Unit 16

Convection 2

16.1 Other formulations of the instability

- It is convenient to look at the stability criterion in terms of temperature gradients instead.
- Using Eq. (15.8) and the ideal gas law $P = \rho RT/\mu$ (ignore gradients in mean molecular weights FOR NOW) we can show that
$$\rho' = \rho + \frac{\rho}{p}\frac{\mathrm{d}P}{\mathrm{d}r}\delta r - \frac{\rho}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\delta r. \tag{16.1}$$
- For instability, again, we then require that $\delta\rho < 0$, or
$$\rho^* - \rho' = \left(\frac{1}{\gamma} - 1\right)\frac{\rho}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\delta r + \frac{\rho}{T}\frac{\mathrm{d}T}{\mathrm{d}r}\delta r < 0. \tag{16.2}$$
- Now the instability condition becomes
$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} > \frac{\mathrm{d}T}{\mathrm{d}r}, \tag{16.3}$$
where the adiabatic temperature gradient is given by
$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} = \left(1 - \frac{1}{\gamma}\right)\frac{T}{P}\frac{\mathrm{d}P}{\mathrm{d}r}. \tag{16.4}$$
- This says that if the temperature gradient decreases too steeply out through the star there will be convection.
- To simplify in analogy with what we did before (Eq. 15.13), it is convention to write the inequality as
$$\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P} > \frac{\Gamma_2 - 1}{\Gamma_2}. \tag{16.5}$$
- We reintroduce
$$\nabla = \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}, \qquad \nabla_{\mathrm{ad}} = \frac{\Gamma_2 - 1}{\Gamma_2}, \tag{16.6}$$
so that the instability condition is
$$\nabla > \nabla_{\mathrm{ad}}. \tag{16.7}$$
Again, the RHS is 2/5 in the ionized ideal case.
- This is known as the **Schwarzschild criterion**. The inequality is also referred to as superadiabaticity.

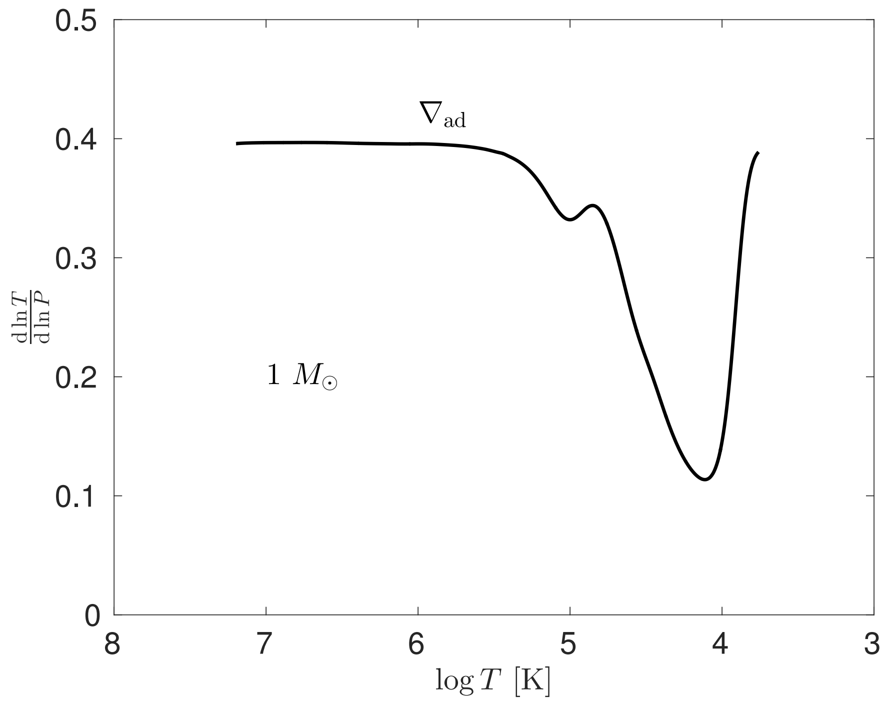

Figure 16.1: The adiabatic temperature gradient of a solar-like model. Note the decrease in the outer layers.

16.2 Physical conditions for convection onset

- Where in various types of stars does convection occur?
- Consider first energy transport by radiation. Recall Equation (15.3) which is reproduced here:

$$\nabla_{\rm rad} \equiv \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\rm rad} = \frac{3}{16\pi acG}\frac{P\kappa}{T^4}\frac{L}{m},$$

- Another convenient form from substituting an equation of state is

$$\nabla_{\rm rad} \equiv \frac{3k_{\rm B}}{16\pi acGm_{\rm u}}\frac{\kappa}{\mu}\frac{L}{m}\frac{\rho}{T^3}. \tag{16.8}$$

- These are the gradients required to transport all the luminosity (L) by radiation. Keep in mind that $L = L(r)$ and $m = m(r)$.
- The radiative gradient $\nabla_{\rm rad}$ is a spatial derivative connecting P and T in consecutive mass shells.
- The adiabtic gradient $\nabla_{\rm ad}$ describes the thermal variation of one mass element upon an adiabatic compression.
- Anyway, for determining where an instability is, we must evaluate

$$\nabla = \nabla_{\rm rad} > \nabla_{\rm ad}, \tag{16.9}$$

  which, if satisfied, allows for convection to take place.
- So, a superadiabatic temperature gradient induces convective motions.
- The efficiency of convection (heat transport) increases when convection itself drives the temperature gradient very close to an adiabat.

- Let that sink in: Once convection is set up, it tends to drive ∇ very close to ∇_{ad}. So it goes from superadiabatic to nearly adiabatic.
- Examining the condition in Eq. (16.9), we see that convection may occur if
  - L/m is large. Think of this as $L/m \sim \mathrm{d}L/\mathrm{d}m = \varepsilon$ at small m (near the core). The energy generation rate is huge in massive stars in the cores, so many massive stars have convective cores.
  - κ is large. This is satisfied in the outer parts of less-massive stars, or with low surface temperatures and ionization zones of hydrogen (Sun).
  - ρ/T^3 is large. Happens typically in the outer parts of relatively cool stars. In fact, this ratio increases rapidly as the effective temperatures go down.
  - $\nabla_{\mathrm{ad}} = 1 - 1/\gamma$ is small. Satisfied in the ionization zone of hydrogen, in outer parts of cool stars where γ gets small (increased specific heat). See Figure 16.1.
- $n = 1.5$ polytropes are good models for convective regions, as this corresponds to $\gamma = 5/3$.

16.3 Depth of outer convection zones

- We've seen where convection tends to set in, but how large are these regions and what do they depend on?
- To first order the depth depends on T_{eff}, since this determines where ionization layers are.
- Stars with lower surface temperatures achieve H ionzization at deeper depths with higher pressure.
- So for cool main-sequence stars, outer convection zones can extend deep, even to the center.
- For hotter stars, up to about 8000K, ionization occurs higher and higher in the atmosphere, and the convective shells of these stars can be very thin.
- The depth can also depend on **chemical abundances**.
- Consider a star with relatively high He abundance.
- The larger mean molecular weight corresponds to lower pressures at a given layer.
- He ionization, which can cause a convective instability, thus happens at a deeper layer, or higher temperature.
- Convection zones will be deeper for He-rich stars.
- For metal-poor stars, the opacity is reduced and therefore the radiative temperature gradient gets reduced, reducing the convective instability.
- Metal-poor stars will have shallower convection zones than metal-rich ones.

16.4 Semiconvection

- Let's return to the Brunt-Väisälä frequency again from Equation (15.14):

$$N^2 = g\left(\frac{1}{\gamma P}\frac{\mathrm{d}P}{\mathrm{d}r} - \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right).$$

- We want to rewrite this in a very convenient form, and this time we **will** take into account composition gradients in the gas to be as general as possible. The form is

$$N^2 = \frac{g^2\rho}{P}(\nabla_{\rm ad} - \nabla + \nabla_\mu), \tag{16.10}$$

where (as some have been defined before)

$$\nabla = \frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}, \qquad \nabla_{\rm ad} = \left(\frac{\mathrm{d}\ln T}{\mathrm{d}\ln P}\right)_{\rm ad}, \qquad \nabla_\mu = \frac{\mathrm{d}\ln \mu}{\mathrm{d}\ln P}. \tag{16.11}$$

EXAMPLE PROBLEM 16.1: Show how you can get from Equation (15.14) to Equation (16.10). Start with

$$\begin{aligned} N^2 &= g\left(\frac{1}{\gamma}\frac{\mathrm{d}\ln P}{\mathrm{d}r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right), \\ P &= \frac{\rho T}{\mu}, \\ \mathrm{d}\ln P &= \mathrm{d}\ln\rho + \mathrm{d}\ln T - \mathrm{d}\ln\mu. \end{aligned}$$

Answer:

$$\begin{aligned} N^2 &= g\left(\frac{1}{\gamma}\frac{\mathrm{d}\ln P}{\mathrm{d}r} - \frac{\mathrm{d}\ln\rho}{\mathrm{d}r}\right), \\ P &= \frac{\rho T}{\mu}, \\ \mathrm{d}\ln P &= \mathrm{d}\ln\rho + \mathrm{d}\ln T - \mathrm{d}\ln\mu, \\ N^2 &= g\left(\frac{1}{\gamma}\frac{\mathrm{d}\ln P}{\mathrm{d}r} - \frac{\mathrm{d}\ln P}{\mathrm{d}r} + \frac{\mathrm{d}\ln T}{\mathrm{d}r} - \frac{\mathrm{d}\ln\mu}{\mathrm{d}r}\right), \\ &= g\left(\frac{\mathrm{d}\ln P}{\mathrm{d}r}\left(\frac{1-\gamma}{\gamma}\right) + \frac{\mathrm{d}\ln T}{\mathrm{d}r} - \frac{\mathrm{d}\ln\mu}{\mathrm{d}r}\right), \\ &= g\left(-\nabla_{\rm ad}\frac{\mathrm{d}\ln P}{\mathrm{d}r} + \frac{\mathrm{d}\ln T}{\mathrm{d}r} - \frac{\mathrm{d}\ln\mu}{\mathrm{d}r}\right), \\ &= g\frac{\mathrm{d}\ln P}{\mathrm{d}r}\left(-\nabla_{\rm ad} + \frac{\mathrm{d}\ln T}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}\ln P} - \frac{\mathrm{d}\ln\mu}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}\ln P}\right), \\ &= g\frac{\mathrm{d}\ln P}{\mathrm{d}r}\left(-\nabla_{\rm ad} + \nabla - \nabla_\mu\right), \\ &= \frac{g}{P}\frac{\mathrm{d}P}{\mathrm{d}r}\left(-\nabla_{\rm ad} + \nabla - \nabla_\mu\right), \\ &= -\frac{g^2\rho}{P}\left(-\nabla_{\rm ad} + \nabla - \nabla_\mu\right), \\ N^2 &= \frac{g^2\rho}{P}\left(\nabla_{\rm ad} - \nabla + \nabla_\mu\right). \quad \square \end{aligned}$$

- Ignore the composition gradient for a second. We recover the "standard" stability relation: if the temperature gradient is larger than the adiabatic one (Schwarzchild), the BV frequency becomes complex.
- If it's the reverse, the BV is positive and the medium is stable to convection.
- Now however, we have the possibility that the Schwarzchild criterion is satisfied ($\nabla > \nabla_{\rm ad}$), yet the medium remains stable because the composition gradient makes it positive again.

- This is the *Ledoux criterion*, and when this is the case we have weak convection, or **semiconvection**.
- This typically would not occur in a convection zone, why? Because convection mixes material and composition gradients are removed.
- But in areas of nuclear burning where gradients do exist, and at the "edges" of convection zones, this situation can arise. Large peaks in the μ-gradient (also caused by g increases) can cause large jumps in N.
- Can be thought of as difficulty in moving "heavier" material (high μ) up - it doesn't want to do that.
- Becomes important for red-giant stars and their gravity modes mixing (boosting frequency) with acoustic modes.

16.5 Mixing length theory

- How does the convection actually transfer the energy?
- We have previously shown how convection can take place by considering a blob displaced from its equilibrium position.
- The blob is hotter than its surroundings.
- This blob rises with a velocity up to a point where a new hydrodynamic instability sets in, whereby the motion becomes turbulent and the blob dissolves, depositing its heat in the surroundings.
- The exact details of this process are still unknown and are a subject of much research. For solar convection, the most sophisticated numerical simulations carried out on the fastest computers take an order of magnitude more computation time than the solar "time" they are trying to simulate, and that is just for a small section of the Sun.
- In comparison, computing the solar model over the Sun's lifetime, with a simple model of convection, takes a couple minutes.
- Remember that the radiative flux $L/4\pi r^2$ is

$$F_{\rm rad} = \frac{4ac}{3}\frac{T^3}{\kappa\rho}\frac{{\rm d}T}{{\rm d}r} = \frac{4acG}{3}\frac{mT^4}{\kappa P r^2}\nabla,$$

  using hydrostatic equilibrium. ∇ is the actual stratification.
- Neither $F_{\rm rad}$ or ∇ are really known, since radiation only carries some of the star's flux.
- But we did note that the temperature gradient required to carry **all** of the stellar luminosity is $\nabla_{\rm rad}$. If this includes convection, the total flux then is

$$F_{\rm tot} = F_{\rm rad} + F_{\rm conv} = \frac{4acG}{3}\frac{mT^4}{\kappa P r^2}\nabla_{\rm rad}. \tag{16.12}$$

- A very reasonable convective flux for a parcel in pressure equilibrium is its heat content multiplied by the mass flux:

$$F_{\rm conv} = \rho v c_P \Delta T. \tag{16.13}$$

- All that's left to do is find ρ, v and ΔT of the blob.
- $\Delta T = T_i - T$ is the excess heat of a rising parcel of gas with respect to its surroundings.

- Assume a parcel moves a distance ℓ before dissolving into the background material, so that at any given time, a typical parcel will have moved $\ell/2$. The temperature difference between the parcel and the gas is

$$\Delta T = T_i - T = \left(\frac{\mathrm{d}T_i}{\mathrm{d}r} - \frac{\mathrm{d}T}{\mathrm{d}r}\right)\frac{\ell}{2}. \tag{16.14}$$

- Multiply by the pressure scale height $H_P = -\left(\mathrm{d}\ln P/\mathrm{d}r\right)^{-1}$ and divide by T

$$\frac{\Delta T}{T} = (\nabla - \nabla_i)\frac{\ell}{2H_P}. \tag{16.15}$$

- If the parcel remains in pressure equilibrium, assuming a general equation of state, then changes in its density are

$$\frac{\mathrm{d}\rho}{\rho} = -\delta\frac{\mathrm{d}T}{T} = -\delta(\nabla - \nabla_i)\frac{\ell}{2H_p}, \tag{16.16}$$

  where

$$\delta = -\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln T}. \tag{16.17}$$

- Since the density difference implies a buoyancy force $-g(\rho_i - \rho)$, there is work that goes into moving the parcel

$$-g(\rho_i - \rho)\frac{\ell}{2} = g\rho\delta(\nabla - \nabla_i)\frac{\ell^2}{4H_p}. \tag{16.18}$$

- Suppose half of the work goes into the kinetic energy of the parcel, and the other half goes into the surroundings, moving it aside. The velocity is then

$$v^2 = \frac{\ell^2}{4}\frac{g\delta(\nabla - \nabla_i)}{H_p}. \tag{16.19}$$

- We get for the convective flux then that

$$F_{\mathrm{conv}} = \rho c_p T (g\delta)^{1/2}\frac{\ell^2}{4}H_p^{-3/2}(\nabla - \nabla_i)^{3/2}. \tag{16.20}$$

- It is possible to compute ∇_i, because as the parcel moves its internal energy will change because of radiative losses and adiabatic expansion/contraction. One finds

$$\frac{\nabla_i - \nabla_{\mathrm{ad}}}{\nabla - \nabla_i} = \frac{6acT^3}{\kappa\rho^2 c_p v\ell}. \tag{16.21}$$

- The overal problem can now be solved. Five new equations for the 5 unknowns F_{rad}, F_{conv}, v, ∇, and ∇_i. The *mixing-length parameter* $\alpha = \ell/H_P$ is used as a free parameter, chosen to match observations.
- Solutions can be found in the literature. Limiting cases are interesting. In dense regions of a star, $\nabla \to \nabla_{\mathrm{ad}}$. A gradient just over the adiabatic limit is all that is necessary to transport all of the luminosity.
- Near stellar photospheres, $\nabla \to \nabla_{\mathrm{rad}}$, and convection is ineffective so $F \to F_{\mathrm{rad}}$.
- In between these limits, the mixing-length equations need to be solved.
- Near the surface, convection does not transport energy efficiently, since convective flux is proportional to the the density which is quite low.
- It is also proportional to the mixing length which has to be small. So the only way for convection to dominate is if δ can get large. This means it has to be very far from adiabatic.

- We know this because we see *granulation*, which typifies the size of convective elements, about $10^{-3} R_\odot$.
- The convective time scale

$$t_{\rm conv} \simeq \left[\left(\frac{{\rm d}\ln T}{{\rm d}\ln r} \right)_{\rm ad} - \frac{{\rm d}\ln T}{{\rm d}\ln r} \right]^{-1/2} t_{\rm dyn}. \tag{16.22}$$

- This timescale is a dynamical time scale, but since the gravitational acceleration is reduced because of the difference in density that comes into all of this, the timescale is increased by that pre-factor.
- That prefactor in the Sun is of order 10^{-6}, and we find convective time scales $t_{\rm conv} \simeq 0.2$ yr.
- This is still small compared to evolution. Matter mixes over such a time scale in the convection zone.

16.6 Convective overshoot

- We briefy mention this here because it has very important implications for stellar modeling.
- The parcels we consider unstable to convection will eventually reach a stable layer.
- However the momentum causes them to overshoot the boundary into this stable layer.
- For example, this is why we see granulation in the solar photosphere (which is stable).
- One important consequence of this overshoot is the deposition of convectively mixed material into stable regions, which can affect the thermal structure as well as evolutionary properties.

16.7 An entropy formulation

- Convection can also be understood in terms of entropy.
- For reversible processes, ${\rm d}Q = T{\rm d}S$ and so

$$T\,{\rm d}S = {\rm d}U + P{\rm d}V. \tag{16.23}$$

- Using standard thermodynamic relations, it can be shown from here that

$$\frac{{\rm d}S}{{\rm d}r} = c_P(\nabla - \nabla_{\rm ad})\frac{{\rm d}\ln P}{{\rm d}r}. \tag{16.24}$$

- So if the star is radiative, ${\rm d}S/{\rm d}r > 0$ and the entropy increases outward.
- If the star is convective, ${\rm d}S/{\rm d}r < 0$. If the convection is efficient the gradient is very close to being adiabatic, meaning the entropy is very nearly constant throughout convection zones.

16.8 An energy formulation

- Start with Equation (16.4) and rewrite using ideal gas law and hydrostatic equilibrium:

$$\left(\frac{{\rm d}T}{{\rm d}r}\right)_{\rm ad} = \left(1 - \frac{1}{\gamma}\right)\frac{\mu}{\rho R_g}\frac{{\rm d}P}{{\rm d}r} = -\left(1 - \frac{1}{\gamma}\right)\frac{g\mu}{R_g}. \tag{16.25}$$

- Remembering that $\gamma = c_P/c_V$ and $c_P - c_V = R_g/\mu$,

$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} = -\left(\frac{c_P/c_V - 1}{c_P/c_V}\right)\frac{g\mu}{R_g}, \tag{16.26}$$

$$= -\left(\frac{\frac{c_P - c_V}{c_V}}{c_P/c_V}\right)\frac{g\mu}{R_g}, \tag{16.27}$$

$$= -\left(\frac{c_P - c_V}{c_P}\right)\frac{g\mu}{R_g}, \tag{16.28}$$

$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\mathrm{ad}} = -\frac{g}{c_P}. \tag{16.29}$$

- This form shows how the parcel of gas is changing as it rises adiabatically and expands

- This can also be derived using energy by considering energy release and work against gravity

- To see this, consider again a parcel of gas at position r that is displaced adiabatically to $r + \mathrm{d}r$. The temperature at the new depth is $T + \mathrm{d}T$.

- The temperature gradient $\mathrm{d}T/\mathrm{d}r$ will be negative in our case since T decreases as we move further from the center. So when the parcel arrives at the new location, it is in cooler surroundings, so the parcel has more internal energy than the local gas.

- If we let time elapse a bit and the parcel releases energy to the surroundings, the amount of thermal energy released at fixed pressure is $C_p\,\mathrm{d}T$ $\mathrm{erg\,g^{-1}}$. Is this energy release important?

- The other factor at work here is gravity, and we have to ask if the work done against gravity to move the parcel upwards balances the release of thermal energy.

- The work done against gravity is $g\,\mathrm{d}r$ per gram of material. Therefore, the total amount of energy to displace 1 gram of material is

$$\Delta E = g\,\mathrm{d}r + C_p\,\mathrm{d}T. \tag{16.30}$$

  Note that the sign can be either positive or negative (since $\mathrm{d}T$ will usually be negative). Let's say that the gravitational energy exceeds the thermal energy released, so that the change in energy is positive. In this case, it has taken more work to displace the gas than what is released as thermal energy, and so the final energy state is higher. Therefore, the parcel will sink down to its initial position in a low energy state. There is no reason for it to move. Is is *convectively stable*.

  In the opposite case, the release of thermal energy is more than enough to compensate for the work against gravity. There will even be some excess, and the parcel will still be hotter than its surroundings. It may continue on rising. This is a *convective instability*, as we have already studied.

- So, the boundary between the two cases is when $g\,\mathrm{d}r = -C_p\,\mathrm{d}T$, which occurs when the temperature gradient takes on a particular value of the *adiabatic temperature gradient*

$$\left.\frac{\mathrm{d}T}{\mathrm{d}r}\right|_{\mathrm{ad}} = -\frac{g}{C_p}. \tag{16.31}$$

  For convection to occur, $\mathrm{d}T/\mathrm{d}r < \mathrm{d}T/\mathrm{d}r|_{\mathrm{ad}}$. Since both gradients are negative, this can also be written for instability

$$\nabla > \nabla_{\mathrm{ad}}. \tag{16.32}$$

  This is the same conclusion we reached earlier.

COMPUTER PROBLEM 16.1: [80 pts]: This assignment deals with convection and radiation zones in main-sequence stars over a range of different masses. You are going to quantitatively illustrate where we expect such zones to exist in stars to provide an overall understanding of stellar structure during the period of core-hydrogen burning.

What to do

Your overall goal is to prepare a main plot (described in more detail below) showing where these zones exist. For this purpose, you need to run a bunch of `MESA` models for different mass stars. We will use these models for future studies so make sure to keep the directory(ies) neat and handy.

1. The models should be within the stellar mass range of $M_* = 0.3\,M_\odot - 20\,M_\odot$. The number of models you compute is up to you, but the "mass grid" should be sufficiently populated so you don't miss any important details, and so that you can make the required plots with enough number of points and precision. Somewhere between 10 and 20 models seems reasonable. One of the models should be $M_* = 1\,M_\odot$, of course!
2. The models are to be of solar hydrogen mass fraction and metallicity ($Z = 0.02$). For each mass, you will run the code until the core looks like the core of today's Sun, $X \approx 0.35$, which is the model you will want to use for what follows. You will also want the history data (really just $T_{\rm eff}$ and L) for the ZAMS (zero-age main sequence) point, where $X \approx 0.7$. You shouldn't need to run pre-main sequence models.
3. That's all you need to do with `MESA`. Now you need to carefully explore the resulting data and determine the locations of all the convection and radiation zones for each star mass. Use the criteria discussed in class.

What to hand in

After reading the below information you will find you need to hand in at least 7 plots, as well as text pertaining to items 4 and 6.

1. A nice H-R diagram of your stars. The symbol size should represent the star's radius, and the color the star's mass at the time you stopped your models. Also give an indication of age somehow discretely so we have an idea of the range there. Finally, plot a small symbol for where each star began on the ZAMS, so we see how much "evolution" occured. These symbols do NOT need to show the mass or radius information as in the more evolved points. [15 pts]
2. A plot that shows where the internal convection and radiation zones are for each star. [25 pts]
   - This should be a publication-quality plot (clearly labeled, easy to read, useful for future reference, etc.).
   - The plot is to be as follows: the x axis is $\log M_*/M_\odot$ and the y axis is $m(r)/M_*$.
   - The interior radiation and convection zones should be clearly demarcated by however you wish to do it.
   - Finally, somehow denote the dominant H-burning process for everywhere that nuclear burning is taking place. In many cases, you will have some regions where PP is dominant and some where CNO is (in the same star).
3. A second plot of exactly the same information but instead where r/R_* is the ordinate. These should be on separate pieces of paper. [5pts]
4. A summary paragraph or two of how you obtained the parameters in your plot, i.e., how you determined where the zones are from the stellar profiles. Mention here any issues you ran into or anomalous cases (if any). You don't need to do much interpretation here, just write down what you did to gather the information that went into these plots. [10pts]
5. Pick two of your models. One should be $M_* \leq 1.1 M_\odot$ and one $M_* \geq 3 M_\odot$. For each of these 2 models you will make 2 more plots, so 4 in total here. One of the plots for each mass should have the three quantities ∇, $\nabla_{\rm ad}$, and $\nabla_{\rm rad}$ plotted as a function of interior $\log T$, but reversed, so $\log T$ decreases to the right. The other plot for each mass will be the Brunt-Väisälä frequency (squared), or N^2 in appropriate units as a function too of reversed $\log T$. Make sure it's scaled properly as to be visible. [10pts]

6. Write another page or two on the above 4 plots. Describe what is happening and connect what you observe in these figures to the findings shown in your **main** plot. Compare and contrast the behavior for the 2 different masses. Try to describe and decipher any unusual "features" you see, such as bumps or spikes, in each case. Connect them to things we may have discussed in class, not forgetting, for example, our study of the opacities. Indeed, you may find it necessary to generate other supplementary plots of other quantities in the models to get your point across here. Feel free to draw on and label these plots by hand to point things out. Be thorough. Also in this part, be sure to answer the following question for each of these 2 stellar mass models:

   - In the interior regions where $\nabla_{\rm rad}$ is large, what is your idea as to the physical reasons causing this?

   Use Equation (16.8) from the notes as a guide if needed. [15 pts]

Things to keep in mind

1. Your main plot is going to be judged by the faculty to determine who made the best publication-quality image (oh, and it should be correct too). Therefore the plot should be full page and nice, and on a separate page from your other stuff! You also may want to keep it secret from your classmates so they don't steal your ideas. An emailed .pdf or .eps of your plot would also be good and I can collect them all into one document to show in high quality on a computer screen. The one judged best will receive a special prize extra credit on the assignment.
2. You may want to consider your initial stellar mass grid spaced out regularly in terms of $\log M_*/M_\odot$, rather than in linear $M_*/M_\odot$. But do what you want. Much of the changes to convection properties occur in the $1-2\ M_\odot$ range.
3. You may need to consider adding quantities to what `MESA` saves in the profiles. This is easily done using the .list files. Your best bet might be copy the default profile_columns.list file into your working directory and uncomment the things you'll need. If you do that, comment out the line in your inlist_project that just uses the custom_profiles.list. Let me know if you need help.
4. You likely don't need to create any pre-main-sequence models because the metallicity we are using is standard.
5. It may or may not be reasonable to interpolate certain quantities to make your mass grid denser for your plots. It's up to you.
6. There are very fast and easy, or long and complicated ways of obtaining the profile quantities you need for the plots. Hopefully you find the fast/easy ways.
7. Try to understand what's going on in each model. This will be helpful for grasping the evolution of stars of different masses.