Slopes and y-intercepts

When measuring two variables (x and y) and trying to deduce a relationship between them, we frequently plot one against the other, creating an xy (or scatter) plot. We can then compute the slope (m) and y-intercept (b) for the combined data set.

The slope of a line is its tilt, or the amount that it rises or falls. A flat (horizontal) line has a slope of zero, while a line at a 45° angle has a slope of unity (as you move forward, you rise by the same amount). Lines which rise have slopes larger than zero, while lines that fall have negative slopes, and the higher the slope, the steeper the line.

When you draw a line through a set of points, you need to make two decisions. You need to decide how tilted the line will be (should it be horizontal, vertical, or something in between?), so you need to choose a slope. Increasing the slope makes the line tilt upwards from the horizontal toward vertical (and for a negative slope value, making the slope more negative makes the line tilt downwards from the horizontal toward vertical). You also need to decide how high or low to place the line on a page, so you need to choose a y-intercept value. If you increase the y-intercept, the whole line shifts upwards on the page (towards larger y values), and if you decrease the y-intercept, the whole line shifts downwards on the page

The slope m is defined mathematically as the change in y (Δy, or "delta" y) divided by the change in x (Δx, or "delta" x), along a best-fit line, and the y-intercept is the y-coordinate of the line when $x = 0$. These two properties are sufficient to define a line. Two points will also define the coordinates of the entire line, so if we know the positions of point 1 (x_1, y_1) and point 2 (x_2, y_2), we can determine the slope and y-intercept of the line. The general form of the equation for a line with slope m and y-intercept b is

$$y = mx + b, \quad \text{where}$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \quad \text{and}$$

$$b = y_1 - mx_1 = y_1 - \left(\frac{y_2 - y_1}{x_2 - x_1}\right) x_1.$$

If you select any value for x and then calculate $y = mx + b$, the point (x, y) will lie right along the line. The actual x and y data points from an experiment should appear near to the line, but they will not all line up perfectly on top of it, because the underlying relationship between them can be broadened by both natural variation and by measurement errors.