Unit 20

The Terminal-Age Main Sequence and Subgiant Branch

20.1 Description of evolution movies

- From here on you can use the evolution movies to study the various stages that will be discussed.
- There are 2 stars considered of different masses, $M_1 = 1\ M_\odot$ and $M_7 = 7\ M_\odot$.
- Each star has 2 associated movies, one of the time evolution of general properties, and one depicting the time evolution of interior profiles.
- They will be referred to as 1Ma, 1Mb, 7Ma, 7Mb, where "a" and "b" denote the history or profile movie, respectively.
- A particular model in a movie will be denoted by, e.g., 1Mb-200, so the 200th model of movie 1Mb (not megabyte!)

20.2 TAMS

- The terminal-age main sequence (TAMS) is roughly the stage when the all the hydrogen is exhausted from the very center of the star (not necessarily though throughout the whole core).
- This is about at model 280 for the 1 $M_\odot$ star and model 1200 for the 7 $M_\odot$ star.
- Observationally, we can only really talk about stars greater than about $0.8 M_\odot$, since less massive stars are still on the main sequence (of course, we can use theory to talk about lower-mass stars).
- To summarize the previous material, as stars evolve on the main sequence they go **above** the ZAMS up and to the right or left depending on mass.
- Notice that this is only the case for chemically inhomogeneous models (if a star remained mixed and the mean molecular weight increased with time throughout, it would evolve below the ZAMS for a given mass, as we saw in our homology relations earlier).
- When the central hydrogen content reaches about $X_c \approx 0.05$ (see points 3 in Figure 20.1 and 2 in Figure 20.2) for stars above about $1.1 M_\odot$, the opacity is dropping (increased He), and the envelope luminosity is greater than the energy generation in the core (not much H left!)

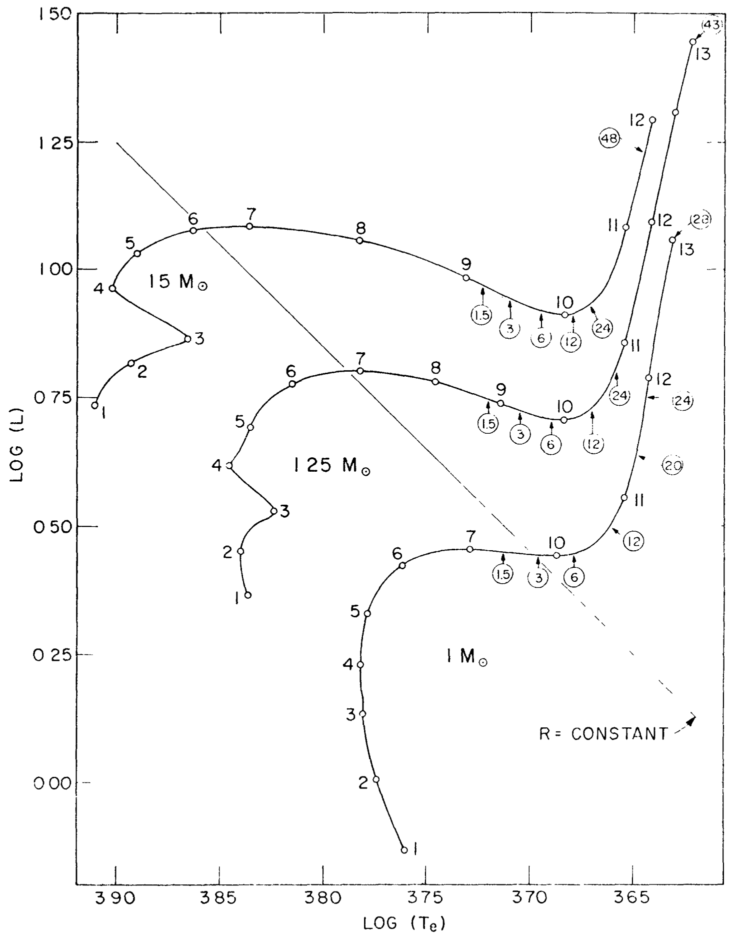

Figure 20.1: Evolutionary tracks for low-mass Pop. I stars. Basically, points 1-3 are the ZAMS to TAMS. From Iben [1967b].

- The star shrinks on a Kelvin-Helmholtz time scale to make up for the excess luminosity, then the effective temperature increases a bit (see § 20.3). This is a called *overall contraction.*
- This causes the little wiggle (or "hook") on the HR diagram. Low-mass stars do not show this because they do not need to contract so much because the luminosity was never that great. See Figure 20.1 (points 3 to 4) and 7Ma-1200-1220.
- The main difference is the higher-mass stars have a convective core.
- The higher-mass cores deplete H over larger regions, and thus the contraction is more drastic as to maintain nuclear burning at the right level.
- Nonetheless, near the TAMS as $X_c \to 0$ for all masses:
  - Core is mainly filled with inert helium (too cool to burn, needs 10^8K)
  - But there is a large T_c and μ
  - Core is isothermal since $\varepsilon \to 0$ and then $\mathrm{d}T/\mathrm{d}r \to 0$ (see Equation (13.18)).
  - The temperature at the core boundary is high enough, however, to ignite leftover hydrogen
  - The contraction has pulled in H to hotter and denser regions (still the shell), so the shell ignites!
  - The shell burns H and adds He to the core, whose mass increases and it contracts more, heating it up (eventually to ignite He later on)

- All of this emphasizes the **Shell-burning law**: When a region within a burning shell contracts, the region outside the shell expands; when the region inside the shell expands, the region outside the shell contracts. We will see this behavior again and again.
- Despite many efforts, and the fact that numerical experiments show that this law is true, it is not obviously clear why it is the case.

20.3 Schönberg-Chandrasekhar Limit

- Let's look at what's happening in the core at the TAMS. Can it support the growing mass in the overlying layers from outer core burning?
- In 1942 Chandrasekhar and Schönberg studied hydrostatic equilibrium for an isothermal He core and an ideal equation of state.
- Assume constant core temperature, and that the envelope provides a pressure $P_{\rm env}$. The goal is to compute the core pressure P_c.
- Consider hydrostatic equilibrium and multiply both sides by $4\pi r^3$ and integrate in core (recall Equation (**??**)):

$$\int_0^{R_c} 4\pi r^3 \frac{\mathrm{d}P}{\mathrm{d}r}\mathrm{d}r = -\int_0^{R_c} \rho \frac{Gm}{r^2} 4\pi r^3 \mathrm{d}r = E_{g,\rm c} \tag{20.1}$$

- Integrate by parts and use ideal gas law

$$4\pi R_c^3 P_c - 3\frac{M_c k_{\rm B} T_c}{\mu m_{\rm u}} = E_{g,\rm c}. \tag{20.2}$$

- If we assume that the density is the mean core density $\rho \approx 3M_c/4\pi R_c^3$, then

$$E_{\rm g,c} \approx -\frac{3}{5}\frac{GM_c^2}{R_c}. \tag{20.3}$$

- Solving everything for P_c, we get

$$P_c = \frac{3}{4\pi R_c^3}\left(\frac{M_c k_{\rm B} T_c}{\mu m_{\rm u}} - \frac{1}{5}\frac{GM_c^2}{R_c}\right) \tag{20.4}$$

- The core pressure must match the envelope pressure for equilibrium, and must adjust its radius to do so.
- Can it always do so? Its maximum value is when

$$R_c = \frac{4}{15}\frac{GM_c \mu m_{\rm u}}{k_{\rm B} T_c}, \tag{20.5}$$

which gives

$$P_{\rm c,max} = \frac{10125}{1024 G^3 M_c^2}\left(\frac{k_{\rm B} T_c}{\mu_c m_{\rm u}}\right)^4. \tag{20.6}$$

- As you can see, as the core mass increases, the core pressure will drop and at some point may fall below the envelope pressure.
- The mass at which this happens is the Schönberg-Chandrasekhar limit.
- We know from hydrostatic equilibrium that $P_{\rm env} \propto M^2/R^4$.

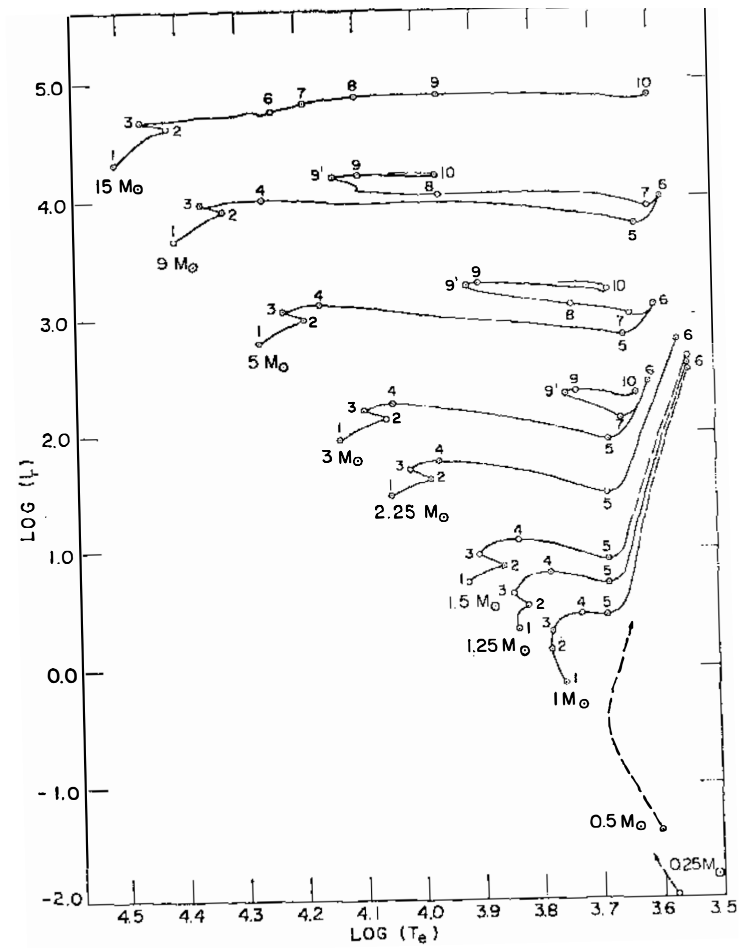

Figure 20.2: Paths in the HR diagram for a range of masses and solar metallicity. From Iben [1967a].

- From homology, we can find that $P_{\text{env}} \propto T_c^4/M^2$
- So the pressure at the surface of the core is independent of the core size.
- Using the right coefficients, it is then easy to show that

$$\frac{M_c}{M} \approx 0.37 \left(\frac{\mu_{\text{env}}}{\mu_c}\right)^2. \tag{20.7}$$

- If $\mu_{\text{env}} = 0.6$ and $\mu_c \approx 1$ (at solar composition), then the limit is roughly

$$\frac{M_c}{M} \approx 0.13. \tag{20.8}$$

- Above this limit, which will likely occur for stars greater than $3M_\odot$:
  - the isothermal core contracts rapidly
  - the density increases, the temperature increases and nuclear reactions speed up in the shell
  - This pushes in both directions and mass is lost in the shell, and burning is in a thin shell
  - Even though the energy rate increases, the luminosity decreases a bit because of the mass loss in the shell

  - Since the timescale is faster than the nuclear one, the stars become redder very quickly
  - This leads to observational Hertzsprung gap (points 4 to 5 in Figure 20.2)
  - Not many stars have time to be "observed"
  - These are subgiant stars on the way to the bottom of the red-giant branch
  - After the wiggle for our 7 $M_\odot$ model, the core He mass is about 1 $M_\odot$, above the limit computed above.

- For low-mass stars ($\leq 1.3 M_\odot$), the helium core is somewhat degenerate and higher pressures are present, so this limit is not applicable and the approach to the RGB is slower.
- For higher-mass stars, the contraction happens very quickly and isothermal cores never actually have time to set in.

20.4 The subgiant branch

- To summarize the above once again, in general, the move across the H-R diagram to the right defines the subgiant branch (SGB).
- These are models 300-330 for the 1 $M_\odot$ star, and 1200-1500 for the 7 $M_\odot$ star.
- In general, cores are shrinking, envelopes are expanding, and surface temperatures are being reduced.
- Higher-mass stars have He core masses above the C-S limit.
- The envelope is adjusting to a new H-burning shell.
- The luminosity is larger as the burning takes place at a higher temperature than it was in the core
- With a large luminosity the shell has a difficult time radiating it (it will eventually become convective)
- But right now it absorbs the luminosity, heats up, and expands
- The Virial theorem shows some of the energy goes into expansion, not all of it makes it to the surface
- The slope of the luminosity in this move across the HR diagram depends on mass
- This stage should happen over the timescale of shell burning, a nuclear timescale ...
- But other influences may affect it, as will be discussed