Unit 25

Last Stages of Evolution: Low-Mass Stars

Here we discuss the late stages of evolution after the thermal pulses to the white dwarf.

25.1 Planetary Nebula

- AGB stars of low mass continute to brighten as their (outer) H shell approaches the surface.
- The thermal pulse number is set by the mass of the H envelope and the mass of the CO core.
- If the CO core exceeds about $1.4M_{\odot}$, non-degenerate, non-explosive carbon burning sets in and the AGB phase is over.
- Due to mass-loss processes during the AGB pulses, no star is really able to reach that core mass, because the H-burning shell stops when it's at about $10^{-3}M_{\odot}$ below the surface.
- The mass loss can be quite large, even $10^{-5}\,\mathrm{M_{\odot}\,yr^{-1}}$
- These superwinds that are created speed off at $10\,\mathrm{km\,s^{-1}}$
- The mass loss could be explained by pulsations (Mira variables).
- The gas compression, and subsequent cooling and formation of molecules and dust grains, can trap the outgoing radiation and get carried away.
- Anyway, however it happens, when the pulses stop, and the star evolves to hotter effective temperatures.
- The maximal luminosity depends on the star's initial mass (and that of its envelope) and how much mass it has lost.
- The H shell burning region approaches the surface and the effective temperature increases.
- An even faster stellar wind is produced, up to $2000\,\mathrm{km\,s^{-1}}$, which bumps up against the previous shell ejecta - this produces interesting features in the planetary nebula.
- The shell is dusty and optically thick (masers).
- The envelope is irradiated by UV radiation from remaining hot central star (core).

- The gas gets ionized and recombines quickly, giving distinct emission lines when the stellar remnant reaches about $30,000$K.
- A thin H-burning shell continues until the bluest point on the evolutionary track.
- Then, the H-rich envelope and He-rich layer contract quickly. A few scenarios are now possible:
  1. All nuclear burning shuts off and the star cools as a WD.
  2. The heating of the He from contraction leads to a thermal runaway, and the star goes back near to the AGB (born again). Then does the same stuff and cools as a WD.
  3. The heating of the envelope causes a H-burning runaway and the star is a *self-induced nova*. The process can be dynamic and blow off all H layers to become a DB white dwarf. Or, the process can be quiescent and it will burn H and start to cool down, possibly leading to another nova event.

25.2 White Dwarfs

- Recall all the discussion in Sec. 10.4.
- Degenerate matter obeys polytropic relations $P \sim \rho^\gamma$.
- For non-relativistic particles, $\gamma = 5/3$.
- For relativistic particles, $\gamma = 4/3$.
- The cores of evolved degenerate stars like white dwarfs are dominated by electron pressure rather than ion pressure.
- That's because $\mu_e \approx 2$ and $\mu_{\rm ion} \approx 12$, and $P \propto \mu^{-1}$.
- Why are white dwarfs special?
- Let's simply consider approximations to equilibrium with averages over the star, so

$$\frac{P}{M} = \frac{GM}{4\pi R^4}. \tag{25.1}$$

- For a polytrope, replacing density by its average value

$$P \sim \left(\frac{M}{R^3}\right)^\gamma. \tag{25.2}$$

- The "pressure" term f_p from equilibrium and the EOS, and the "gravity" term f_g from equilibrium, are

$$f_p \sim \frac{M^{\gamma-1}}{R^{3\gamma}}; \qquad f_g \sim \frac{M}{R^4}. \tag{25.3}$$

- Their ratio must be 1 for equilibrium

$$f = \frac{f_g}{f_p} \sim M^{2-\gamma} R^{3\gamma-4}. \tag{25.4}$$

- This is $M^{1/3}R$ for $\gamma = 5/3$, and $M^{2/3}$ for $\gamma = 4/3$.
- So consider a star less than some critical mass $M < M_{\rm crit}$ and non-relativistic electrons. The star can get into an equilibrium by just adjusting R so that $f = 1$.

- If we increase M so that $f > 1$ (more gravity), R must decrease to regain equilibrium (hence more massive WDs are smaller)
- Now consider relativistic electrons.
- We can only get equilibrium by setting the mass to a certain value $M = M_{\text{crit}}$.
- If $M < M_{\text{crit}}$, $f < 1$, and the pressure term is dominant and so the star expands so that the electrons become non relativistic.
- But if $M > M_{\text{crit}}$ and $f > 1$, the gravity term forces the star to contract. But this does not help, because f is independent of R!
- The star collapses without "finding" an equilibrium.
- Clearly, M_{crit} is some limit.
- So again, consider a total degenerate equation of state then (recall Eq. 7.10)

$$P \approx \frac{R}{\mu_e}\rho T + K_\gamma \left(\frac{\rho}{\mu_e}\right)^\gamma . \tag{25.5}$$

- γ depends on density and relativistic effects, being $\gamma = 5/3$ for $\rho \ll 10^6$ and $\gamma = 4/3$ for $\rho \gg 10^6$.
- Using polytropic relationships once can derive a critical mass that governs the future behavior of the core of these dense stars

$$M_{\text{crit}} = \left(\frac{K_{4/3}}{fG}\right)^{3/2} \mu_e^{-2}, \tag{25.6}$$

  where f is the ratio of the mean density to the central density.
- The cricital mass is then identified as the Chandrasekhar mass (Equation (10.43)):

$$\frac{M_{\text{Ch}}}{M_\odot} = \frac{5.836}{\mu_e^2} = 1.456 \left(\frac{2}{\mu_{\text{e}}}\right)^2 . \tag{25.7}$$

- It's also important to see how the central temperature and density depend on this critical mass:

$$\frac{\rho_c}{\mu_e} = \frac{1}{8}\left(\frac{K_{4/3}}{K_{5/3}}\right)^3 \left(\frac{M_c}{M_{\text{crit}}}\right)^2 \approx 2.4 \times 10^5 \,\text{g}\,\text{cm}^{-3} \left(\frac{\text{M}_\text{c}}{\text{M}_\text{crit}}\right)^2 , \tag{25.8}$$

$$T_c = \frac{1}{R}\frac{K_{4/3}^2}{K_{5/3}} \left(\frac{M_c}{M_{\text{crit}}}\right)^{4/3} \approx 0.5 \times 10^9 \left(\frac{M_c}{M_{\text{crit}}}\right)^{4/3} \text{K}. \tag{25.9}$$

- For core masses below critical, maximum temperatures cannot exceed about 500 million K.
- In white dwarfs it is believed that the electrons are relativistic in the central part, but non-relativistic in the outer part.
- This changes the above results quantitatively, but not qualitatively.
- The mass of the core compared to the critical (Chandrasekhar) mass can be distinguised by 4 cases:

Case 1: If $M_c < M_{\text{crit}} \approx M_{\text{Ch}}$ and if there is no significant envelope (from mass loss or just a small original mass), so that M_c will NOT approach M_{Ch} during shell burning, then the core becomes degenerate, will cool, and the star becomes a white dwarf. T_c peaks. If it is a member of a binary system, then it can accrete enough mass to ignite carbon, which will detonate He and destroy the star in a runaway, producing at Type I supernova

Case 2: Initially if $M_c < M_{\rm crit}$ but there remains an envelope such that shell burning M_c can grow to $M_{\rm Ch}$, the core becomes degenerate and cools. However, ρ_c increases with M_c and carbon will be ignited. This will happen for $4 \lesssim M/M_\odot < 8$, and the stars will likely become white dwarfs as well, but C-O white dwarfs.

Case 3: If $M_{\rm crit} < M \lesssim 40 M_\odot$, degeneracy does not happen. The core can heat up even more (because of no degeneracy) and further nuclear reactions can occur. Eventually the core collapses leading to formation of a neutron star and ejection of the envelope. This is a Type II supernova. See next section.

Case 4: If $M_c \geq 40 M_\odot$, the core also will burn C non degenerately. Black hole. See next section.

- In reality, almost all stars born with about $8M_\odot$ or less will lose significant mass and will not reach the interior conditions to ignite carbon.
- Most WDs are observed at $0.6M\odot$ with very little variation.
- The higher-mass stars will lose all of their envelope and become CO WDs.

25.3 Futher WD properties

- It can be shown through simple Virial arguments that white dwarfs cool (approximately) according to the Mestel law
$$\Delta t \propto \left(\frac{L}{M}\right)^{-5/7} \approx \frac{4.5 \times 10^7}{\mu_i} \left(\frac{LM_\odot}{L_\odot M}\right)^{-5/7} \quad \text{[year]}. \tag{25.10}$$
- The Δt represents the time for some change in luminosity.
- So higher mass WDs cool more slowly, due to more storage of energy.
- Increasing the ionic mean molecular weight decreases the evolutionary time, since there are fewer ions in that case.
- Roughly, for a WD to reach 1/1000th of the solar luminosity, it would take 1 billion years.
- A more precise cooling law would treat the ions more properly, since the steady decreae in temperature causes Coulomb interactions to become more important.
- The specific heats ratio grows and the ions form a lattice: this is *crystallization*.
- The crystallization obviously affects the EOS, and therefore the cooling times.
- The envelopes of WDs canonically predicta He layer of $\sim 10^{-2} M_{\rm tot}$ above the CO core, surrounded by a H envelope of mass $\sim 10^{-4} M_{\rm tot}$.
- Mass loss causes these notions to change.
- Metals are rarely observed due to atomic diffusion processes.
- Typically there are about 4 H WDs for every 1 non-H WD, but this varies widely with effective temperature, as evolutionary processes are still ongoing.
- The outer layers determine the opacity, and hence the cooling times.
- As WDs cool, they can develop convection zones as well.
- WDs can also be He core stars from an RGB progenitor that lost its envelope.
- There are also O-Ne WDs from higher-mass progenitors.

25.4 Type Ia supernovae

- Type I do not have visible hydrogen in its spectrum.
- The progenitor of these events are CO WDs that have accreted mass from a companion to exceed the critical mass .
- The companion is likely on the RGB or AGB where mass can be transferred effectively.
- In one scenario two similar stars evolve through a common envelope, and the two CO WDs merge through angular momentum loss from gravitational waves.
- The resultant object is higher than the critical mass.
- This is the *double-degenerate* channel.
- No observational evidence of these massive systems has yet to be shown.
- In the *single-degenerate* case, the events that happen depend strongly on the mass-accretion rates onto the low-mass star.
- For low rates, H burning on top of He layers produces an electron degenerate scenario, which can undergo explosive flashes.
- These are typically the classical *Novae*.
- Building up enough mass this way to reach the critical limit takes longer than a Hubble time, however.
- At moderate rates it can be shown that an explosion would occur for *sub-Chandrasekhar* mass objects.
- If enough He is accreted (either through H-burning above or from an He-rich companion), violent He-ignition can occur which can detonate the CO core.
- Basically, WDs are unable to regulate their temperature the way they did in earlier stages (expanding and cooling). So runaway fusion occurs.
- The explosion models must give about 10^{51}erg, as well as the production of heavy elements, such as lots of nickel.
- The explosive event is a shock wave whose speed depends on densities and abrupt thermal changes.
- Typical SNIa light curves rise rapidly and after about 20 days the light fades monotonically, with slighly different behavior in different bands.
- Almost all of them have a maximum absolute magnitude of -20 which declines linearly within 15 days after maximum.
- Thus, they are excellent standard candles.

Unit 26

Last Stages of Evolution: High-Mass Stars

26.1 Nuclear burning

- Now we consider stars greater than $15M_{\odot}$.
- If we only focus on the core, the processes in the late stages follow the simple causes and effects of

$$\ldots \rightarrow \text{nuclear burning} \rightarrow \text{fuel exhaustion} \rightarrow \text{core contraction} \rightarrow \text{core heating} \rightarrow \ldots \tag{26.1}$$

- At 500 million K, carbon burning can take place in the core, converting into Mg, Ne, and Na.
- Neutrinos from pair annihilation contribute a substantial amount to the luminosity (energy loss), even more than the nuclear burning.
- So the star contracts to make up the loss with gravitational energy.
- The convective C core is less massive for more massive stars, as neutrinos cause a considerable amount of mass loss.
- Eventually the burning moves to a shell of C and s-elements are produced.
- The core is now about 70% O, 25% Ne, and the rest Mg.
- Ne burning sets in .
- Oxygen burning can set in at 1.5 billion K.
- Eventually, the photons released in such reactions have such high energies that they can photo-disassociate surrounding nuclei.
- A large number of neutrons begin to be produced.
- Silicon burning sets in through interactions with alpha particles.
- Then it moves into the shell.
- The process stops at the iron group, and the last reaction is ^{52}Fe capturing an alpha particle to make nickel.
- Further reactions would **require** energy to proceed. All reactions now are balanced by their inverse reaction.

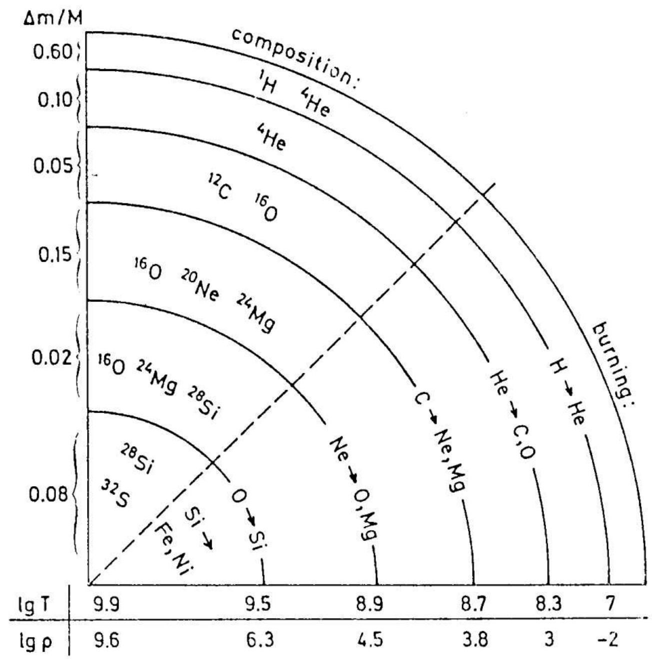

Figure 26.1: Illustration of the "onion-skin" structure in the interior of a highly evolved massive star. From Kippenhahn and Weigert [1990].

- Figure 26.1 shows how the core builds up its layers this way.
- For a $15M_\odot$ star, the burning of each successive element is rapid:

$$\mathrm{H}(10^7); \mathrm{He}(10^6); \mathrm{C}(10^3); \mathrm{Ne}(10^1); \mathrm{O}(10^1); \mathrm{Si}(10^{-1}), \tag{26.2}$$

  where the times are in years.
- Since all this happens so quickly, the surface is basically "frozen in" and the star does not move from right to left until it explodes.

26.2 Type II supernova - core collapse

- The core of the massive star is hot, $T_9 \approx 10$, and electrons are relativistic.
- Simply, the ratio of specific heats, γ, drops below 4/3 and the star is in an unstable configuration.
- After silicon burning, electrons are captured by protons

$$p^+ + e^- \longrightarrow n + \nu_e, \tag{26.3}$$

  producing a lot of electron neutrinos and neutrons.
- Neutrinos carry away energy, cool the core, and pressure drops.
- Photo-disintegtration produces many free α particles.

$$^{56}\mathrm{Fe} \longrightarrow 13^4\mathrm{He} + 4\mathrm{n} - 100\ \mathrm{MeV}. \tag{26.4}$$

- The loss of free electrons also reduces the pressure.

- The core collapses from overlying weight on the scale of a few seconds.
- This collapse halts when the neutron degeneracy pressure kicks in ($\rho \approx 10^{15}\,\mathrm{g\,cm^{-3}}$) - note that this is nuclear matter density! And 40km in size!
- Energy release of about 10^{53}erg from change in gravitational energy $GM^2/\Delta R$. Where does it go?
- This is as much light as a galaxy shines at for decades.
- In one scenario, most of the light is not released however, but goes into the kinetic energy of a shock.
- This propagates outward into the outer core region that is still collapsing.
- Naively, this might blow off the outer layers, but that does not happen.
- In the more accepted scenario, the increased core density causes it to be optically thick to neutrinos.
- They begin to deposit their energy into the material.
- This causes the outward shock that blows off the star's layers.
- This is the core-collapse Type II event.
- Heavy nuclei are created through neutron capture (s and r processes)
- Just to note, the neutrino cross section is extremely small, and its mean free path is

$$\ell_\nu = \frac{1}{n\sigma_\nu} \approx \frac{1}{\mu_e A}\left(\frac{\rho}{\mu_e}\right)^{-5/3} 1.7 \times 10^{25}\,\mathrm{cm}. \quad (26.5)$$

- $\mu_e = 2$, $A = 100$, and a density of about 10^{10}, $\ell_\nu \approx 10^7$cm, which is contained withing the collapsing core
- So neutrinos do not escape without interaction.
- About 1% of the energy goes into the outward motion, and 1% of that gets released as photons
- So only about 10^{49} erg of energy gets radiated over a few months
- We observe these supernova because hydrogen lines are present.
- The collapse occured when a H-rich envelope still existed.
- If the initial star was $M \geq 25M_\odot$, the remnant is likely a neutron star.
- For higher masses, it is too much to be supported by neutron degeneracy pressure.
- The object then becomes a **black hole**.
- If the initial star was over $100M_\odot$, or a core He mass of about $40M_\odot$, a different scenario might take place.
- After He burning the thermal environment produces electron-positron pairs, reducing the specific heat so that $\gamma < 4/3$.
- The star immediately starts to collapse and subsequent burning is not enough to halt the collapse.
- The star produces a black hole in a *pair-instability* supernova.

26.3 Neutron star

- Masses between 1.2 and $2.5M_{\odot}$ and $R \approx 10$km
- The mass-radius relationship for neutron stars in each case is (derived from our polytropic equations):

$$M = \left(\frac{15.12\,\mathrm{km}}{\mathrm{R}}\right)^3 \mathrm{M}_{\odot};\ \mathrm{non-relativisitic} \quad (26.6)$$

$$M = 5.73\mathrm{M}_{\odot} \equiv \mathrm{M}_{\mathrm{Ch}}^{\mathrm{NS}};\ \mathrm{relativisitic} \quad (26.7)$$

- However, tne maximum mass of a neutron star depends on the existence of a general-relativistic instability (interactions between nucleons)
- This likely takes place well before the M_{Ch} for a neutron star, hence the $\sim 3M_{\odot}$ limit
- Most neutron star observations are in a very narrow range of masses $M_{\mathrm{ns}} = 1.35 \pm 0.04M_{\odot}$.
- The neutron-degenerate material creates mostly an isothermal environment.
- They cool down faster than WDs.

26.4 Black hole

- For Type II remnant $> 2.5M_{\odot}$, or a progenitor $> 25M_{\odot}$, a black hole is produced.
- The radius, determined from the escape velocity, is:

$$R = 2\frac{GM}{c^2} = 2.95 \times 10^5 \frac{M}{M_{\odot}}\ [\mathrm{cm}]. \quad (26.8)$$