Unit 19

Evolution on the main sequence

19.1 Low-mass stars

- The time of arrival on the main sequence is known as the ZAMS - zero-age main sequence.
- "Where" it ends up depends only on mass and chemical mixture.
- The lower mass limit is roughly about $0.1M_{\odot}$.
- The upper mass limit is about $100M_{\odot}$.
- The mean molecular weight changes a lot. Consider fully ionized H in core at beginning at ZAMS (see Equation (5.10)):
$$\mu = \frac{4}{3 + 5X - Z} \simeq 0.61. \tag{19.1}$$
- As all of it gets converted to He, we then have
$$\mu = \frac{4}{3 + 5(0) - Z} \simeq 1.3. \tag{19.2}$$
It more than doubles!
- This change (increase) in mean molecular weight causes changes in other things. It can be shown from homology that the hydrogen-burning luminosity $L_{\mathrm{H}} \propto \mu^7$.
- Also note that the opacity is reduced, as He is less opaque than H under the same conditions.
- The number of free particles also decreases, as does the pressure.
- A low-mass star slightly contracts its core and heats up. Firstly, ρ increases by the core contracting. As this happens gravitational energy gets released according to the Virial theorem, which partly goes to increasing the thermal energy of the core - increased T.
- This must increase the pressure to account for the "heavier" material. Indeed, according to the equation of state, if P increases, and μ increases, then ρT must certainly increase.
- The nuclear energy generation increases and then so does the luminosity.
- This causes a slow increase of the star's luminosity over the whole MS phase.
- Also, as the core contracts, the surface radius increases, slightly for low-mass stars, more rapidly for high-mass stars. This can affect the effective temperature (see below).

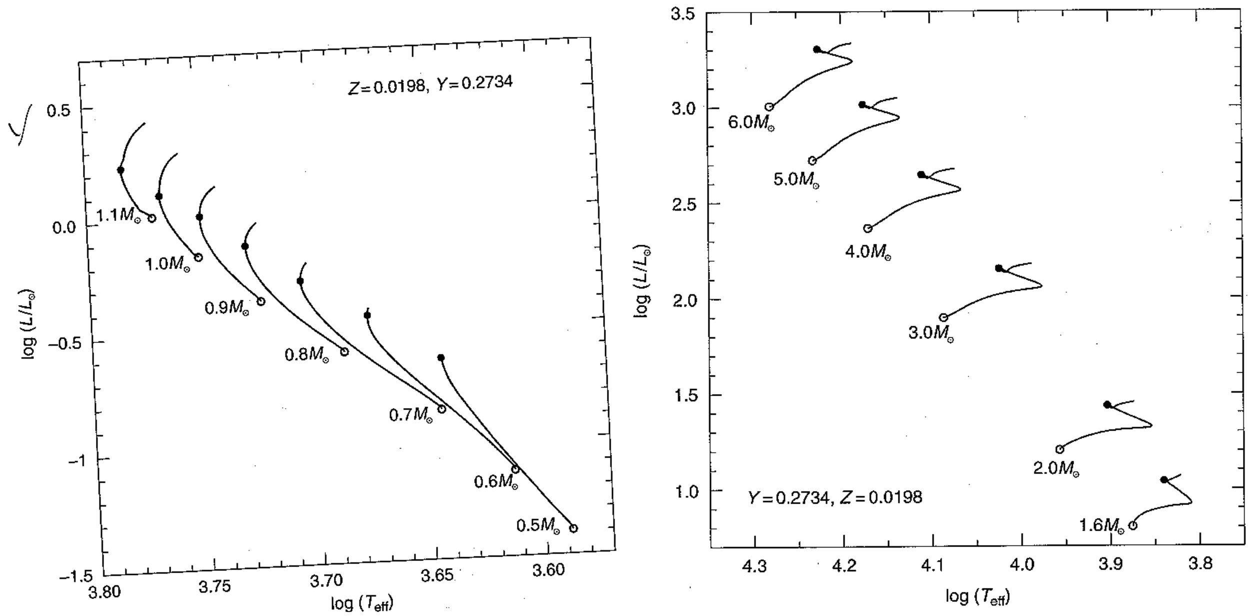

Figure 19.1: (Left): Low-mass star evolution on the main sequence. (Right): The same but higher-mass stars.From Salaris and Cassisi [2006].

- See Figure 19.1 for the MS evolution.
- Figure 19.2 (left) shows the core hydrogen mass fraction as a function of time. The burning region extends out to a significant radial distance.
- How much has the solar luminosity changed over time? Calculations (including homology ones) show that the Sun's luminosity compared to its ZAMS luminosity L_0 is $L \simeq 1.26L_0$. This means it was about 25% less luminous than it is today, which has/had implications for the Earth.
- All of the main properties of the Sun from its ZAMS point until today are shown in Figure 19.3.
- The important things to note are the increase in density and decrease in X. One would think the temperature increase too would really increase the nuclear energy generation rate ϵ_c.
- But as we saw (Equation (3.9)), it not only depends on T but also on X^2, so it is somewhat halted by the decreasing hydrogen amount over time.

19.2 High-mass stars

- The main difference in these stars is the increased temperature in the core, as in Figure 19.5.
- Thus, the CNO cycle is the dominant luminosity source.
- This has the effect of concentrating the the luminosity production in the inner 10% of the mass for a $10M_\odot$ star, compared to about 30% for a $1M_\odot$ star.
- The other effect is a steep temperature gradient in the inner regions due to the high flux. Thus, convection kicks in.
- This region becomes fully mixed chemically, as in Figure 19.2, right.
- The outer regions are radiative, since the ionization regions are very far out in the atmopshere compared to low-mass stars.

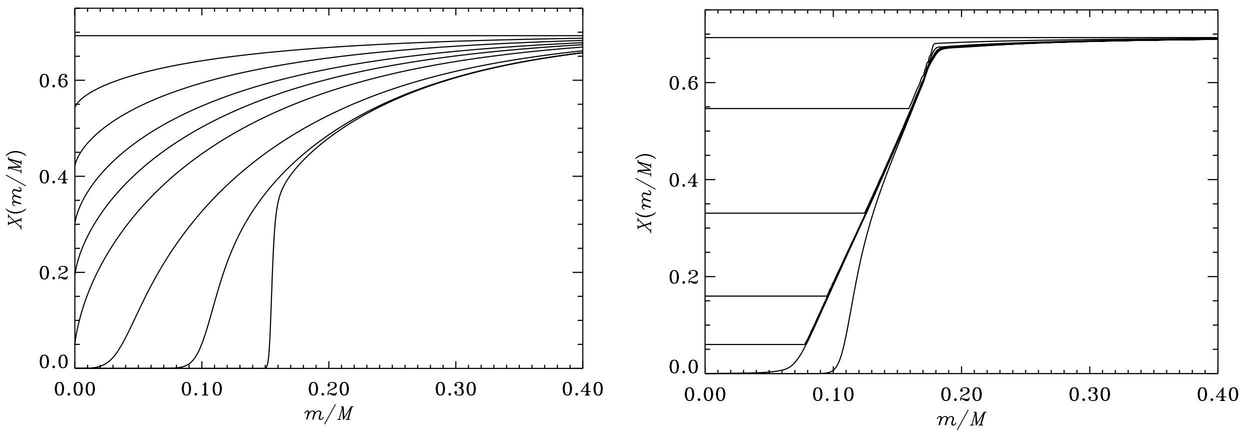

Figure 19.2: (Left): Hydrogen profiles showing the gradual exhaustion of hydrogen in a $1M_\odot$ star. The homogeneous initial model consists of a mixture with a hydrogen abundance by mass of 0.699. X as a function of the mass fraction $m/M_\odot$ is plotted for nine models which correspond to ages of 0, 2.0, 3.6, 5.0, 6.2, 7.5, 9.6, 11.0 and 11.6 times 10^9 years, after the onset of hydrogen burning. The model at 5.0×10^9 years corresponds roughly to the present Sun, whereas the last two models are in the shell hydrogen burning phase. (Right): The same but for a $2.5M_\odot$ star. The lines show the hydrogen profiles for models of age 0, 1.5, 3.1, 4.0, 4.4, 4.6, and 4.8 times 10^8 years. From Christensen-Dalsgaard [2008].

- As evolution occurs, the star gets brighter because of the strong dependence of L on μ.
- At the same time, the effective temperature shows a monatonic decrease, as in Figure 19.1, right. This is due to the increasing radius, which increases faster than the luminosity (especially compared to lower-mass stars).
- If the core of the star grows in size due to convective overshoot, it will also extend the MS lifetime and make the star brighter.
- Figure 19.2 (right) shows the core hydrogen mass, and note the shrinking convective core of the higher-mass star over time.
- One of the main reasons for this is the reduced opacity as H is converted into He and the electron scattering processes decreases.
- Also note in the higher-mass star in Figure 19.2 that since hydrogen burning is negligible at the edge of the convective core during the main-sequence phase, the hydrogen profile established during this phase reflects the decrease in the extent of the core. In contrast, the last model is in the hydrogen shell-burning phase, the helium core has grown substantially beyond the smallest extent of the convective core.
- Figure 19.4 shows a comparison of core mass size for a $1M_\odot$ and $20M_\odot$ model in both relative and absolute visualizations.
- Note in this figure that even though the luminosity saturates at a high value very close to the center, the convective region of the core is quite extensive.
- The size of the convective core increases as the mass of the star increases too, due to the higher central temperatures.
- We can estimate the main-sequence lifetime of a star. If $\varepsilon_{\rm H}$ is the energy per unit mass per unit time of hydrogen burning, we know that

$$\tau_{\rm MS} \propto \frac{q_c \varepsilon_{\rm H} M}{L}, \tag{19.3}$$

  where q_c is some fraction of the stellar hydrogren mass that actually participates in nuclear burning.

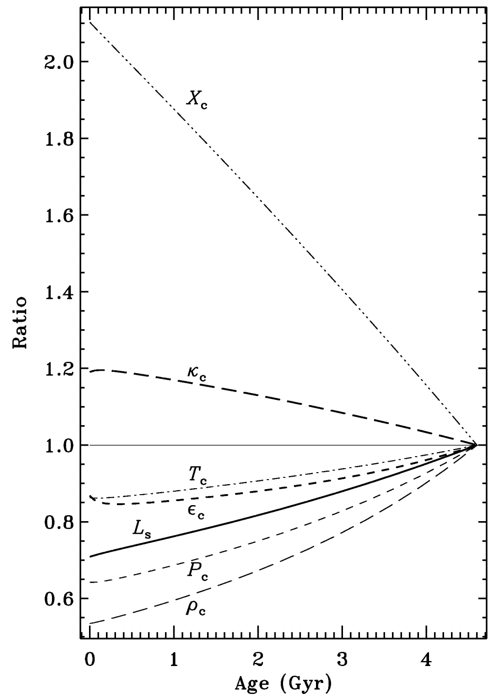

Figure 19.3: The changes in the solar properties at the center of a solar model. All variables are normalized with respect to the present Sun. From Christensen-Dalsgaard [2008].

- If ε_H and q_c are roughly independent of total stellar mass, and we assume that $L \propto M^{\gamma}$ as we showed before, then
$$\tau_{MS} \propto M^{-(\gamma-1)}, \tag{19.4}$$
where we found $\gamma \approx 3-5$, and the relation is written to emphasize that the exponent is always negative, and main-sequence lifetime is inversely related to mass.

A note about very low mass stars

- Stars below about $0.3 M_{\odot}$ are fully convective on the MS.
- They have large opacities due to low temperatures and very high densities.
- The densities are high because the stars need to contract to build up high enough temperatures for nuclear fusion.
- Only the PP-I chain can operate, so helium-3 is never really destroyed.
- But at the lower mass limit (high ρ), electron degeneracy kicks in.
- Conduction is very efficient here, which then cools the core below the minimum ignition temperatures.
- Some lithium or deuterium is burned, but these objects become brown dwarfs and cool down like white dwarfs.
- The difference between them and WDs is that they are fully mixed chemically, and their degenerate electrons do not move relativistically.

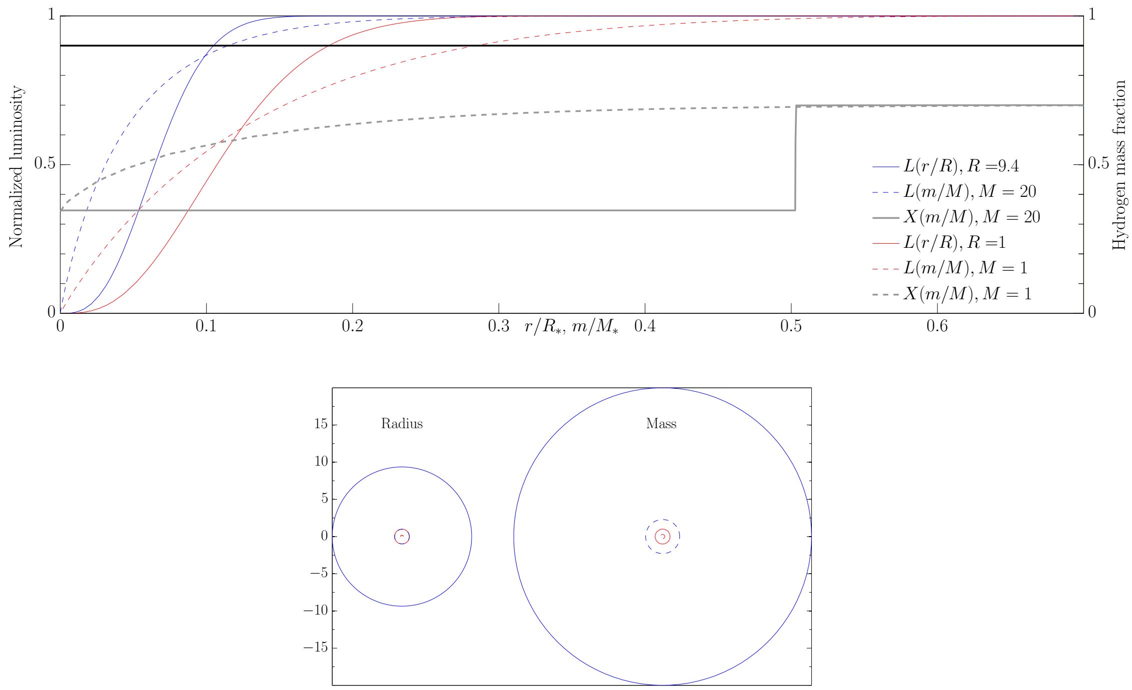

Figure 19.4: Comparison of core sizes for 2 models ($1M_\odot$ and $20M_\odot$) both with $X_c \approx 0.35$ on the main sequence. The top panel shows the normalized luminosity as a function of normalized radius and mass. A horizontal line at $0.9L_{\rm max}$ will be used to define the approximate core boundary. The blue lines are for the massive model, and red for the less massive one. The low-mass model has a core that is fractionally larger than the core in the high-mass model. The gray solid and dashed lines are the hydrogen mass fraction, given on the right y axis. In the bottom panel, the surface (solid lines) and core (dashed lines) boundaries are shown to scale in absolute masses and radii. On the left for the $9.4R_\odot$ model (blue), we see the core boundary in this space is almost exactly the same size of the entire low-mass star (red). In terms of mass, the core size of the massive star is larger than one solar mass.

19.3 Summary of main-sequence properties

- In general, the star regulates its nuclear burning rate to maintain hydrostatic equilibrium.
- If the rate increases for some reason, the star expands, thereby decreasing its temperature and density, reestablishing equilibrium.
- Mass rules of thumb:
  - Below about 1.3 $M_\odot$ convective envelopes, above, radiative envelopes
  - Below about 1.2 $M_\odot$ PP chain, above, CNO cycle
  - Below about 1.5 $M_\odot$ late-type stars (F, G, K, M), above, early-type (O, B, A)
- Structure rules of thumb:
  - Low-mass cores
    * PP chain is sufficient to balance gravity
    * Luminosity is not too steep, and energy flux is moderate
    * Radiation is enough to carry out the luminosity from the core
    * Core is radiative
    * Since the PP chain has a low temperature dependence, the region of burning is relatively a large mass fraction of the star, as in Figure 19.2, left.

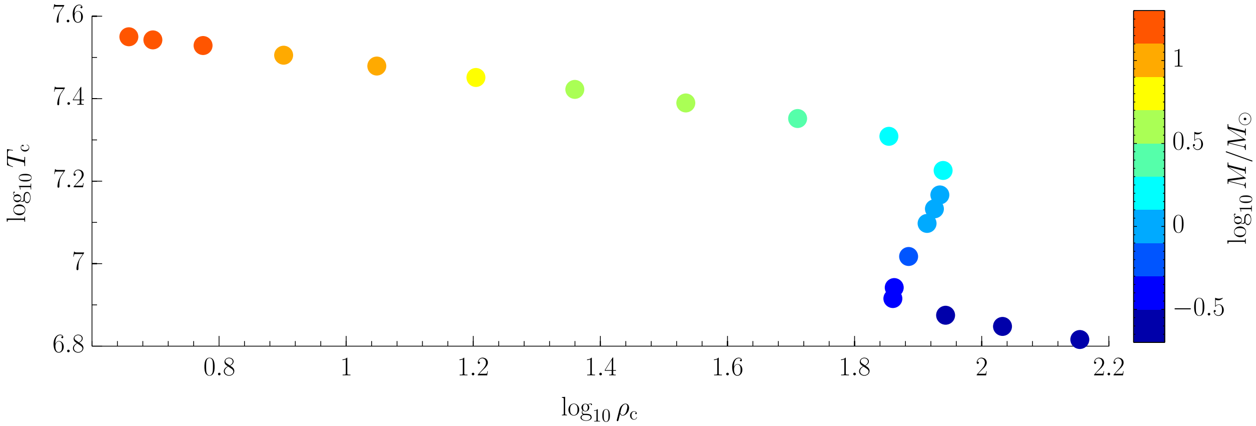

Figure 19.5: The central temperature and density for various `MESA` models of mass given by the colorscale. All models are just on the main sequence when $X_c = 0.68$.

- High-mass cores:
  - CNO cycle is necessary to balance gravity
  - High-temperature sensitivity ($\sim T^{20}$) means a very central energy generation region
  - Luminosity is very steep in core with a high flux
  - Temperature gradient is very steep, convection sets in
  - A convective core develops, which is very efficient
  - So efficient that here it equals the adiabatic gradient
  - Core mixing due to convection removes gradients in composition
  - The core temperature and density for different masses is shown in Figure 19.5.
  - Note the strong variations at a little above 1 solar mass stars, where convective cores start to appear.
- Low-mass envelopes:
  - Opacity is rather large because of hydrogen and helium ionization zones and corresponding bound-free transitions
  - Convection is needed to carry the radiative flux through the region; steep temperature gradient
  - Below about $0.3\,M_\odot$ the entire star is convective
- High-mass envelopes:
  - Hydrogen and helium ionized so rather low opacities
  - The radiative flux is carried out by radiation. Radiative envelope.
  - In very massive stars $> 10\,M_\odot$, some opacity peaks due to ionized iron and nickel can cause thin convection zones near the surface

- MS location rules of thumb (as we saw in homology relations):
  - Higher He content results in more luminous and hotter MS tracks.
  - The MS lifetime decreases with increasing He content.
  - Higher metallicity makes the star cooler due to increased opacity.
  - Alpha elements (O, Ne, Mg, Si, S, Ca, Ti, ...) that are enhanced in metal-poor stars produce fainter, cooler MS tracks.
  - Changing the mixing length, or convective efficiency, affects the MS.
  - There is no effect on the luminosity, but an increased effeciency sets up a lower thermal gradient.

  - This increases the effective temperature, and therefore the radius decreases.

- Evolution (on main sequence) rules of thumb:
  - ZAMS to TAMS lifetimes are much shorter for high-mass stars
  - Tracks on HR diagram are vertical for low-mass, and diagonal for high-mass stars
  - Luminosity increases due to increase in molecular weight in core
  - Low-mass stars have abundance changes in core that is smooth
  - High-mass stars have discontinuous changes due to convective mixing (although core shrinking can smooth out the discontinuities)
  - Below about $0.3\,M_\odot$ the core is completely mixed and stars live long
  - Near the end of main-sequence evolution:
    * Nearly isothermal core with zero luminosity
    * Core is very hot from high μ
    * At core boundary temperature is high enough for shell burning to occur
    * The high T and large volume of the burning region leads to high shell L
    * No thermal equilibrium, envelope expands