Unit 14

Opacity sources

14.1 Kramer's Laws

- The opacity κ determines how the flux is transported by radiation, and so is an important quantity.
- Computing opacities is hard stuff, only a few groups in the world have succeeded (LANL, LLNL). These computations require full quantum-mechanical treatments.
- In general, opacities can be approximated as power laws of the form

$$\kappa = \kappa_0 \rho^n T^{-s} \ \ [\mathrm{cm}^2\,\mathrm{g}^{-1}], \tag{14.1}$$

  similar to how we treated energy generation rate and its dependence on these parameters.
- The goal is to compute *total* Rosseland mean opacities from all possible sources, including the following four:

1. Scattering of photons from electrons
   - Photons, or electromagnetic radiation, can cause free electrons to oscillate at the same frequency if the energy is low enough. These electrons then radiate this energy. This is a scattering process.
   - For relativistic free electrons, this is known as Compton scattering, but for conditions inside most stars, this is Thomspon scattering.
   - For temperatures below about 1 billion Kelvin (thermal energies below electron rest mass energy), the cross section for scattering off electrons is frequency independent.
   - One finds for the opacity

   $$\kappa_{\mathrm{e}} = \frac{n_{\mathrm{e}}\sigma_{\mathrm{e}}}{\rho} \qquad \mathrm{cm}^2\,\mathrm{g}^{-1}, \tag{14.2}$$

     where the cross section is given in the Thompson scattering prescription

   $$\sigma_{\mathrm{e}} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = 0.6652 \times 10^{-24}\,\mathrm{cm}^2. \tag{14.3}$$

     (Note the classical electron radius buried in there).
   - Scattering off electrons will only occur in a highly ionized gas where a sufficient number of electrons are present. Looking back at Equation (5.14) and considering a gas with inconsequential metals gives

   $$\kappa_e \simeq 0.2(1+X). \tag{14.4}$$

- If there are many metals or ionization is incomplete then the electron densities have to be computed more carefully.
- Below 10,000 K hydrogen is not ionized at low pressure and so this opacity does not contribute to the Rosseland mean.
- For this case, the exponents $n = s = 0$.

> **EXAMPLE PROBLEM 14.1:** Derive Equation (14.4) in the full ionization case.

2. Free-free absorption

   - A single free electron cannot absorb a photon while still conserving energy and momentum.
   - If an ion is nearby, this absorption is possible however.
   - In this case, it can be shown that the opacity

   $$\kappa_{\mathrm{f-f}} \approx 10^{23} \frac{Z_c^2}{\mu_\mathrm{e}\mu_\mathrm{I}} \rho T^{-3.5}, \tag{14.5}$$

     where Z_c is some average nuclear charge. Typically a quantum-mechanical gaunt factor needs to be included in the coefficient.
   - Opacities of this form that scale as $\kappa \sim \rho T^{-3.5}$, or $n = 1$ and $s = 3.5$, are known as Kramers opacities.
   - The inverse of this process, when an ion changes the momentum of an electron which then emits a photon, is known as Bremsstrahlung emission
   - Note that for reference, in the core of the Sun the product $\rho T^{-3.5} \approx 5 \times 10^{-23}$. So these opacities are of order 1 to 10 or so.

3. Bound-free absorption

   - Here an ion absorbs a photon which frees a bound electron.
   - Causes continuum opacity at wavelengths bluer than the ionizing photon.
   - It can be shown that the opacity

   $$\kappa_{\mathrm{b-f}} \approx 10^{25} Z(1 + X)\rho T^{-3.5}. \tag{14.6}$$

     This also has a Kramers form.
   - At temperatures below about 10^4K, this should be used with caution since photons are not energetic enough to ionize a gas.
   - This source of opacity can be larger than free-free absorption by a few orders of magnitude.

4. Bound-bound absorption

   - This is photon-induced transitions of electrons between bound levels in an atom or an ion.
   - At large temperatures, most photons are ionizing so this contribution is smaller to the total opacity (maybe 10%).
   - Very difficult computations since there are millions of absorption lines possible. Need oscillator strengths as well as equation of state.

5. H^-

   - We just note that in stellar atmospheres (like our Sun) contributions from negative hydrogen and molecules and grains dominate the opacity.
   - Low energy photons starting in the infrared can be absorbed in this bound-free transition.

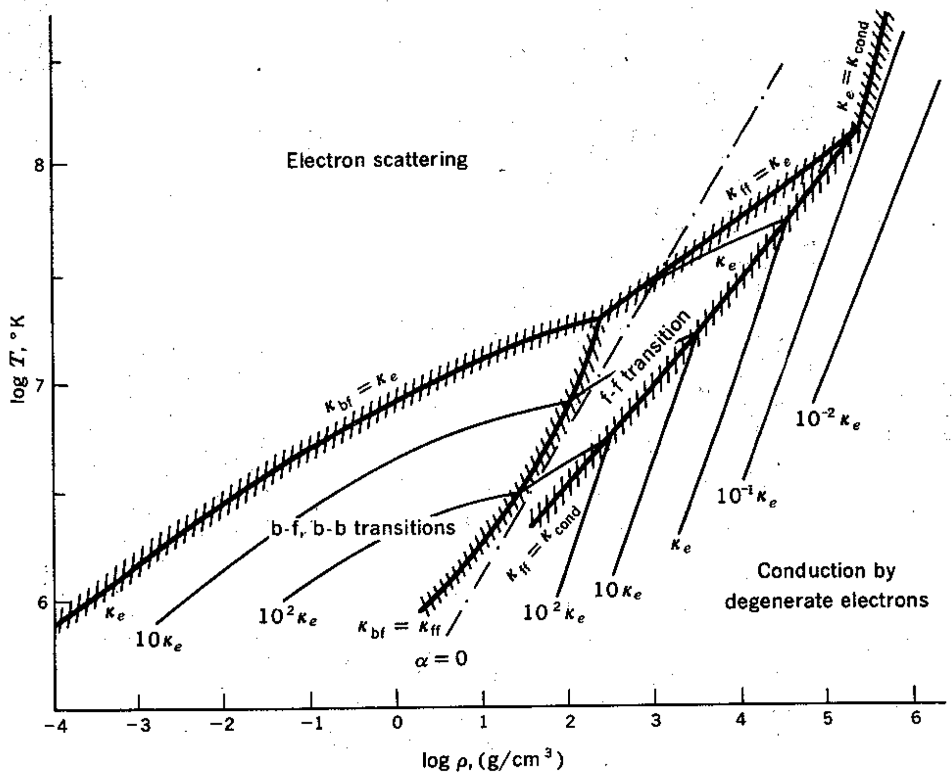

Figure 14.1: Opacity contributions in the $\rho - T$ plane. The values are given in units of electron scattering opacity κ_e. Conduction is not a radiative opacity, and will be discussed in Sec. 15.2. From Clayton [1983] after Hayashi et al. [1962].

- What is needed for the H^- opacity are free electrons, which at low temperatures might not be completely abundant from ionization of H.
- Other sources of these electrons are the outer ones from metals, so this is sensitive to metallicity.
- An approximate relation in the range of about 3000-6000K and low densities is

$$\kappa_{\mathrm{H}^-} \approx 2.5 \times 10^{-31} \left(\frac{Z}{0.02}\right) \rho^{1/2} T^9 \tag{14.7}$$

14.2 Consequences

- Figure 14.1 shows the contributions from different opacity sources in a stellar interior.
- At low temperatures and partial ionization, b-b and b-f absorption dominates from the bound electrons.
- As ionization occurs at higher temperatures, free-free opacity takes over, but as T continues to increase, $\kappa_{\mathrm{f-f}}$ decreases and scattering from free electrons dominates.
- In reality, all of these processes are occuring at any one time and place, so contributions must be added correctly.
- Note that the sum of the Rosseland mean opacities of each component is not the same as the Rosseland mean of the sum
- Figure 14.2 shows opacity data from the OPAL project.[1] We see that the Sun's opacity can reach 10^5 or so near the surface where atoms recombine.

[1] http://opalopacity.llnl.gov/

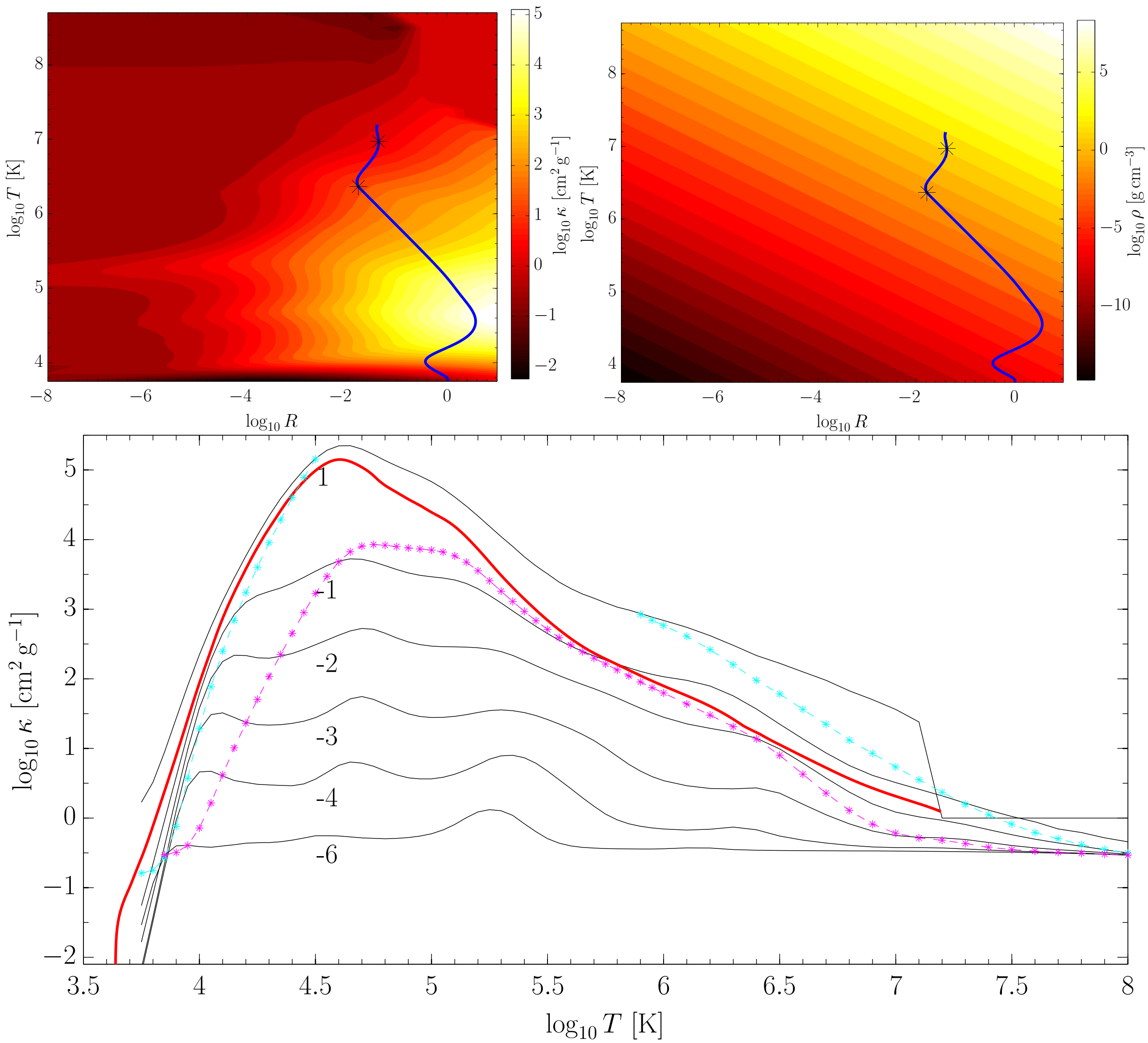

Figure 14.2: (Left top): OPAL Rosseland mean opacity data for a solar composition mixture. The x-axis quantity, R, is defined as $R = \rho T_6^{-3}$, where $T_6 = T \times 10^{-6}$ as usual. Since R can be multivalued for any ρ, T pair, on the right panel is plotted the density, for reference. In each case the blue curve are the values for a standard interior solar model, and the 2 black stars represent the boundaries of the core and convection zone ($0.2R_\odot$ and $0.7R_\odot$, respectively). The direction of increasing T is toward the stellar center. (Bottom): Cuts through the opacity values for specific values of $\log R$, given by the labeled curves. The red curve is the opacity from a current solar model. The cyan and the magenta symbols represent the 0.5 and 0.99 ionization curves for pure hydrogen, respectively, as in Figure 12.2. (Specifically the data are from computation 200503120009, Table 73. The calculations consider about 20 species.)

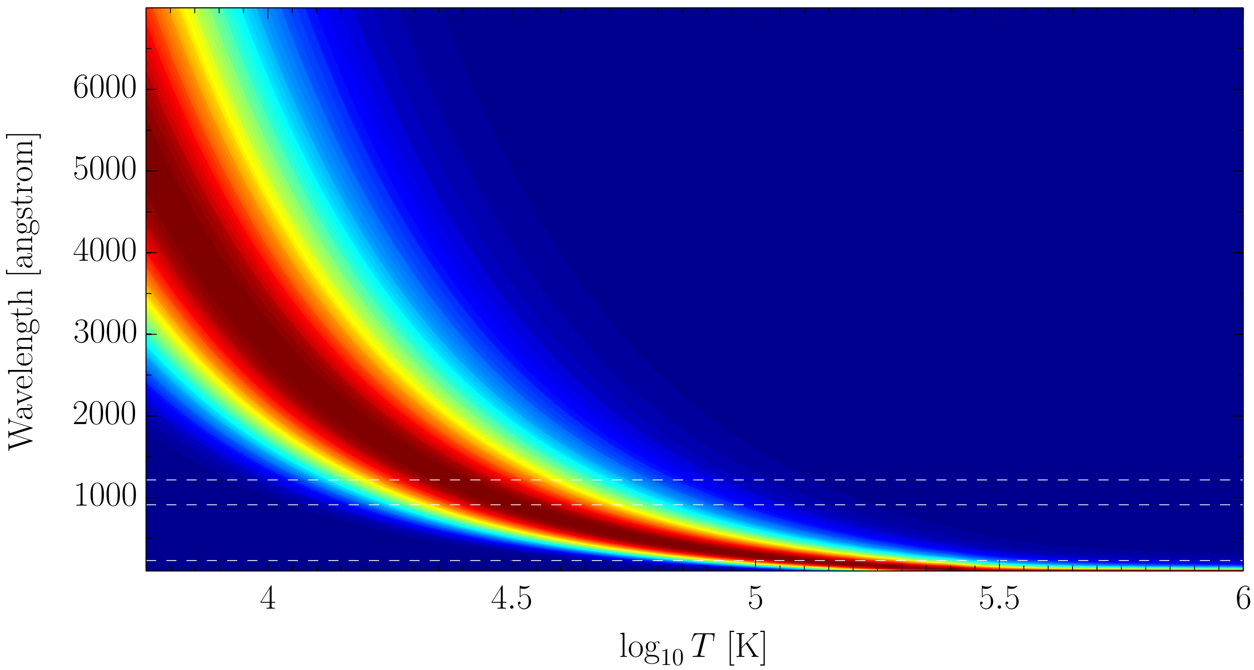

Figure 14.3: Blackbody curves for a range of wavelengths and temperatures. The curve at each temperature is normalized to its maximum value across wavelengths. Wavelength guides from top to bottom are at $1216\,\AA$, $912\,\AA$, and $228\,\AA$.

- Figure 14.2 can be understood in a few limiting cases:
  - For the low density, high temperature case (small R), and for the high temperature regime at any density, all H and He is ionized and the electrons are free particles. Here we expect electron scattering, where Equation (14.4) is dominant. Indeed, $\kappa_e \approx 0.34\,\mathrm{cm}^2\,\mathrm{g}^{-1}$ for an $X = 0.7$ composition, which agrees with the values over most of the temperature range ($\log_{10} -0.34 \approx -0.47$).
  - Below 10,000 K the opacities all converge to small values. Here, all electrons go to the ground state of H and He. Incoming photons can only cause a bound free transition if their wavelengths are shorter than $912\,\AA$ (for H) or $228\,\AA$ (for HeII). In fact, the whole Lyman series (edge) only extends to $1216\,\AA$ (for H). But at this temperature, the blackbody curve is peaked at about $2880\,\AA$, and there just aren't many photons with such high energies available. The stellar region in which this applies is near the surface, but the Sun does not have such conditions as the plots show. Also see Figure 14.3.
  - At intermediate temperatures we see increased opacity, particularly as the density increases (larger R). Even for the lowest density curve we see a bump around temperatures of 5.0-5.5. Wien's law tells us the wavelengths are in the $100 - 300\,\AA$ range. This suggests the HeII Lyman edge, even though there aren't many He ions. The bump at about 4.5-4.75 at larger densities could be due to a bound-free absorption of HeI. In any case, these large radiative opacities are found in the H and He ionization zones in stars.
  - Note the Kramers shape to the opacities, which decrease at higher temperatures.
  - Keep in mind however that these are mean opacities, independent of individual wavelengths.