- Anyway, we continue with a polytrope $n = 1.5$ where from before

$$K = K_{\text{poly}} = N_{3/2} G R M^{1/3}. \tag{18.19}$$

- Matching these two $K$s and solving for the effective temperature gives

$$T_{\text{eff}} = N_{3/2}^{\xi} \left(\frac{G\mu}{k_{\text{B}}}\right)^{\xi(\eta+1)} \left(\frac{\kappa_0^{\text{ph}}}{\alpha+1}\right)^{-\xi\eta} R^{\xi(1-2\eta)} M^{\xi(\eta+1/3)}, \tag{18.20}$$

  where $\xi = 1/[1+\eta(\beta+1)]$.

- Solving for the radius $R$ and using $L = 4\pi\sigma R^2 T_{\text{eff}}^4$ gives an expression for the luminosity, as well as other similar-type relations we've computed before. The results are

$$\begin{aligned} T_{\text{eff}} &\simeq 2400\,\text{K} \left(\frac{M}{M_\odot}\right)^{0.2} \left(\frac{R}{R_\odot}\right)^{0.06}, \\ \frac{L}{L_\odot} &\simeq 0.03 \left(\frac{M}{M_\odot}\right)^{0.8} \left(\frac{R}{R_\odot}\right)^{2.2}, \\ \frac{L}{L_\odot} &\simeq 0.03 \left(\frac{M}{M_\odot}\right)^{-7.0} \left(\frac{T_{\text{eff}}}{2400\,\text{K}}\right)^{40.0}. \end{aligned}$$

- Note that
  - The dependence of the temperature on parameters is very weak: these stars all have similar temperatures.
  - This describes the contraction of convective stars as they form.
  - This is known as the *Hayashi track*.
  - The slope of the luminosity (the sign) is wrong a bit, but it's pretty good.
  - There cannot be stars to the right of the Hayashi track (with a lower effective temperature).
- A glance at the approach to the main sequence along the Hayashi track is shown in Figure 18.2.

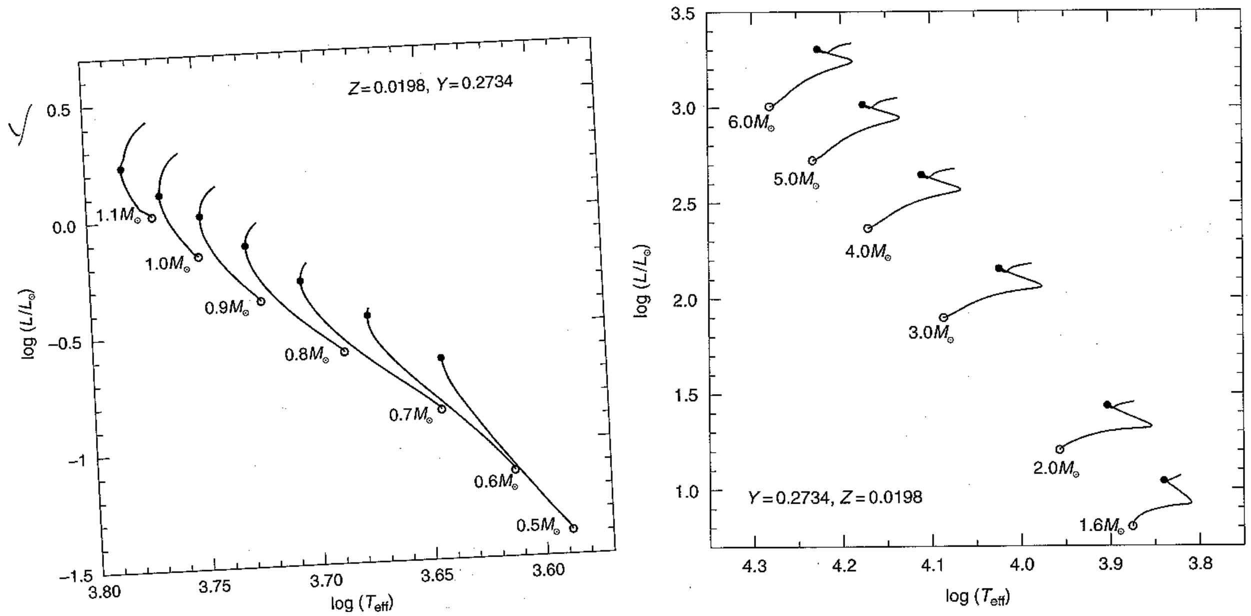


**Figure 18.3:** (Left): Low-mass star evolution on the main sequence. (Right): The same but higher-mass stars.From Salaris and Cassisi [2006].

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## 18.3 Evolution on the main sequence

### 18.3.1 Low-mass stars

- The time of arrival on the main sequence is known as the ZAMS - zero-age main sequence.
- "Where" it ends up depends only on mass and chemical mixture.
- The lower mass limit is roughly about $0.1M_\odot$.
- The upper mass limit is about $100M_\odot$.
- The mean molecular weight changes a lot. Consider fully ionized H in core at beginning at ZAMS (see Equation (5.10)):
$$\mu = \frac{4}{3+5X-Z} \simeq 0.61. \tag{18.21}$$
- As all of it gets converted to He, we then have
$$\mu = \frac{4}{3+5(0)-Z} \simeq 1.3. \tag{18.22}$$
It more than doubles!
- This change (increase) in mean molecular weight causes changes in other things. Note that the opacity is reduced, as He is less opaque than H.
- The number of free particles also decreases, as does the pressure.
- A low-mass star slightly contracts its core and heats up. Firstly, $\rho$ increases by the core contracting. As this happens gravitational energy gets released according to the Virial theorem, which partly goes to increasing the thermal energy of the core - increased $T$.