Common xy Relationships

There are many families of relationships which occur between two variables. Let's examine a few, to get comfortable with the idea of reading an equation defining a relationship between x and y and visualizing the resultant curve.

Figure 1 shows the behavior of three families of lines. The first panel shows how y varies when it is equal to x raised to some power. These relationships can be very useful in connecting the orbital radius and velocity of a planet or asteroid. We have plotted the cases $y = x^2$, $y = x$, and $y = \sqrt{x} = x^{1/2}$. If y is raised to a power greater than unity, then y will be larger than x as long as $x > 1$. If y is raised to a power less than unity, then y will be smaller than x as long as $x > 1$.

If you are presented with a set of these lines and are unsure which line is which, check a few values by hand. For example, to verify that the red line illustrates $y = x^2$, we note that if $x = 4$, $y = x^2 = 4^2 = 4 \times 4 = 16$, and then observe that only the red line has a value anywhere near to 16 when x is equal to four. If we had calculated the model value for $x = 1$, would we have been able to distinguish between the models?

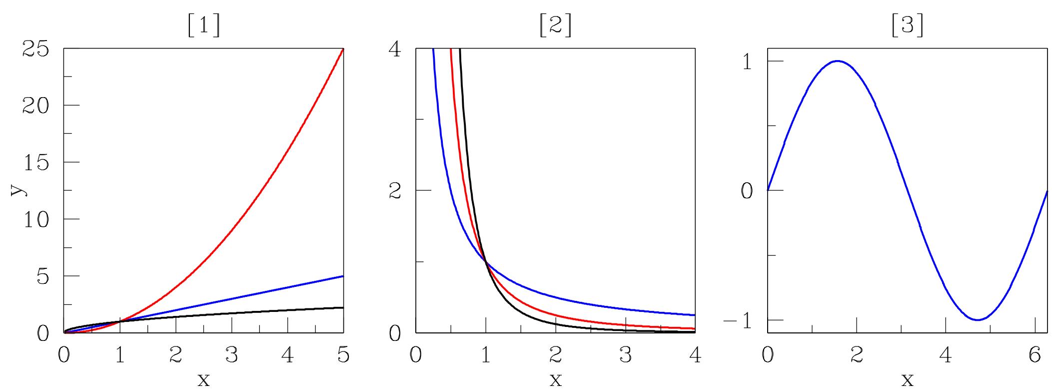

Figure 1: These three panels illustrate a number of common relationships. Panel [1] shows the lines formed by the relations $y = x^2$ (red line), $y = x$ (blue), and $y = \sqrt{x}$ (black). Would it be easier to distinguish between these three lines near to $x = 1$, or $x = 5$? Panel [2] shows the lines representing $y = 1/x$ (blue), $y = 1/x^2$ (red), and $y = 1/x^3$ (black). Though the family of lines has a common form (dropping steeply as x increases, and leveling off for $x > 1$), we can distinguish between the lines by checking the value of y where $x = 0.5$, $x = 3$, or anywhere else besides $x \approx 1$. Panel [3] shows a single line which represents a sine wave; y starts at zero and varies between -1 and 1, as x covers the range from zero to 2π. As x continues to increase, the values for y repeat, so $y(x) = y(x + 2\pi)$.

The middle panel shows the behavior of y when it is equal to the inverse of x raised to some power. We have plotted $y = 1/x$, $y = 1/x^2$, and $y = 1/x^3$. We can again differentiate between models (tell which is which) by comparing y-values where the lines diverge.

The right panel shows a trigonometric function, $y = \sin(x)$. When viewing the orbit of a planet around a distant star, a sine wave is the perfect model. (As the planet moves around the star it appears to move towards us, across the sky, away from us, and then back across the sky, once each local year.) If we multiply a set velocity V by a sine wave, the product will vary from V down to zero and then to $-V$, and then return to V, matching the observed velocities of the distant planet.

You should be comfortable with these types of functions, and able to distinguish between their forms on a plot by eye.

Match the curves (red, blue, and black) in the first and second panels in Figure 1 and the blue curve in the third panel with the following statements. In each case identify (by color and by panel) which of the seven lines is the best fit to the statement. Remember to read the figure caption carefully, for additional information about the difference between models.

1. As the distance x between an observer and an object doubles, its apparent size y decreases by a factor of two. (Red / Blue / Black) line on panel (1 / 2 / 3).

2. The y-values for $x = \pi$ and $x = 3\pi$ are identical, as this pattern of y-values repeats. (Red / Blue / Black) line on panel (1 / 2 / 3).

3. If windmill blade length x doubles, output y of a wind-powered electrical generator increases by a factor of four. (Red / Blue / Black) line on panel (1 / 2 / 3).

4. If $x = 10$, $y = 100$. (Red / Blue / Black) line on panel (1 / 2 / 3).

5. If x, the distance between the Earth and Moon, doubled, the gravitational force between them, y, would be one-fourth as great. (Red / Blue / Black) line on panel (1 / 2 / 3).

6. If someone walking for one hour at a constant speed covers three miles, they can cover six miles in two hours. (Red / Blue / Black) line on panel (1 / 2 / 3).