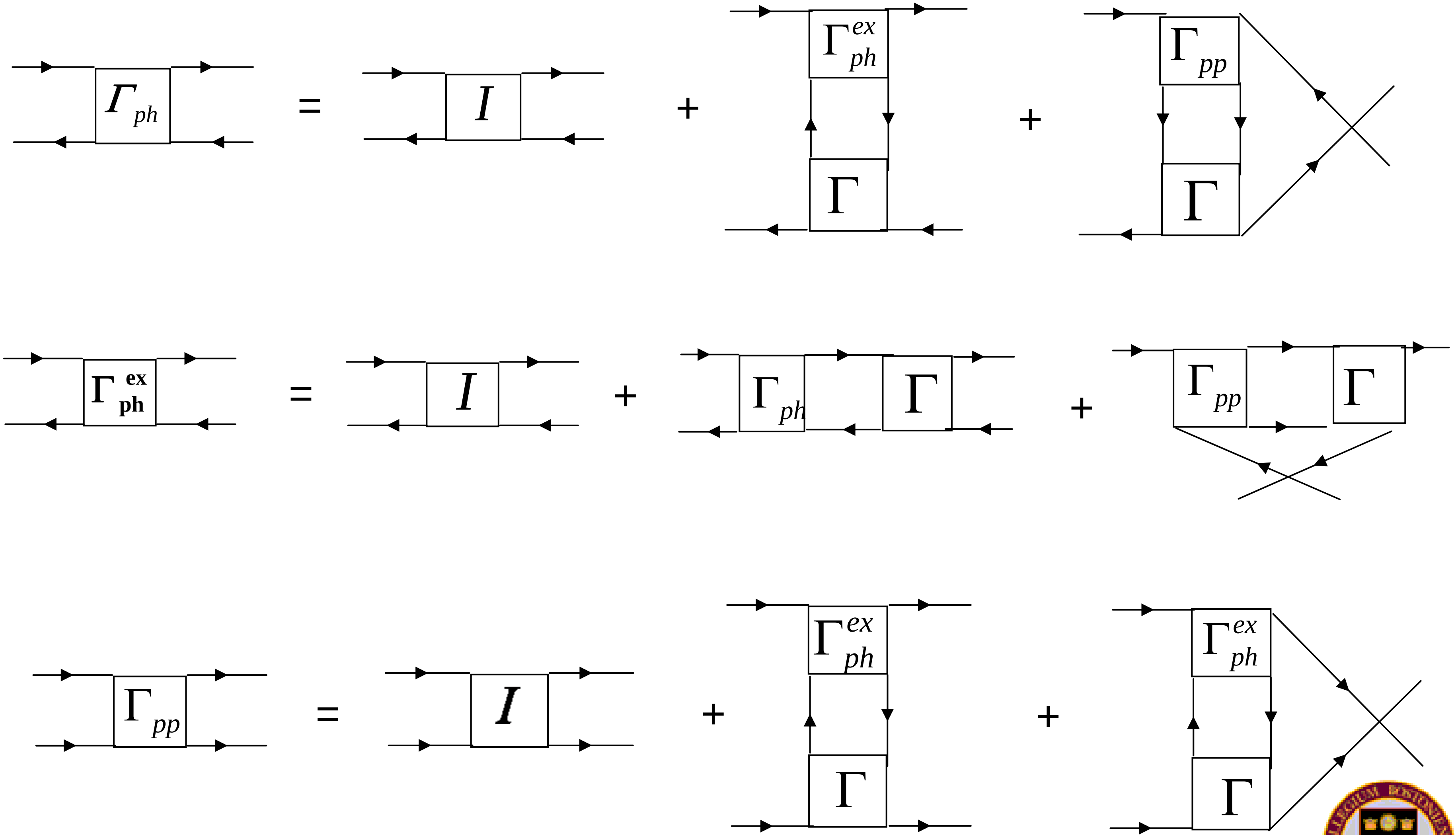
Induced interaction term



2-body particle -hole reducible scattering vertex

exchange ; particle-hole

Crossing-symmetric/Parquet Equations: Diagram Forms



Induced interactions’ *inside*

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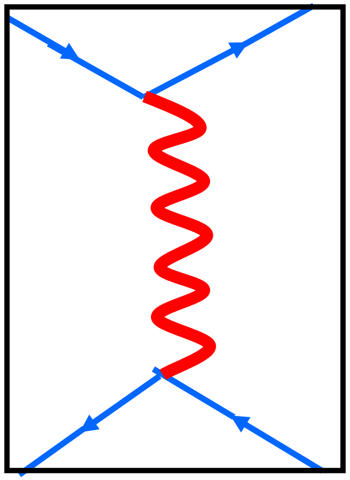

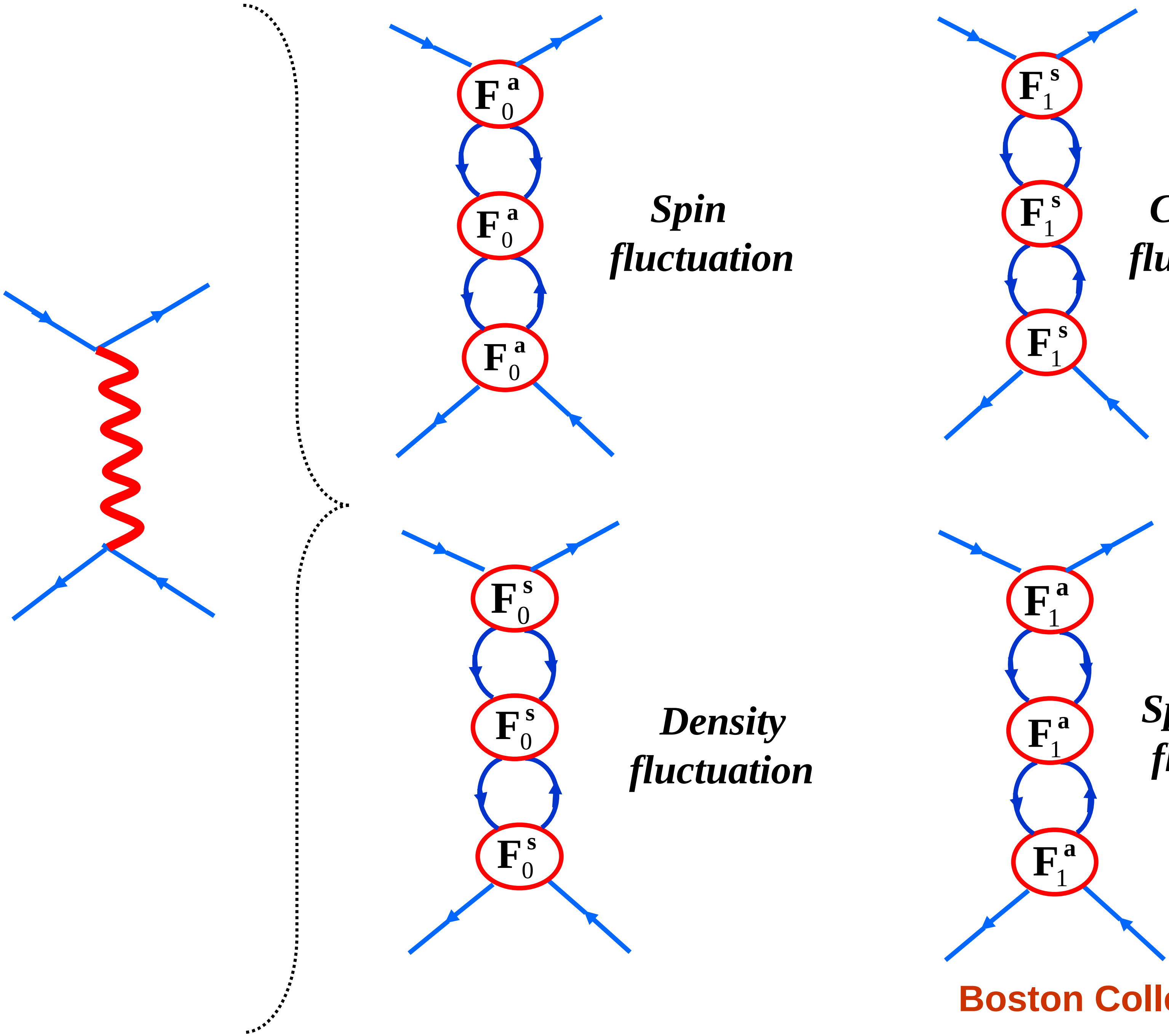

Spin fluctuation

Current fluctuation

Density fluctuation

Spin current fluctuation

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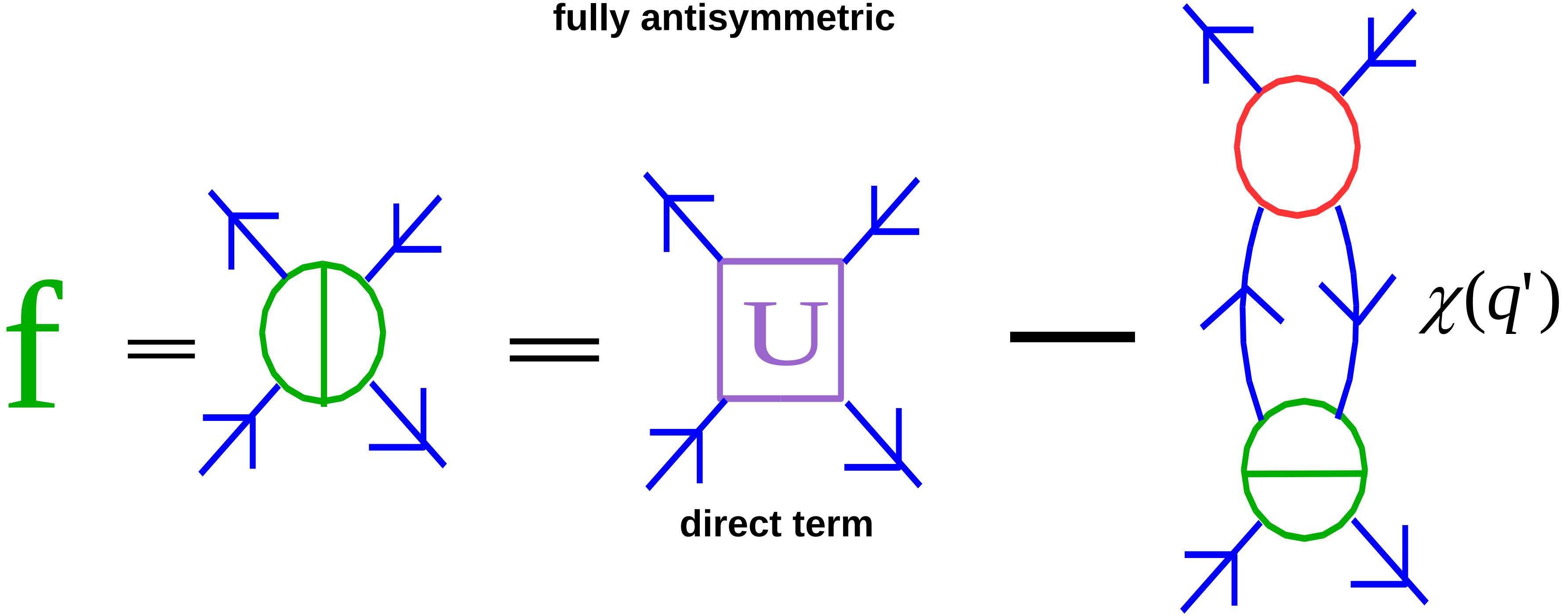
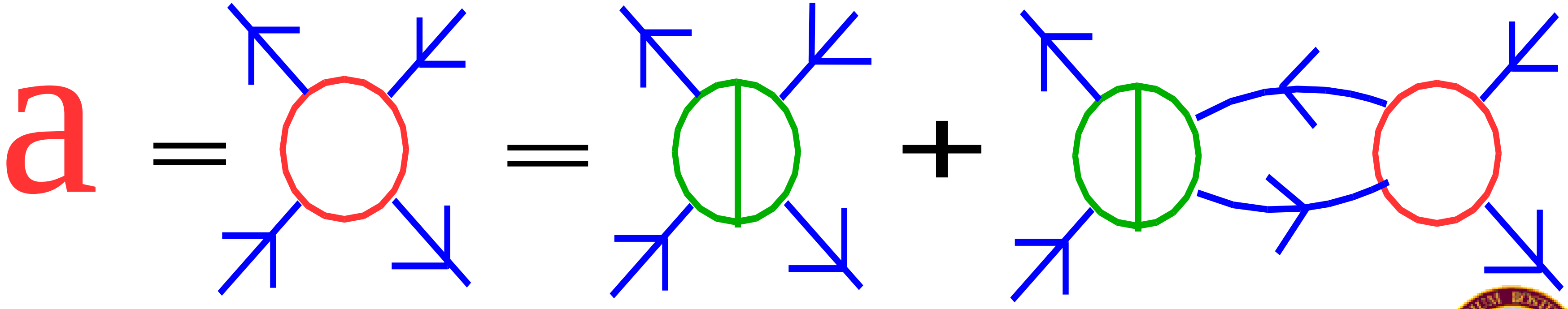
Induced Interactions – *quantum fluctuations*



fully antisymmetric

direct term



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Induced interactions - equations

$$\boxed{\begin{aligned} F_0^s &= D_0^s + \frac{1}{2}\frac{F_0^s\chi(q')F_0^s}{1+F_0^s\chi(q')} + \frac{3}{2}\frac{F_0^a\chi(q')F_0^a}{1+F_0^a\chi(q')} \\ F_0^a &= D_0^a + \frac{1}{2}\frac{F_0^s\chi(q')F_0^s}{1+F_0^s\chi(q')} - \frac{1}{2}\frac{F_0^a\chi(q')F_0^a}{1+_{q'}F_0^a\chi(q')} \end{aligned}}$$

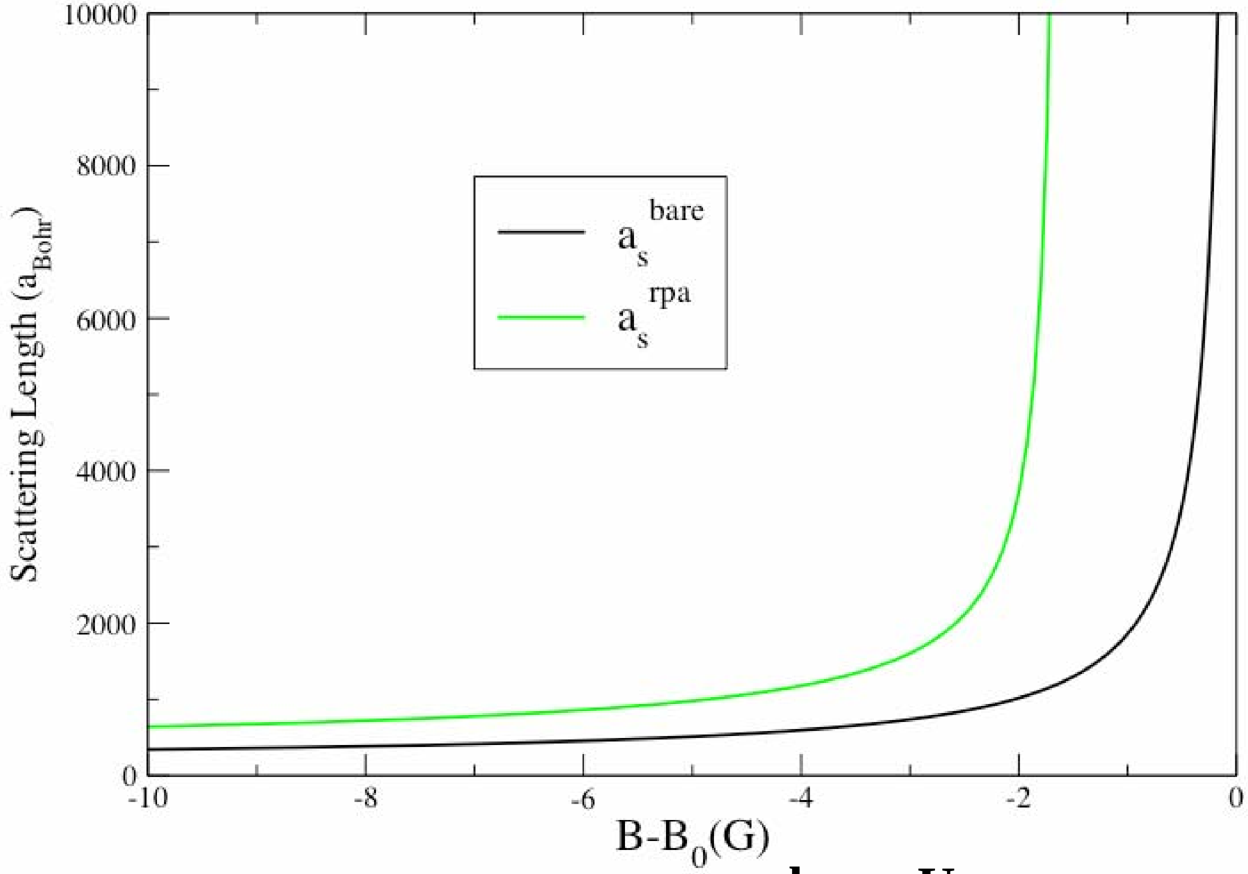
local limit (l=0)

- particle-hole exchange momentum $q'^2 = |p-p'|^2 = k_F^2(1-\cos\theta)$

$$\chi(q') = \frac{1}{2}\left(1+\frac{q'}{4k_F}\ln\left|\frac{k_F - q'/2}{k_F + q'/2}\right|\right)$$

Lindhard function

Results for this model in the fermion condensates



Feshbach

large U →

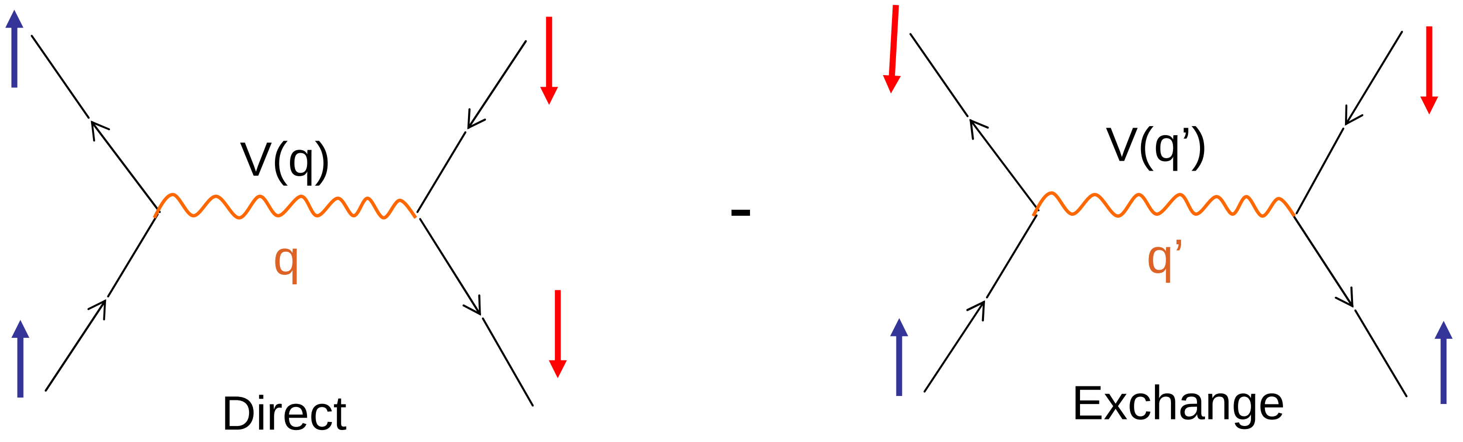
$$a_s^{bare} = 174\,a_{Bohr}\left(1-\frac{\Delta B}{B-B_0}\right)$$

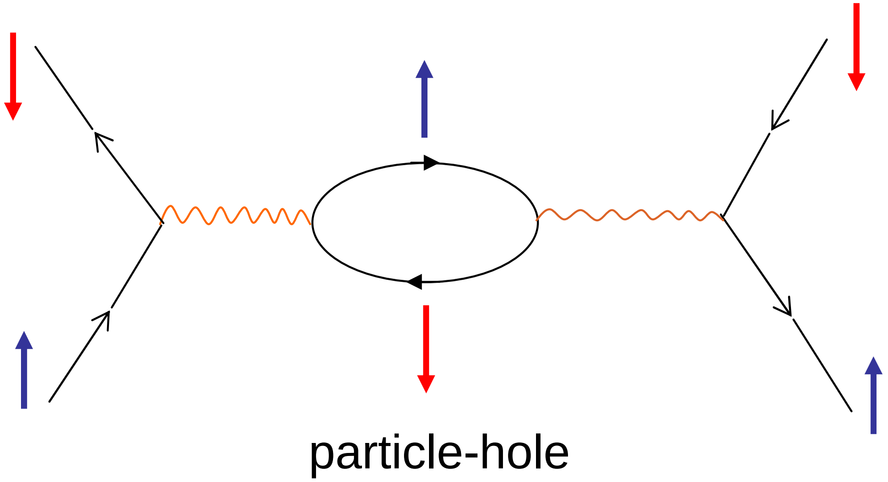
$$U = \frac{4\pi\hbar^2}{m}a_s^{rpa}$$

s-wave singlet scattering length

$$A_0^{\text{sing}} = A_0^s - 3A_0^a$$

detour: RPA and exchange





Beyond Hartree-Fock

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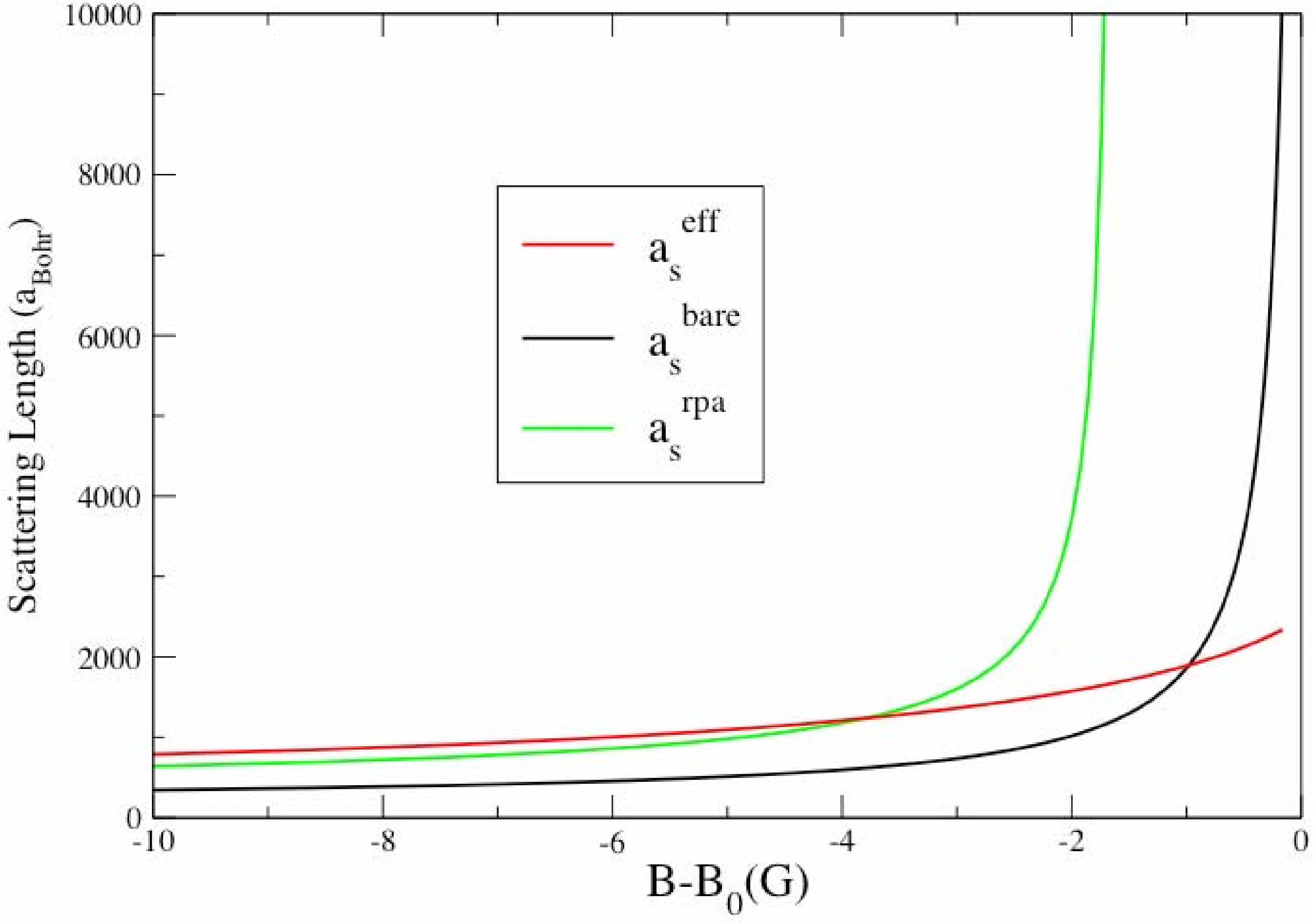
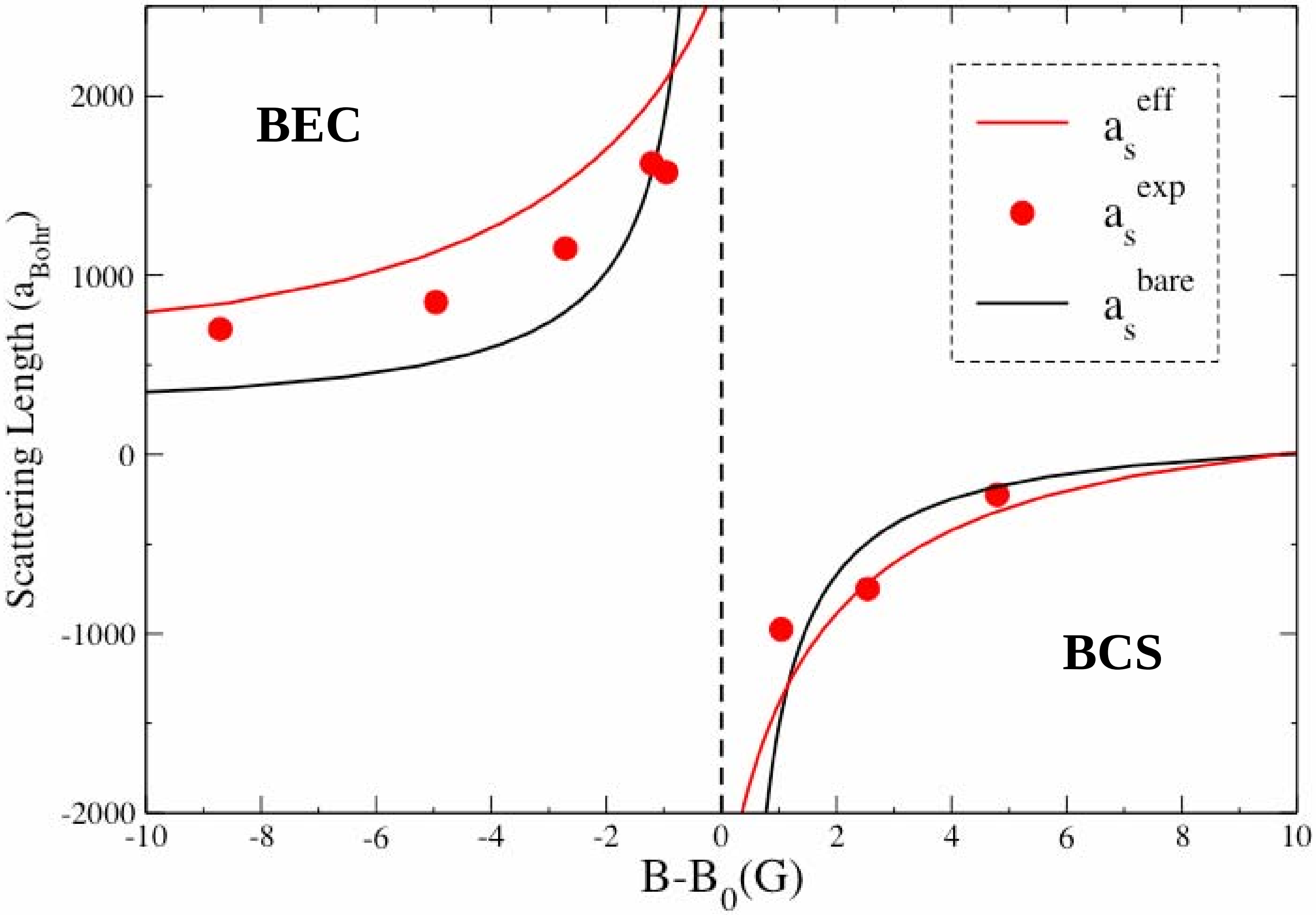


$$U = \frac{4\pi\hbar^2}{m^*} a_s^{eff}$$

- effective theory does not show divergence near Feshbach resonance

Experiment



large U → ← **large -U**

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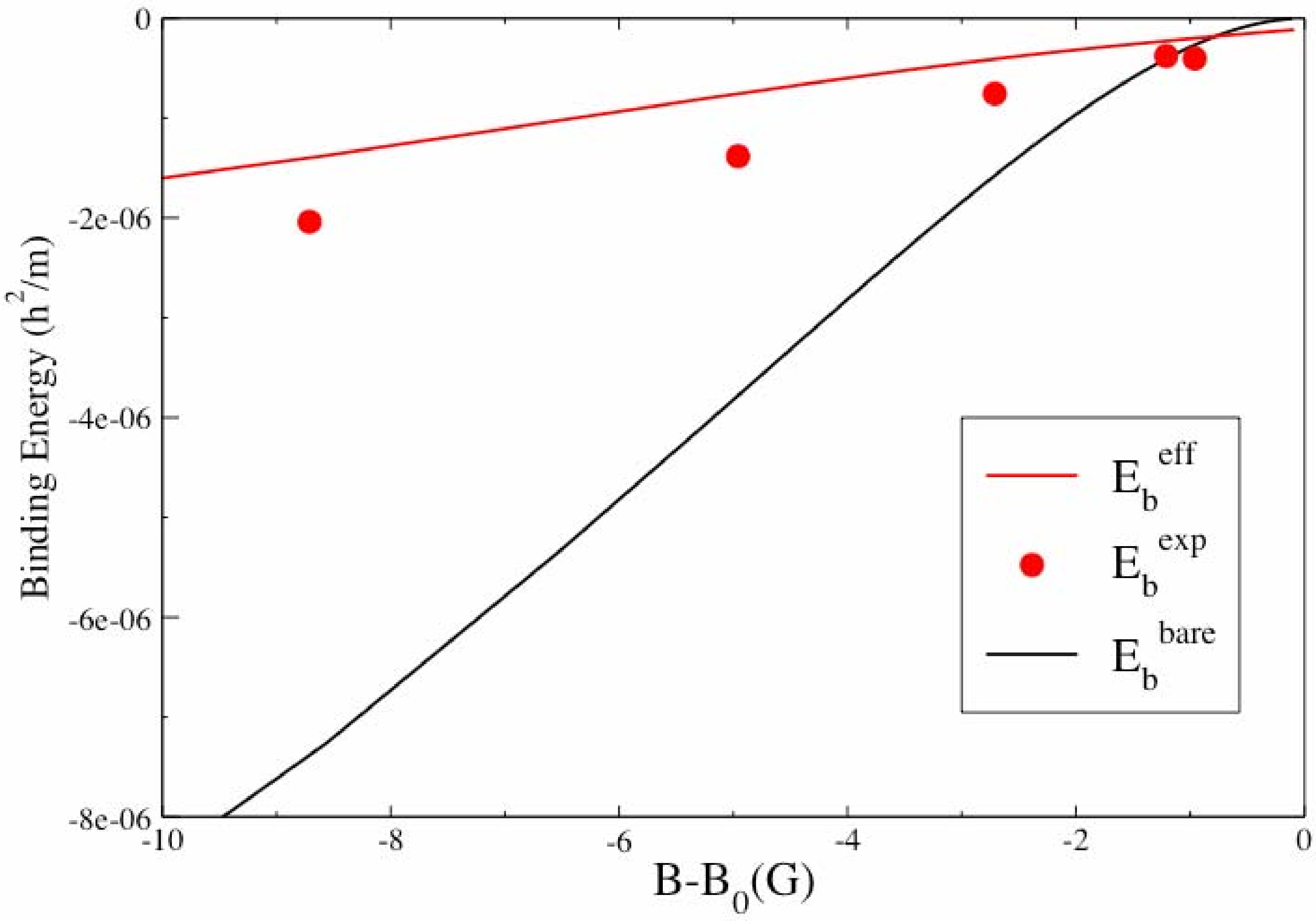
Binding energy



$$E_b^{eff} = \frac{\hbar^2}{m^* \left(a_s^{eff}\right)^2}$$

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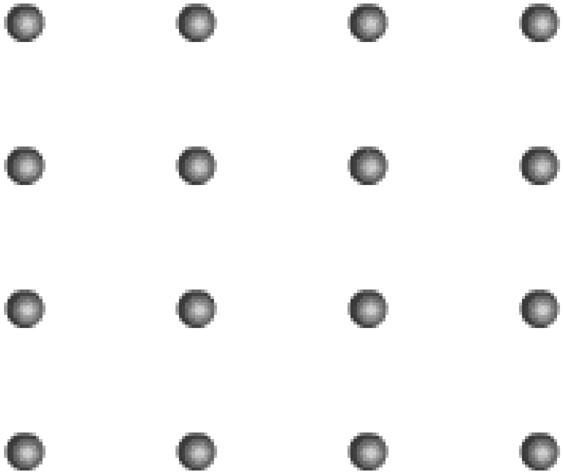
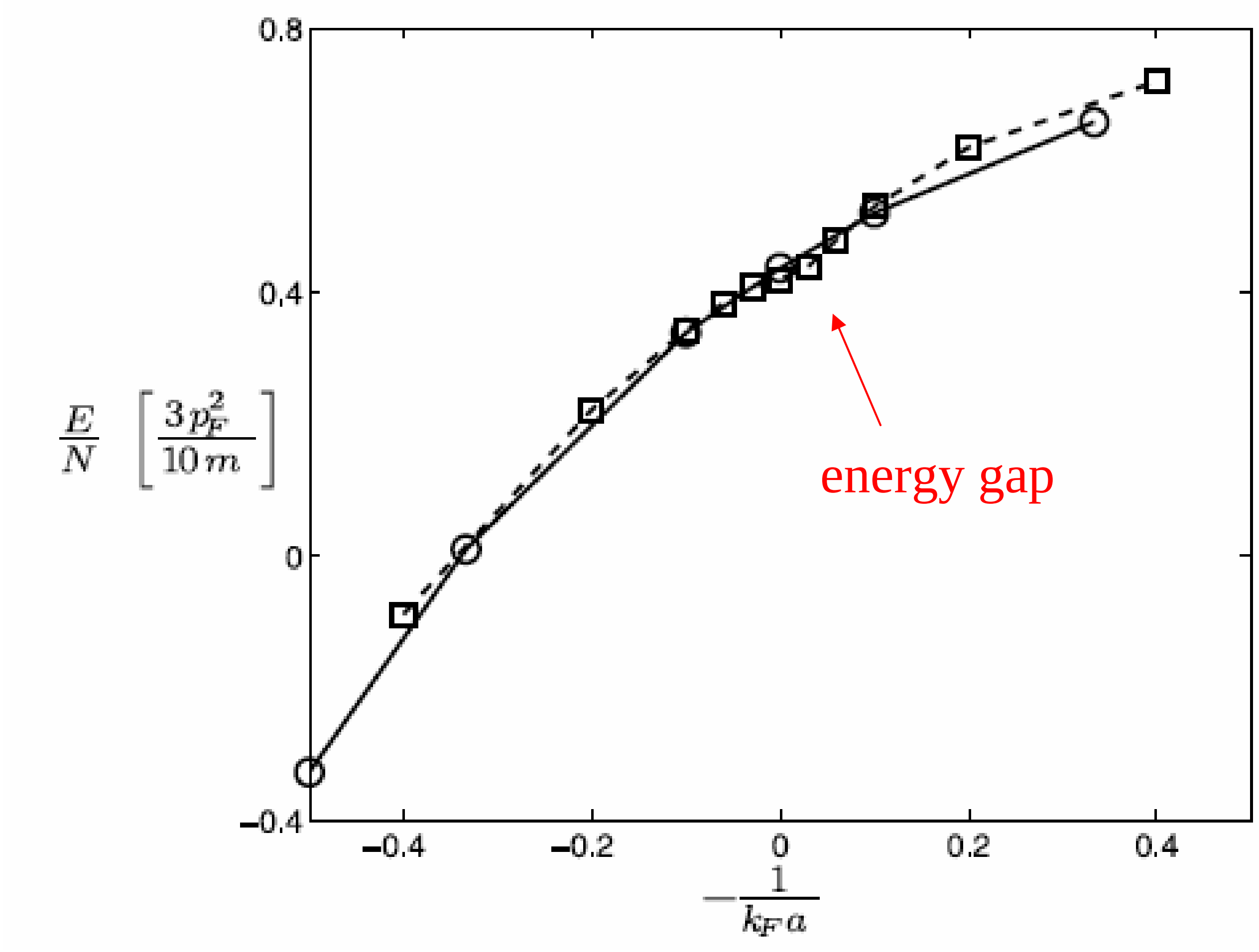
Superconductivity

- formation of Cooper pairs of electrons due to phonon coupling



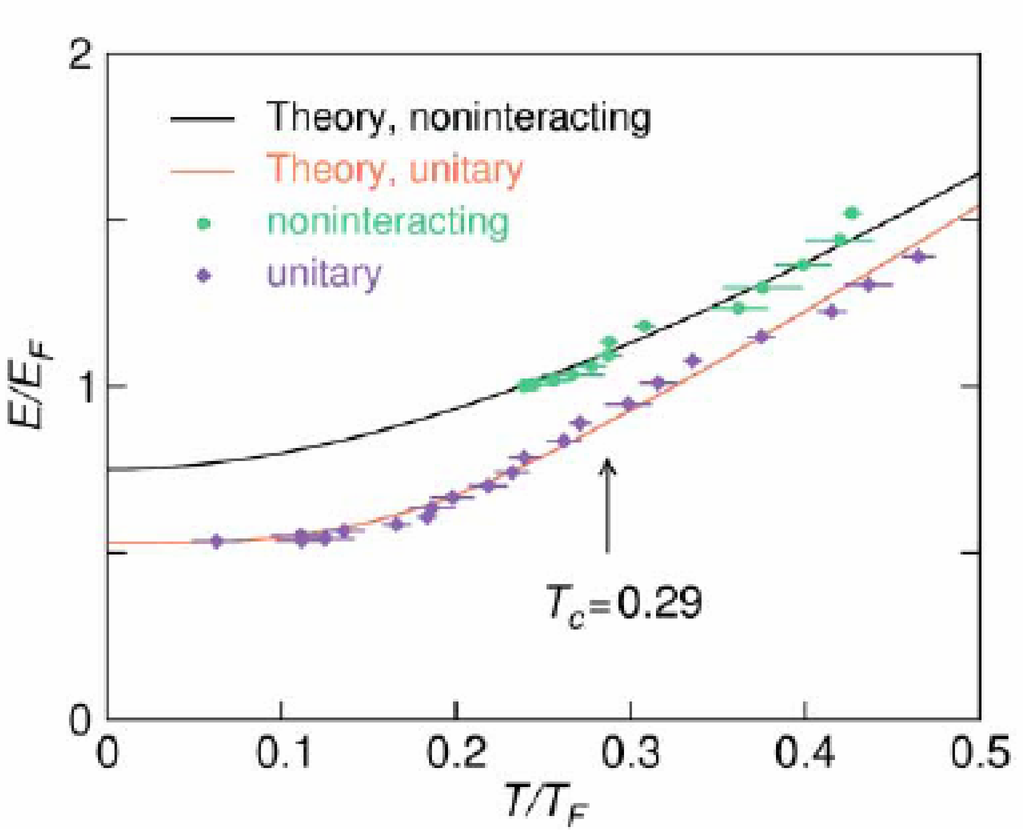
- for singlet s-wave, pairs have opposite spin and momentum
- an energy gap opens up at the Fermi level
- jump in specific heat, ultrasonic attenuation, electromagnetic absorption signature

Is there evidence for superfluidity on the BCS side in the fermion condensates?



energy gap

Is there evidence for superfluidity on the BCS side in the fermion condensates?



Heat capacity jump

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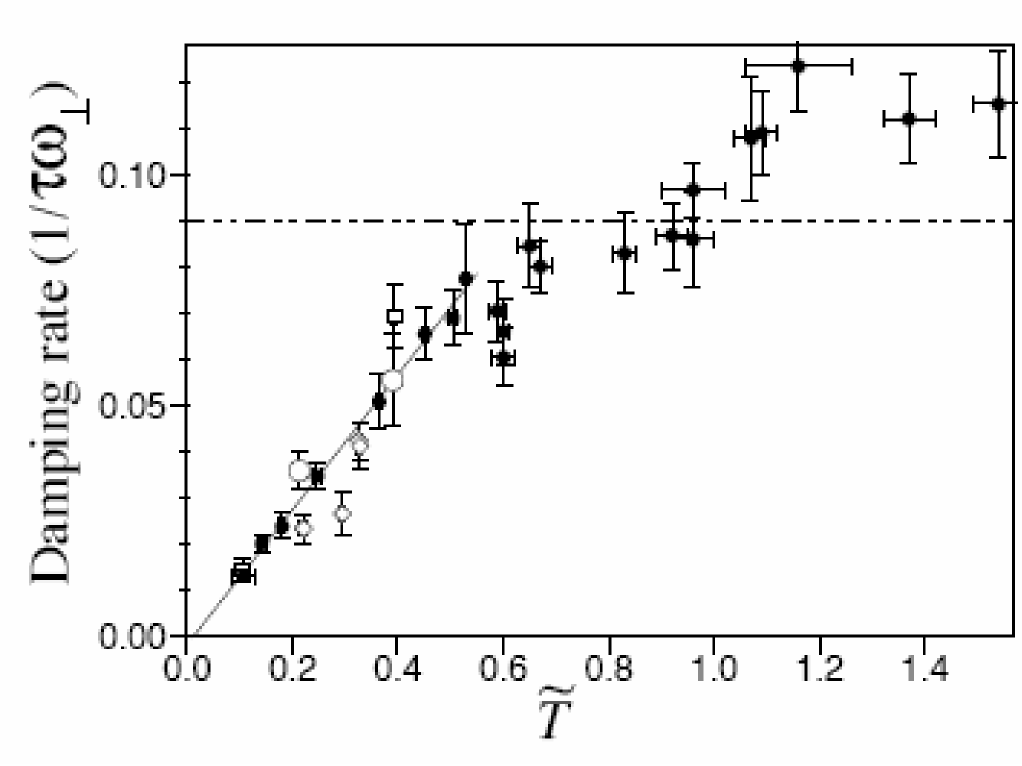
Is there evidence for superfluidity on the BCS side in the fermion condensates?



2 transitions!

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Theory

- Most of the theory field has only considered singlet pairing for superfluidity because the mean-field models favor this
- We believe that near the resonance where the scattering lengths get large, there are higher angular momentum channels available for scattering and hence pairing. P-wave superfluidity should become stronger.
- Thus there should be competition among competing pairing channels, and the observation of 2 transitions supports this

Pairing in the induced interaction model

- **Pairing Channel Scattering Amplitudes**

$$T^{t,s}(\theta,\vartheta) = \sum_{l,m} T^{s,t}_{l,m}(\theta = \pi,\vartheta)\,(-1)^{l}\, P_m \cos(\vartheta)$$

$$T^{t}(\theta,\vartheta) = A^{s}(\theta,\vartheta) + A^{a}(\theta,\vartheta)$$: **Triplet**

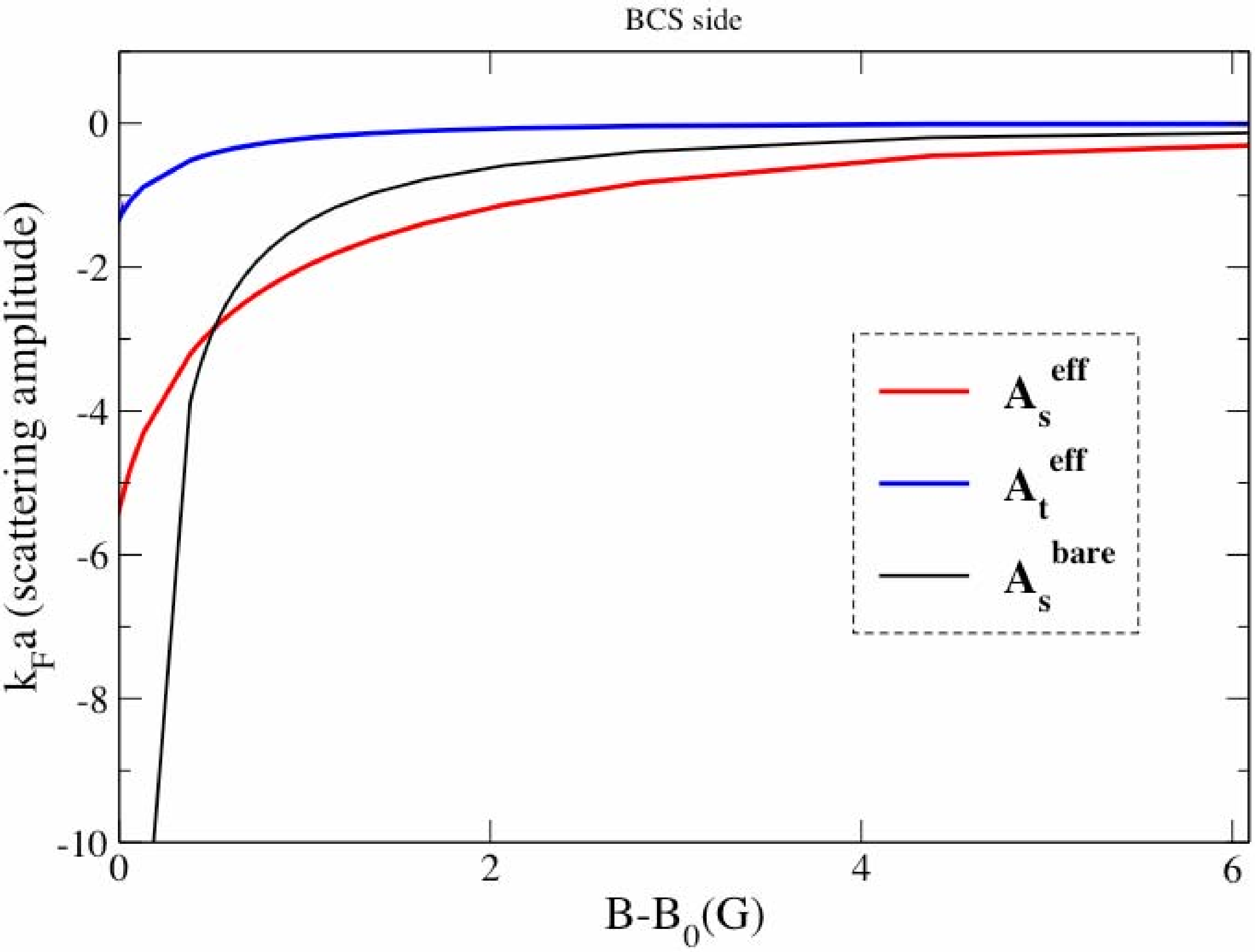
$$T^{s}(\theta,\vartheta) = A^{s}(\theta,\vartheta) - 3A^{a}(\theta,\vartheta)$$: **Singlet**

- **Pairing Interaction in Angular Momentum Channels**

$$\lambda_l = \frac{1}{8}\int_{-1}^{+1} d(\cos\vartheta)\, T^{t,s}(\theta = \pi,\vartheta)\, P_l(\cos\vartheta)$$

Even *l* : *l*-channel Singlet

Odd *l* : *l*-channel Triplet



Same order of magnitude for singlet and triplet pairing channels!

Transition temperatures

- induced interaction model $\longrightarrow$ $T_c \cong 1.13 T_F e^{-1/\lambda_l}$

experimental

singlet $T_c^s \approx 0.7 T_F$

triplet $T_c^t \approx 0.2 T_F$

$$\boxed{\begin{array}{l} T_c^1 \approx 0.5 T_F \\ \\ T_c^2 \approx 0.25 T_F \end{array}}$$

Discussion

- the physics near a Feshbach resonance is highly correlated and many-body theory is appropriate (not 2-body)

- Fermi liquid theory is quite valid at such low temperatures and even strong couplings

- there must be competing superfluid pairing channels in the strong coupling regime, and there is need for a full wave function variational approach to determine the low energy ground state (in progress)

Current - future work

- We are solving the Landau kinetic equation in the hydrodynamic limit to look for sound modes in the system which could be used as a signature to a superfluid transition (also interesting in the normal state)
- Hamiltonian variational approach
- General scattering properties in the induced interaction model - Levinson’s theorem
- Specific heat calculation for transition
- Non-local solutions to the induced interactions to see what effect higher angular momentum channels have in the scattering lengths

Thank you !

Danke!

Merci!

Gracias!

Grazie!