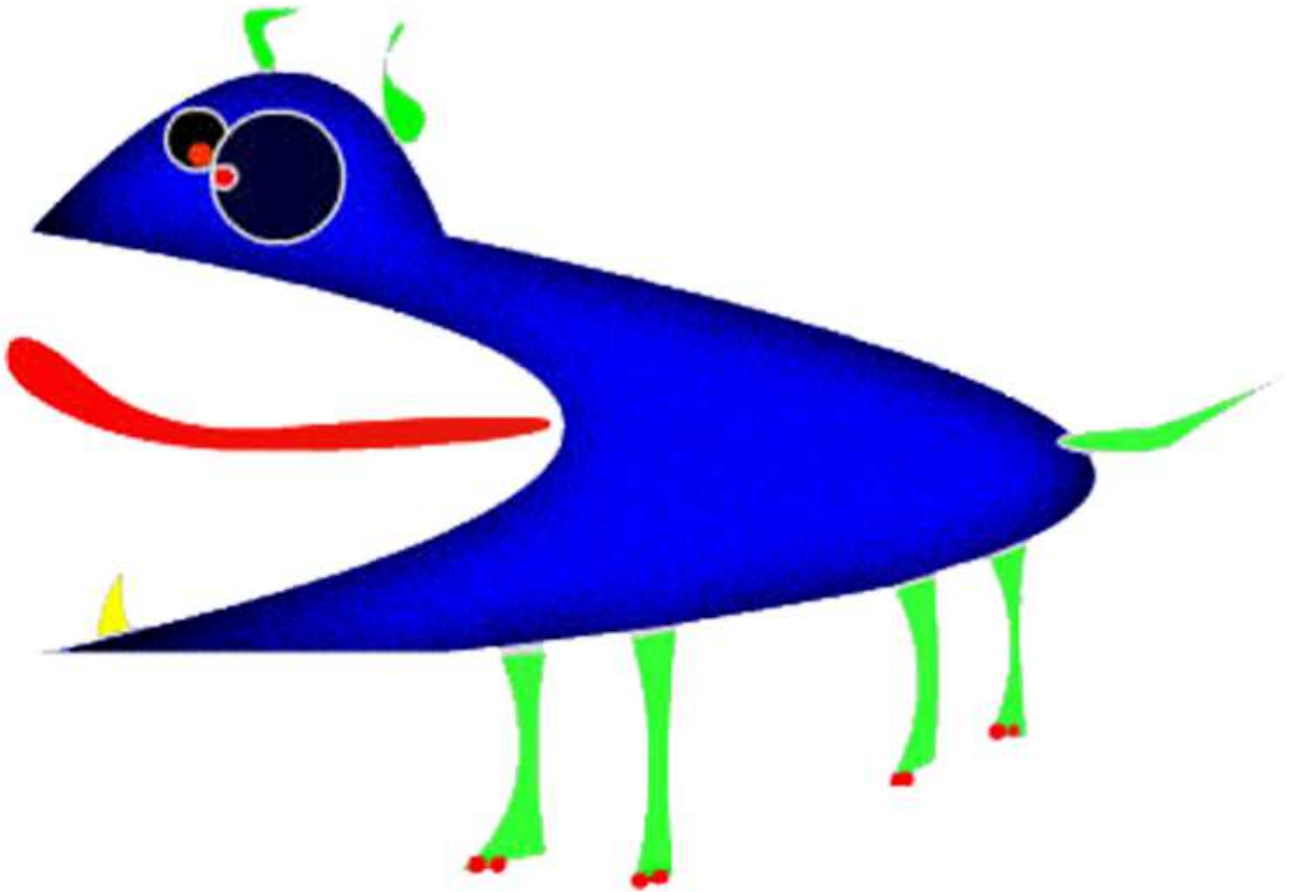


ASTR110G  
Astronomy Laboratory Exercises



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## ASTR110G Laboratory Exercises

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# Lab 1

## Fundamentals of Measurement and Error Analysis

### 1.1 Introduction

This laboratory exercise will serve as an introduction to all of the laboratory exercises for this course. We will explore proper techniques for obtaining and analyzing data, and practice plotting and analyzing data. We will discuss a scientific methodology for conducting experiments in which we formulate a question, predict the behavior of the system based on likely solutions, acquire relevant data, and then compare our predictions with the observations. You will have a chance to plan a short experiment, make observations, and collect data for yourself, and you will also work with data sets that have been collected in advance.

Pay careful attention to the general rules that will be introduced for dealing with data, so that you can apply them every week. You will be exploring key mathematical relations and manipulating some of the same data sets that you will focus on in future weeks. By learning basic techniques now you will be able to focus on astronomy, and better understand the Universe surrounding you, when we take a second look later. Use this document as a general reference for the experimental and data analysis work that you do throughout the entire course, and be prepared to reread parts of it as you work on other exercises.

#### 1.1.1 Goals

The primary goals of this exercise are to become comfortable planning and conducting simple observations, and to use real data to test hypotheses which relate to behavior of the natural world. We will introduce a set of online tools for this purpose and define key statistical measures which allow us to discuss trends, and you will be guided through their basic usage.

### 1.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed here.

<http://astronomy.nmsu.edu/geas/labs/html/home.shtml>

You will need the following items to perform a simple experiment:

- either a half-cup of pinto beans OR a bag of marshmallows OR several handfuls of gravel OR 30 bushes or trees OR a herd of amiable dairy cows OR access to a parking lot full of cars OR a few shelves of books OR 30 friendly people OR two bags of tortilla chips OR 30 coins of a single denomination
- appropriate tools for measurement, such as a ruler, a tape measure, string, a stopwatch, and/or a kitchen or jeweler's scale

You will not need all of these items; you will select a particular experiment based on your interests and the materials you have available to you (see page 4 for details). For certain experiments, you may find a friendly assistant (one with no particular training, but who is careful) to be very helpful.

You will also need a computer with an internet connection, to analyze your experimental data.

### 1.1.3 Primary Tasks

This lab is built around four activities: 1) planning and conducting a short experiment with common household items, 2) examining existing data to uncover a basic connection between seasonal changes and the height of the Sun in the sky at noon, 3) analyzing data, including error estimates, and 4) making appropriate conclusions based on evidence.

You will also be presented with an overview of our plotting and data analysis tools, and we will review the process for creating laboratory reports and sharing them with your instructors. You should find yourself reading certain sections of this chapter again during future weeks, as part of your preparation for other experiments.

### 1.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. The 19 questions in Section 1.2 (§1.2) are each worth 1 or 4 points each, while the three data tables are worth 10 points together for a total of 38 points. The five questions in §1.3, the four questions in §1.4, and the five questions in §1.5 are each worth 2 points, for a total of 28 points. The three final (Post-Lab) questions in §1.6 are worth 3 points each, for a total of 9 points. Note that §1.8 contains 5 extra credit points.

Table 1.1: Breakdown of Points

Activity	Experiment	Errors	Fits	Trends	Questions	Summary
Section	§1.2	§1.3	§1.4	§1.5	§1.6	§1.7
Page	3	19	23	28	31	33
Points	38	10	8	10	9	25

### 1.1.5 Lab Reports

You will write a laboratory report for each laboratory exercise using the Google Documents application, an online tool which allows you to write reports and share your work with others. To learn how to set up a free account and work with lab report templates, return to the GEAS project lab URL and select “Getting Started > Google Documents” from the masthead. The filename for your first lab report should be defined by your last name and first initial followed by “\_01”. (Albert Einstein’s lab report would be called einsteina\_01.)

### 1.1.6 Timeline

Week 1: Read §1.1–§1.5, and complete activities in §1.2 through §1.4. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Complete activities in §1.5; complete §1.2.4 if you were unable to do so during Week 1. Finish final (Post-Lab) questions in §1.6, write lab summary, and submit completed lab report.

## 1.2 Performing a Simple Experiment

We are going to illustrate experimental procedure by performing a simple experiment, making repeated measurements, and analyzing the data. Read through all of §1.2 before beginning your experiment so that you understand each step of the task and can plan your procedure. Make sure to answer each of the 20 questions contained in this section, and to fill out all three data tables.

### 1.2.1 Planning Your Experiment, and Collecting Data

Begin by selecting a simple experiment, one that we can do with common household objects in a short amount of time. To keep things simple, we will measure a single property, make repeated measurements, and then analyze the distribution of those values. We will analyze the data to determine average values and see how well our measurements agree with each other. We want to become comfortable with this process so that we can apply it confidently

to astronomical data sets later.

We have put together a list of simple, fun experiments, in the form of questions to answer. Select one question from the list shown below, one that you have the resources to conduct and that sounds interesting to you. (If you read through the list and think of an alternate experiment in a similar vein that you would prefer, do contact your instructor and discuss your idea with them. If your idea is a good one, it might even end up on the list of suggested experiments for next year!) You will make a series of 30 measurements in order to study the question.

### List of Simple Experimental Questions

1. How much do individual pinto beans or pieces of gravel weigh?
2. How large are the circumferences of tree trunks or bushes?
3. How far can marshmallows be thrown?
4. What is the point-to-point length of tortilla chips?
5. How many pages make a book (or how wide or heavy are books)?
6. How much milk do dairy cows produce per day?
7. How wide are windshields (or how large are tires)?
8. What is the distance between the pupils of people's eyes?
9. How long does it take you to complete a practice self-review quiz for this class?
10. How heavy (or thick) are coins of a given value?

Each of these questions can be answered by conducting an experiment and making a particular measurement repeatedly. We will need to carefully define the process by which data are collected, collect the data in a uniform and non-biased fashion, and consider the precision of our measurement technique.

You will define and then follow a certain procedure when making measurements. Strive for reproducible results and keep track of the measurement precision, the numerical agreement between multiple measurements made in the same way. Minimize and quantify errors, and work to the highest level of accuracy.

We will describe the distribution of values for the measured quantity by estimating two numbers: the mean, or average value, and the spread, or width, of the distribution around the mean (how close most measured values lie to the mean).

Note that we will not be answering these questions by using archival data (looking up the answers on the internet, for example) but instead by making our own measurements. If you



choose to study how tall people are, you will be measuring their heights (not just asking them how tall they think they are), for example. You will need to make precise measurements for yourself.

Once you have selected a question to answer, you need to plan your experiment. Determine that you have the materials necessary (such as a bag of marshmallows and a tape measure) and a clean, safe location in which to work. (If you decide to measure the properties of cars, make sure you have a well-lit, safe parking area to sample, and that there is no possibility of anyone accusing you of trying to damage the vehicles.)

Think carefully about how you will select the sample of objects to study and how you will make the measurements. If you want to measure tree circumferences, for example, make sure that there are enough trees in your vicinity to sample, and consider ahead of time whether having an assistant would make it easier to wrap a pliable measuring tape or a piece of string around the larger trees. Make your measurements at a particular height from the ground (typically at chest height), so that root growth does not inflate the measured lengths.

Consider how precise your measurement tool is, and whether this will provide you with an acceptable precision in making measurements. For example, if you are measuring the time it takes you to complete an online quiz you should attempt to record the lengths to the nearest tenth of a second, not just the nearest second. A good rule of thumb if you are unsure is that is that your measurement should be made to *at least* one part in one-hundred – if you are measuring a length of ten centimeters, you should record your data to at least a tenth of a centimeter ( $10\text{ cm}/100 = 0.1\text{ cm}$ , also called a millimeter), and ideally, to a smaller length.

1. In the space below, describe your experimental plan. (Remember that you can either write your answer on a printed copy of this page, or you can write directly in your online lab report template.)

Give enough details so that if another student read your notes, he or she could duplicate your efforts in a consistent fashion and obtain a data set that you could use productively in combination with your own data. Later on you will partner with another student and attempt to reproduce each other's results. If measuring the sides of tortilla chips, for example, what brand and style of chip will you use? If using three-sided chips, will you measure all three sides on each chip, or just largest side? Will you use a pliable tape measure (like a cloth one used for sewing) and lay it along each chip side, or measure the shortest distance between the two points along a straight line? What units will you use (centimeters, inches)?

Include a rough guess for the average value that you expect to measure. Keep it simple, and remember that you are simply guessing (in part, to select an appropriate measuring tool) – there are no “wrong answers” to this question. (4 points)

Now that you have a plan, it's time to go ahead and start taking data! You will start by making a few measurements, and perform a rough analysis to check that your results make sense. If necessary, you will modify your technique. You will then complete the data set.

As a first step, perform your measurement seven times, and enter the values into the second column of the table shown below (Table 1.2). In the table title, after the words "Experimental Data I," describe the quantity that you are measuring (such as "Pinto Bean Weights"), and in the column heading for column two, after the word "Value" state the units of the recorded values (such as "centimeter" or "cm") within the parentheses.

Record your values with a single set of units. For example, if you measure lengths in feet and inches convert the measurements into inches before entering them in the table. If you measured a distance of five feet and 4.25 inches, you would use the fact that there are twelve inches per foot to write this as  $5 \times 12 + 4.25 = 64.25$  inches. In this expression the **value** is 64.25, and the **unit** is inches.

You will transfer this information into your online lab report later, but it is probably easiest to first record the data on paper (so that you do not need to have a computer handy).

The word “trial” tells us that we are going to perform the same measurement repeatedly. It is important to be careful, to use the same technique, and to make the measurement to the same level of precision each time. We are trying to sample the underlying, or actual, distribution of a large number of objects (such as all of the pinto beans in the world), but taking only a few measurements. It is thus important to sample as randomly as possible from the set of objects, so if you have a bag of pinto beans, don’t choose the largest (or smallest, or roundest) ones to measure, but simply work with the first seven which come to hand. Of course, if you find one that was chopped in half you should discard it, as it is not a complete pinto bean.

Once you have measured seven objects from your data set, stop and perform an initial data analysis on this sample. We want to determine the average value of the measurements, and how widely they vary over the entire sample. We will thus introduce the concept of a mean value ( $\mu$ , pronounced mu or mew), and of the spread of the distribution ( $\sigma$ , pronounced sigma), also called the standard deviation.

For a more mathematical discussion of mean values and standard deviations, please return to the GEAS project lab URL and select “Getting Started > Mean Values” from the masthead.

Let’s assume that we are conducting the first experiment (weighing pinto beans), as the same technique will apply to any measurement. The mean value of the sample is simply the average of all seven weights. If the seven measurements are 0.124, 0.351, 0.300, 0.323, 0.377, 0.402, and 0.356 grams, then the mean value is simply

$$\mu = \frac{1}{7} (0.124 + 0.351 + 0.300 + 0.323 + 0.377 + 0.402 + 0.356) = 0.319 \text{ gm.}$$

2. Go ahead and enter your seven trial measurements into Table 1.2, and calculate  $\mu$ . Make sure to note your measurement unit within the parentheses on the top line. (1 point)

$$\mu = \frac{1}{7} (\text{_____}) = \text{_____} \text{_____}.$$

Because we have only seven measurements, we want to make sure that an error in measurement does not skew our results dramatically. If you accidentally placed your thumb on the scale while making one measurement, for example, you might end up with an artificially high weight for the pinto bean under examination. We will thus perform the averaging process again, but first discard the lowest and highest measured values.

We begin by sorting the seven values in order from lowest to highest, and then remove the top and bottom values from the list. Take care to remove the *lowest* and the *highest* values, not the *first* and *last* values. (The lowest and highest values are only the first and last values after you sort your list to run from from lowest to highest.)

$$\mu = \frac{1}{5} (0.124 + 0.300 + 0.323 + 0.351 + 0.356 + 0.377 + 0.402) = 0.341 \text{ gm.}$$

Table 1.2: Experimental Data I: \_\_\_\_\_

Trial	Value ( _____ )
Trial 1	_____
Trial 2	_____
Trial 3	_____
Trial 4	_____
Trial 5	_____
Trial 6	_____
Trial 7	_____

(2 points)

Our revised estimate of the average weight of a pinto bean is thus 0.341 grams.

3. Perform the same operation for your innermost five trials from Table 1.2 below. (1 point)

$$\mu = \frac{1}{5} ( \text{_____} ) = \text{_____} \text{_____}.$$

We next want to estimate the scatter in values – are most pinto beans within a tenth of a gram, or within ten grams, of the mean value?

The standard deviation tells us the average difference between the mean value and the individual measurements. It has the same units as your data points, just like the mean value. If lots of points fall much lower and much higher than the mean value, you will have a large standard deviation. If your points cluster tightly around the mean, then your standard deviation will be small.

The standard deviation is calculated by subtracting the mean value from a set of measurements, and then combining the remainders. We have five data points: 0.300, 0.323, 0.351, 0.356, and 0.377 grams. How far do these points lie from the mean value? The difference (higher or lower) between the first value (0.300 gm) and the mean value (0.341 gm) is 0.041 grams. For the other four points, we find differences of 0.018, 0.010, 0.050, and 0.036 grams.

(Note that we express each difference as a positive number, so that a point above the mean cannot “cancel out” a point below the mean.)

Sorting the differences from smallest to largest, we get values of 0.010, 0.018, 0.036 0.041, and 0.050 grams. The standard deviation for this data set is 0.030 grams, which is similar to the average value of these differences.

We can combine our values for the mean value  $\mu$  and the standard deviation  $\sigma$  and say that our observed sample of pinto beans has a weight per bean of  $\mu \pm \sigma$ , or  $0.341 \pm 0.030$  grams. In general you should find that two-thirds (67%) of all data points lie within  $\pm\sigma$  of the mean value for a distribution (and 95% should lie with  $\pm 2\sigma$ ), so we expect that two-thirds of a large sample of pinto bean weights should lie between  $0.341 - 0.030 = 0.311$  and  $0.341 + 0.030 = 0.371$  grams.

4. Do roughly two-thirds of our five bean weights lie between 0.311 and 0.371 grams?  
( Yes / No ) (1 point)

It is a bit of work to calculate standard deviations by hand, so we will go ahead and use a computer program. We will also plot our data as histograms, so that we can examine the distribution of the data points.

### 1.2.2 Learning about Histograms

We will begin by introducing the concept of a histogram, a plot which shows how many measurements fall near to or far from a mean value. If you measure something in the natural world a large number of times, the distribution of measured values will form what we call a normal distribution, or a bell curve. There will be a large number of measurements piled up near to the mean (average) value, and as we move further and further from the mean the number of measurements will drop to zero.

The plot shown in Figure 1.1 is a distribution of the number of pages measured for texts from a shelf of books at a public library. The  $x$ -axis runs horizontally across the page, and tells you how many pages there are in each book. The  $y$ -axis runs vertically up and down the page, and tells you how many books fell into each page range. We group the books into bins, each 15 pages wide, and then count how many books fall within each bin. The most filled bins surround the mean value, and as we move further away from this value, we find that there are fewer and fewer books in each bin.

Notice that the plot has a caption, which describes its content and the primary conclusions drawn from it, and the  $x$ - and  $y$ -axes are clearly labeled, and include units as appropriate. You should format any plots that you place within your laboratory reports similarly.

Figure 1.2 shows the distribution of book lengths again, and superimposes a normal distribution (the blue bell-shaped curve) on top of the data. Notice how the peak of the bell curve, the mean value of the distribution, lies roughly in the middle of the distribution. The

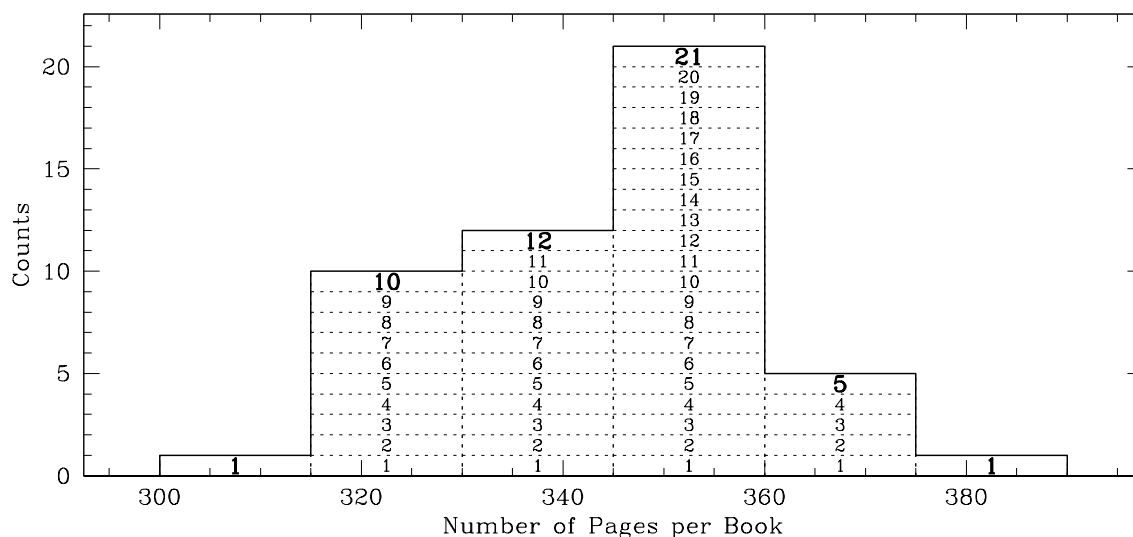


Figure 1.1: The distribution of measured book lengths (in units of pages), for a sample of 50 novels. Only one book had between 300 and 315 pages, 10 had between 315 and 330, 12 between 330 and 345, 21 between 345 and 360, five between 360 and 375, and one final text was between 375 and 390 pages long. The average book length for the sample thus lies somewhere between 340 and 350 pages.

more scattered the values are, with more book lengths lying further away from the mean, the wider the distribution, and the bell shape, will be. In the left-hand panel, the curve and the number of books per bin are only rough approximations of each other. As we increase the size of the sample, however, we can make the bins smaller and still fill each bin with a statistically significant number of points, and the distribution of book lengths will become smoother, and appear more like the underlying bell curve. Figure 1.3 displays a Gaussian drawn to fit these data, with the mean value ( $\mu$ ) and the size of  $1\sigma$  shifts away from the mean labeled for clarity.

Now that we have an idea of the form of a histogram, let's examine a few distributions of measurements made, quite similar to those that you are making for yourself this week. Load the GEAS project lab exercise web page into a web browser (see the URL on page 2 in §1.1.2), and click on the link labeled “Web application #1 (histogram demonstration).” This will cause a new browser window to appear, containing a web application that we will use to examine a few histograms.

This web application is provided in a modern HTML5 format and an older Flash-based format. The legacy Flash-based tool is shown in the video tutorial for this lab exercise. Small cosmetic changes were made for the HTML5 version, but it operates the same way. We recommend that you use the modern HTML5 versions of all of our web applications.

When you first open the web application, look over the entire screen carefully and make sure that you understand the options available to you. There are General Options shown in

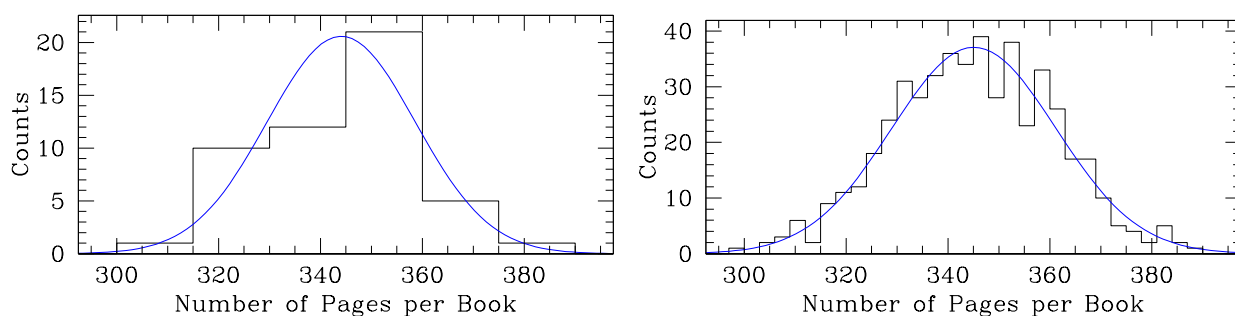


Figure 1.2: The distribution of measured book lengths for the sample of 50 books shown above in the previous figure (left), and for a larger sample of 500 books (right). The smooth bell-shaped curve shows the shape of the estimated underlying distribution in each case, with mean values (344 versus 345 pages) which match the peaks of the curves. Roughly two-thirds of the measurements fall within  $\sigma$  ( $\pm 15$  pages) of the mean value. As the sample size increases, we can make the bins smaller, and the distribution of points shifts closer and closer to matching the smooth bell shape of a normal distribution.

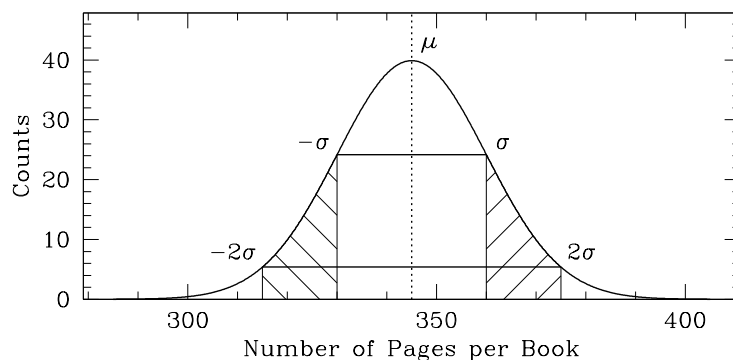


Figure 1.3: A smooth Gaussian, or bell curve, fit to the distribution of book lengths shown in the previous figure. The mean (labeled  $\mu$ ) is the center of the curve, and has the highest  $y$ -value (the peak of the bell shape). A line labeled  $\pm\sigma$  extends from  $\mu - \sigma$  to  $\mu + \sigma$ ; roughly two-thirds of measurements will fall between these positions. A second, lower line labeled  $\pm 2\sigma$  extends further, from  $\mu - 2\sigma$  to  $\mu + 2\sigma$ ; it includes the empty  $\pm\sigma$  region and the two regions between  $-1\sigma$  and  $-2\sigma$  and between  $1\sigma$  and  $2\sigma$  (hatched for emphasis). Almost 95% of measurements fall between  $\pm 2\sigma$ , leaving two small tails to the far left and far right of the distribution made up of points which fall more than  $2\sigma$  from the mean.

the lower right corner, which allow you to print out (make a paper copy) of your plot, or to save a copy as a PNG-format image file on your computer disk (or to attach such a file to an e-mail message that you can send to yourself). By clicking on the button labeled “Help” you will get a quick run-down on the tool options and purpose, and “Credits” will tell you a bit about the creator. You should always click on the “Help” button when you examine a new web application, as your questions may be answered there.

PNG format is simply a particular file format that stores all of the information that makes

up a drawing. (You may use other image formats if that is easier for you; what matters is that the image displays properly within your lab report.) When you save a copy of a histogram (or another image from one of our web applications), your browser may attempt to attach a .txt extension to the file name, rather than letting it end in a .png extension. It is important that your file name end in .png; a name like histogram01.png is good, while a name like histogram01.png.txt is bad. If you are unsure whether your file name was defined correctly, check the name of the file on your local disk. You may need to rename it, to remove an errant .txt extension. As a double-check, try double-clicking once on the name of the saved file. If it opens up and the drawing appears, then your computer has successfully identified the file as having been written in PNG format.

This tool enables you to create histograms that show the distribution of data points for four sample data sets. Under the Experiment Options listed in the upper right corner, you can change the number of measurements that you make (see “sample size”), to observe the effect of taking lots of data, or only a little bit. You can also make your measuring tool (your ruler, or scale) very precise or very vague (see “precision”), to see how important it is to measure properties to a significant number of digits (such as measuring a length to 324.35 cm rather than just writing down 320 cm). The final variable to change allows you to “clip” the data, and discard the lowest and highest measured values from your data set before analyzing the points. (All of the  $N$  data points will still appear on the histogram, but only the innermost  $N - 2$  ones will be used to determine the mean and sigma values.)

Part of doing science is being able to communicate your work clearly to others. Read the title and axis labels on the plot, so that you understand its purpose. The green columns show the number of data points in each bin, where the central bins lie closest to the mean value of the distribution. The taller the column, the more data points it contains. The red curve shows you the shape of the underlying distribution of the measured property, while the green curve shows you the approximate version estimated from the presented data set. Experiment by changing the four variables listed under Experiment Options, to see how your choices affect the level of agreement between the underlying and the observed distributions.

Let us now work through a few questions, in order to test our understanding of histograms. Start by choosing one of the four data sets, and selecting a value for the sample size and the measurement precision. Now change one of these three elements, but before you do so, predict to yourself the effect that the change will have on the plot. Repeat this process several times, until you feel that you have a good impression of how the tool works. Once you have done so, read through and answer the questions listed below.

5. Set the web application to show a distribution of seven tortilla chip lengths, with a precision of 0.05 cm. What happens if you turn data clipping on and off? When you remove the lowest and highest values from the sample, the observed value for sigma (the spread around the mean value) always (decreases / stays constant / increases ). (1 point)
6. Now change the number of measurements from seven to 70,000 chip lengths. When you remove the lowest and highest values from the larger sample, the change in the observed



value for sigma is (smaller / the same / larger ) then/as when this is done with only seven data points. (1 point)

7. Clipping the two outermost values from a distribution of only seven points will almost always cause the estimated value for sigma to drop to well below 100% of the underlying value. Why then do we suggest doing this in some real experiments? (1 point)

8. If a histogram of seven data points looks more like a picket fence than a bell curve, roughly how many values do you need to measure before the sample looks recognizably like a smooth bell curve?

Save a copy of your bell curve histogram to disk as an example, and include it in your Google Document lab report. Click on the button labeled “Save (or Print) Figure” to save a copy of the image to your local computer disk, making sure that the file name ends with the suffix “.png.” To insert this file in your Google document lab report, open the online report, and choose “Insert > Image” from the menu bar labeled “File, Edit, View, Insert, ...” Click on the button labeled “Browse” which appears on the “Insert Image” menu, and then double-click on the name of your PNG-format file. The image will appear within the online document. You can then adjust its size to fit well within the page margins by clicking once on the image, and then shifting the small square symbols which appear at each corner of the image. Pull one of the squares toward the center of the image, and the entire image will decrease in size. (1 point)

9. As you increase the number of measurements, what happens to the relationship between the underlying and the observed values of  $\mu$  and  $\sigma$  – do the observed values become more accurate, getting closer to the underlying values? ( Yes / No ) (1 point)

10. Can you ever determine an observed value for  $\sigma$  which is less than the measurement precision? ( Yes / No ) (1 point)

11. Set the web application to show a distribution of 1,000 book lengths, with a precision of 10 pages, and then change the quantity being measured from book lengths to tortilla chip lengths. What are the key differences between the two histograms, and why are they so different? What can you conclude about the way in which you should decide if your measurement precision is sufficient for a given experiment? (1 point)

### 1.2.3 Plotting and Analyzing your Data

Now that we have worked with a few histograms, let's build one for the seven data points that you collected in Table 1.2. Reload the GEAS project lab exercise web page (see the URL on page 2 in §1.1.2), and click on the link labeled “Web application #2 (plotting tool).” This will cause a new browser window to appear, containing a web application that we will use to plot your data as a histogram. You will be using this same plotting tool throughout the semester, so make sure to examine it carefully and clear up any questions that you have about its operations with your instructor.

First type your last name and access code into the two text boxes at the top of the form. Then carefully read through all of the information listed on the web page.

This web application can be used to plot several types of data. We can create histograms, or we can plot one variable as a function of another (we call this an  $xy$  plot, or a scatter plot). We will discuss  $xy$  plots in §1.5.2; so for now, concentrate on the histogram options.

We are going to create a histogram plot from your seven measurements. Begin by selecting “Histogram” at the top of the form from the pull-down list of three plotting options. Next, type in a title for your plot, and labels for the  $x$ - and  $y$ -axes. These labels should clearly identify the contents and purpose of the plot that you are creating. If you were measuring dairy cow output, for example, you might type a title of “Dairy Cow Daily Milk Production”, and use  $x$ - and  $y$ -axis labels of “Amount of milk per cow (liters)” and “Counts.” Keep each

of your plot titles and axis labels to less than 40 characters (40 letters and spaces) in length. Be careful not to reverse the  $x$ - and  $y$ -axes labels – the  $x$ -axis identifies the quantity being measured, and its units, while the  $y$ -axis tells us how many data points fall within each bin along the  $x$ -axis.

Scroll down until you see the text box labeled “numbers to plot”, and enter your seven measurements into the box, placing one measurement per line. Do not add any commas, parentheses, or other extra characters – just type each measurement on a separate line.

Once you have completed these steps, click on the button labeled “Create Plot” and your plot should appear in a separate browser window. Does it appear as you expected it to look?

A title should appear above the plot, and the  $x$ - and  $y$ -axes should be clearly labeled (review the example plots online if you are unsure how to label your axes). Your plot should show a histogram drawn as a series of light blue columns. Above the columns, the mean and sigma values will appear, labeled as “Mean value of  $\mu \pm \sigma$ .”

12. Record the values below, adding units at the end of the line.

$\mu \pm \sigma =$  \_\_\_\_\_.

Now click on the back button on your browser, to return to the plotting tool. Remove your highest and lowest measured values from the set of seven, and remake your histogram. (You may need to retype your ID code to regain access to the database.)

Record the new values for  $\mu$  and  $\sigma$  below, again with units.

$\mu \pm \sigma =$  \_\_\_\_\_. (1 point)

13. How do the two  $\mu$  estimates compare (by how many  $\sigma$  do they differ)? Did your value for  $\sigma$  change in a predictable fashion? Do your two estimates of  $\mu$  agree with those that you determined earlier by hand, written down on page 8? (1 point)

14. Examine your data, and decide whether your measurements were made with sufficient accuracy. (For example, did you eliminate any potential systematic errors that you identified?) Was the limiting precision good to at least one-hundredth of the mean value? Are

there any changes that you need to make to your plan, now that you have taken a small set of test data? (1 point)

You are now ready to take a larger data set. Fill in Table 1.3 with a set of 30 measurements. (If you are completely satisfied with your first seven measurements, you may repeat them here in the first seven boxes.) Make sure to again label the table with the name for the quantity being measured, and to note the units of the measurements.

15. When you are done, create a new histogram from your larger data set. Save the image file to disk (or e-mail it to yourself), and include it in your Google Document lab report. Be sure to include a plot title, label the  $x$ - and  $y$ -axes, and add a caption to go with the figure in the lab report explaining its contents and your primary conclusions. (You should do this with all of the figures that you include in your lab reports.) (1 point)

### 1.2.4 Reproducing Your Experiments

An important part of a successful experiment is being able to describe it clearly and accurately, so well that another person could read your description and then conduct a successful second set of trials. In order to test how well you defined your simple experiment, your instructor will give your experimental description (your answer to question 1 on page 5) to another student, and give you their description of *their* experiment in return. (*Be sure to enter your experimental description into your lab report as quickly as possible during Week 1, so that you can trade descriptions with another student and still have time to complete the entire lab assignment by the end of Week 2.*)

Once your instructor has given you the description of your partner's experiment, attempt it yourself, recording 7 data values in Table 1.4 and entering these values and your derived mean and spread for the inner 5 points in your lab report. Once you have done so, your instructor will introduce you and your partner to each other via e-mail. Together, the two of you will compare the mean and spread values that you each determined for each of your experiments.

16. For your partner's experiment, your  $\mu \pm \sigma =$  \_\_\_\_\_,

and their (5-value trial)  $\mu \pm \sigma =$  \_\_\_\_\_. (1 point)

Table 1.3: Experimental Data II: \_\_\_\_\_

Trial	Value ( )	Trial	Value
1		16	
2		17	
3		18	
4		19	
5		20	
6		21	
7		22	
8		23	
9		24	
10		25	
11		26	
12		27	
13		28	
14		29	
15		30	

(6 points)

17. For your experiment, your (5-value trial)  $\mu \pm \sigma =$  \_\_\_\_\_,

and your partner's  $\mu \pm \sigma =$  \_\_\_\_\_. (1 point)

18. Discuss the results that you and your partner obtained for both experiments. How close do you think that your mean values should be (in units of  $\sigma$ ) in order for you to say that you described and conducted reproducible experiments? By how many  $\sigma$  do your mean

Table 1.4: Experimental Data III:

Trial	Value ( )
Trial 1	
Trial 2	
Trial 3	
Trial 4	
Trial 5	
Trial 6	
Trial 7	

(2 points)

values actually differ? If the difference is larger than your requirement, brainstorm with your partner and identify the most likely reasons for the discrepancies. (For example, if you measured marshmallow toss lengths and one of you is the star pitcher on the baseball team, you have a probable explanation for why that person's mean toss length was larger than their partner's value.) (*You may each write answers to this question, or you may create a single answer together and place it in both of your lab reports.*) (4 points)

19. Having gone through the experience of having another person attempt to duplicate your experiment based on your written notes, how would you alter your experiment and change your experimental description, if you were to do this exercise again with a new partner? (4 points)

## 1.3 The Role of Errors

Read through all of §1.3 before beginning to answer the questions in the text, so that you understand the material. Make sure to answer every part of the five questions contained in this section.

### 1.3.1 Three Types of Error, or Variation

In scientific parlance, the word “error” often means “uncertainty” rather than “mistake,” and refers to a variety of factors that can combine to blur agreement between measurements. No set of measurements is perfect, and most experimental variables have an expected spread in value (measuring scales are not infinitely precise, and quarter-pounders do not all weigh exactly one-fourth of a pound).

We are going to define three factors (natural variation, measurement error, and systematic error) which can produce changes in measurements, and learn about their effects. Measurement and systematic errors relate to the fact that we cannot design perfect experiments, where only one variable ever changes and properties are measured to infinitely small precision. Natural variation can refer to the intrinsic spread in a quantity (for example, differences in age, gender, and health produce differences in height, so all people are not the same height), or to the inevitable variation in manufactured items (even the United States Mint cannot produce truly identical pennies). Strictly speaking, natural variation is an intrinsic characteristic (how different are a set of objects, if measured perfectly?) and not a source of error, but because it can produce a similar effect we will consider it here.

#### Natural Variation, or Natural Error

Natural variation refers to the intrinsic width of a distribution of a measured property. If you carefully measured the widths of a large set of pennies with a vernier caliper (a measuring

device with two long arms, capable of making very precise measurements of small lengths), you would find that the values clustered around a mean value, but that many tended to be slightly smaller or larger than that central value. The width of the distribution, or the amount that most measurements were above or below the mean, would tell you the natural variation in coin widths, or how much on average pennies differ from each other.

Natural variation is often a key quantity that an experiment will determine. For a well-performed experiment, it should be analogous to the measured value of  $\sigma$ .

### Measurement Error

Measurement error refers to the precision with which a value is measured. If you carefully weighed a large set of books, but only recorded their weights to the nearest pound, then your measurements could all be off by  $\pm 1/2$  pound. If you were examining books that weighed around a pound, for example, then your measurements would all tend to be 0, 1, or 2 pounds (plus a few 3-pounders, if you had a passion for Dostoevsky). If you increased the precision of your measurements by finding a scale that output values to the nearest hundredth of a pound, however, then you would be able to discern much smaller differences in weight between individual books.

### Systematic Error

Systematic error refers to an aspect of an experiment which acts to distort the measurements being made, always in the same fashion (or direction). If you carefully measured the heights of a large set of people, but forgot to have your test subjects take their shoes off, then all of your measurements would be too high (they would all be biased in the same direction). People wearing high heels would have their heights overestimated more than people wearing thin-soled flip-flops, but all of the heights would be overestimates – this is why we call this type of error *systematic*. While natural error and measurement error can produce both large and small measurements, systematic errors shift all of the measurements in one direction (all high, or all low).

In addition to the limiting precision of measurement error and the biasing effects of systematic errors, you might also have more general errors that distort measurements. For example, the wind might carry one thrown marshmallow farther away from you, but force the next to land close by; a careless experimenter might record a measured weight incorrectly; a poorly-designed experiment might ignore additional relevant factors which affect the results. In the interests of simplicity, we shall assume that as part of your experimental design process you consider and attempt to resolve such concerns before performing your experiments.

In this course, we want to identify and eliminate sources of systematic error, and reduce measurement errors to low enough levels that they do not interfere with our determinations of natural variation. As you plan your own experiments, ask yourself how systematic errors might creep into your experimental plan, and how precise your measurements need to be in order to let you measure the desired quantities accurately.

1. Select any one of the ten experimental questions listed on page 4. Identify a systematic error that could affect this experiment, and describe how to eliminate it. (2 points)



2. Compare the value of  $\sigma$  for your 30-point data set, and the precision  $P$  (the smallest difference that you could measure between two values).

Which of these two values represents your natural variation? (  $\sigma$  /  $P$  )

Which of these two values represents your measurement error? (  $\sigma$  /  $P$  )

Is your precision less than (or equal to) one-hundredth of your mean value? ( Yes / No )

$$\frac{P}{0.01 \times \mu} = \frac{\quad}{0.01 \times \quad} = \quad .$$

How does your precision compare to your value for  $\sigma$ ?

$$\frac{P}{\sigma} = \frac{\quad}{\quad} = \quad .$$

In general, would you prefer that  $P/\sigma$  be smaller or larger than one? ( Smaller / Larger )  
(2 points)

### 1.3.2 Error Bars

We have established that any measurement should have an associated error attached to it (for a single data point, this could be the precision of the measurement tool, and for the average value of a set of measurements, this could be the spread  $\sigma$ ). How do we combine measurements when they were taken in different ways, and have different associated errors, and how do we represent the variation in associated errors?

When we have different errors associated with different points, it is often convenient to show these differences, by representing the errors as *error bars* (literally, bars of length  $\sigma$  drawn above and below each data point). In Figure 1.4 we display three measurements, with associated errors added on the right-hand panel. On the left, the three measurements are shown without any indication of relative error. Without any information on the relative importance of the points, we simply average them, deriving the value shown as a red line between the three values. On the right, error bars have been added to each point showing their relative precision. Realizing that the middle data point has much larger errors associated with it than the other two points have, we can minimize its impact, computing an average value which falls between the two points with relatively small error bars associated with them.

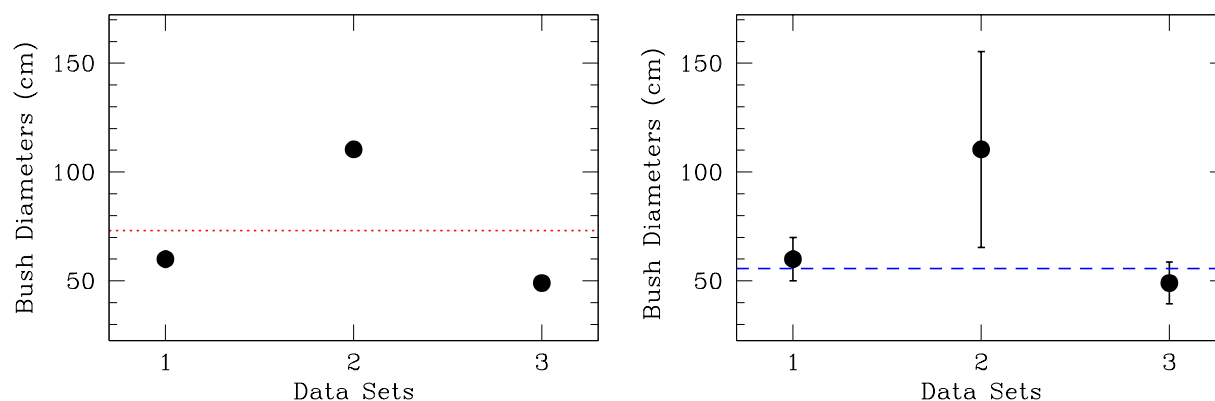


Figure 1.4: Three separate measurements of bush diameters are shown, with and without associated error bars. On the left, the three data points are averaged evenly to produce a representative value which lies in between all three points. On the right, the addition of relative errors allows us to count points 1 and 3 more than 2, and the representative value shifts down to lie between them. The knowledge that point 2 is much less constrained allows us to discount it, and thus to determine a more informed final value.

One might ask whether it would be appropriate to discard the central data point, given how far it lies from the other two, but with only three recorded data points that would be a risky action.

Error estimates, and error bars, allow us to combine data taken under different conditions. Though they cannot identify systematic errors, they are effective tools for factoring measurement errors into determinations of average properties. Consider, for example, measuring 100-meter race times accurately. In 2009, Usain Bolt of Jamaica set a world record with an amazing time of 9.58 seconds, and times for any two of the ten fastest men in the world often differ by a mere 0.01 seconds. To record such a race fairly, the race times should be recorded to the nearest 0.001 seconds (or less). Imagine that you wanted to compute an average race time for a runner, and you had some measurements recorded with Olympic track equipment, and others taken with your grandmother's antique pocket watch. To which times would you assign the most importance (and the smallest error bars)?

3. If the error bars on all of the points in Figure 1.4 were all the same length, how would the derived average values change? The red line on the left panel would ( drop / stay at the same height / rise ), and the blue line on the right panel would ( drop / stay at the same height / rise ). (2 points)
4. If the error bars on point 3 in Figure 1.4 became smaller, the blue line on the right panel would ( drop toward point 3 / stay at the same height / rise away from point 3 ). (2 points)
5. What can error bars add to an analysis? (2 points)

## 1.4 Evaluating Data, and Model Fits

As we begin to study different types of astronomical data, we will need to have a toolbox of basic ways to evaluate different data sets. In one case we might need to combine a large number of repeated measurements of the same quantity (like measuring how much light is emitted from a faint star, by combining telescope observations taken on different nights). Alternatively, we might need to search for a trend between two variables (does the observed phase of the Moon depend on how long after sunset it rises?).

We will describe three different evaluation tools below, each one chosen because it will be useful to us in studying some type of astronomical data in this course. They should seem somewhat familiar to you, because you have already begun to use these concepts!

Keep in mind that our emphasis in this course will be on using these indicators *qualitatively* rather than *quantitatively*. This means that we are more interested in your being able to interpret results than in your calculating fit coefficients by hand. Our tools are designed to provide the analytical information that is most helpful to understand the meaning of your plots. (Simply put, you don't work for them – they work for you!)

Read through all of §1.4 before answering the questions, so that you have a good grasp of all four topics. Make sure to answer each part of the four questions contained in this section.

### 1.4.1 Mean Values and Standard Deviations

It is useful to combine repeated measurements, to determine an average value ( $\mu$ ) and to characterize the variation between measurements (with  $\sigma$ , the spread, or standard deviation). As we learned in §1.2.2, a histogram can also help in checking whether the measurements are consistent with a normal, or bell curve, distribution.

1. In Figure 1.5, how do the means of the two distributions compare (roughly)? Which one has a higher  $\sigma$  value? Identify another significant difference between the two sets of data. (2 points)

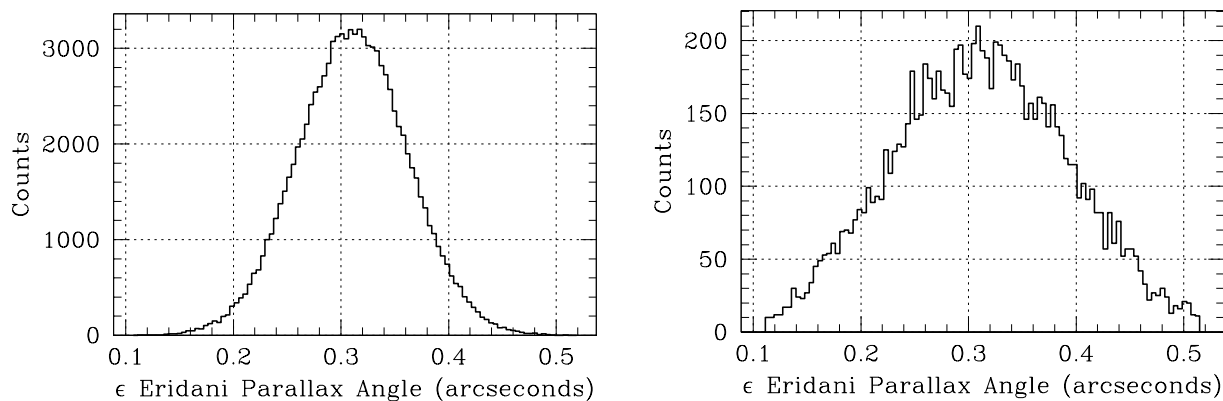


Figure 1.5: Histograms showing the distribution of measured parallax angles for the star Epsilon Eridani, in two different astronomical surveys. Estimate the values for  $\mu$  for each sample, and compare the  $\sigma$  values. How wide is each distribution, in units of arcseconds, at the points where the number of counts has dropped by a factor of two from the peak value?

### 1.4.2 Slopes and $y$ -intercepts

When measuring two variables ( $x$  and  $y$ ) and trying to deduce a relationship between them, we frequently plot one against the other, creating an  $xy$  (or scatter) plot. We can then compute the slope ( $m$ ) and  $y$ -intercept ( $b$ ) for the combined data set (as we will explore in more detail in §1.5). For a quick review of slopes and  $y$ -intercepts, return to the GEAS project lab URL and select “Getting Started > Slopes and  $y$ -intercepts” from the masthead.

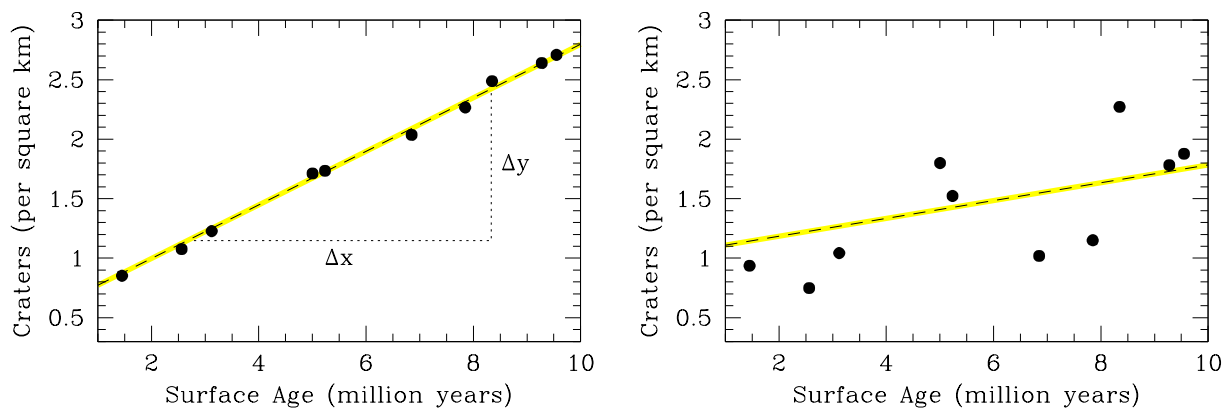


Figure 1.6: Plots showing the relationship between the surface density of craters on Mars and the age of the surface for two samples. Estimate the slope and the  $y$ -intercept from each plot for practice, and examine how scattered the points are around each line.

2. In Figure 1.6, the slope for the left panel sample is ( smaller than / the same as / larger than ) the slope for the right panel sample. The  $y$ -intercept is larger in the ( left / right ) panel sample, and the points in the ( left / right ) panel sample show more scatter (lie further from the line). (2 points)

Just as we add error bars to a point to indicate how confident we are in the value, we can add error measures to slopes and  $y$ -intercepts to indicate how tightly constrained they are. The larger the error bars, the more the slope or intercept could change without substantially changing the quality of the fit (and the further the points lie in general from the line).

Imagine that we want to determine the speed at which a star is moving, in units of kilometers (km) per second, so we measure its position and fit a line to the position as a function of time. The line has a slope  $m = 5.00 \pm 0.05 \text{ km sec}^{-1}$ . A change in the slope of 0.10 (twice the assigned error) is significant, so the slope probably lies between 4.9 and 5.1  $\text{km sec}^{-1}$ .

If the star were instead moving at a speed of  $m = 5.0 \pm 0.5 \text{ km sec}^{-1}$ , then we could only say that the slope probably lies between 4 and 6  $\text{km sec}^{-1}$ , a much broader range of velocities. What if the slope were  $m = 5 \pm 5 \text{ km sec}^{-1}$ ? The speed could lie anywhere between -5 and 15  $\text{km sec}^{-1}$ . The error tells us how well we know the velocity.

3. If you determined that a star was moving at a speed of  $15.0 \pm 0.5 \text{ km sec}^{-1}$  in a particular direction but I found that its speed was  $14.2 \pm 0.5 \text{ km sec}^{-1}$ , could our results be consistent? Roughly how much lower would my velocity have to be for you to say that either our two studies must have observed different stars (you said Asellus Borealis, but I heard Asellus Australis), or something else was significantly different about them? You may assume that a  $5\sigma$  or larger deviation is significant. (2 points)

### 1.4.3 Root Mean Square Deviations

When the points in a plot follow more closely to the fit line, the fit is generally a better match to the data. This tells us that there is a better agreement between the fit and the raw data points, because the line represents a more accurate physical model of the relationship between  $x$  and  $y$ , and/or because the errors on the individual data points are smaller. In Figure 1.6, for example, the points in the left panel are less scattered than those in the right panel and so we have more confidence in the derived slope and  $y$ -intercept.

When we examine repeated measurements of a single quantity (see §1.2.1), we determine a mean value ( $\mu$ ) and then a spread around it ( $\sigma$ ). The smaller the value for  $\sigma$ , the closer we expect a set of measurements to lie to  $\mu$ . In a sense,  $\sigma$  tells us how accurate the mean value is, and when we compare mean values from two experiments and ask if they are consistent

(in agreement), we calculate their difference in terms of  $\sigma$  values. A difference of  $2\sigma$  or less is reasonable, while a difference of  $5\sigma$  indicates that something is probably different about the two data sets (or that something has gone wrong in one, or both, of the experiments).

We can use the same idea for  $xy$  plots. Rather than asking how tightly constrained a mean value is, we examine the slope and  $y$ -intercept values and attach  $\sigma$ -like spreads to them. When we fit a line to a set of points, we minimize the distance in  $y$  between each data point and the line. We want the combination of these offsets to be as small as possible. Figure 1.7 shows two possible lines drawn through the same set of points. Which line minimizes the offsets to the data points in  $y$ ?

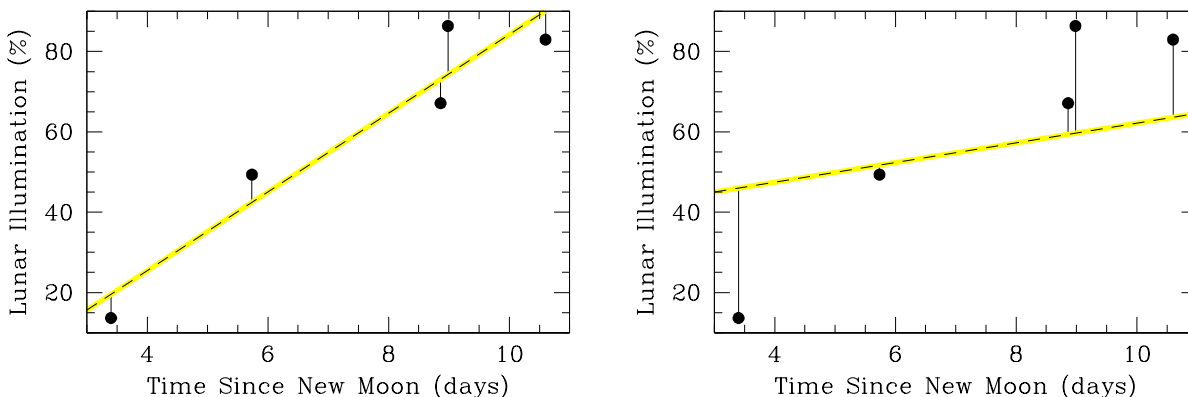


Figure 1.7: Two plots showing a relationship between the amount of time since the last new Moon and the percentage of the Moon’s surface which appears illuminated from Earth. In the left panel the line has been placed to minimize the offsets from  $y$  (shown as vertical lines) from the set of points, while the offsets on the right panel are larger and the fit is less good.

We can quantify, or express as a number, the effect of these offsets in  $y$ , by adding up the net effect. We first square each offset, then sum them, and average the sum to determine what we call the “root mean square” deviation (the “rms” value). First, we need to calculate the  $y$ -position that each point would have if it lay exactly on the line (rather than a bit above or below), and find the difference between this position and that of the actual point. The offset  $o$  is defined as

$$o = y - (mx + b).$$

We square each offset so that points above and below the line will not cancel each other out (by having offsets which are equal in length but have opposing signs), and then divide by the number of points to find the average value. Finally, we take the square root to end up with a quantity which has the same units as  $y$ , and which does not increase or decrease just because we vary the size of the sample.

This rms value is a measure of how well the line fits the data. A large value indicates that there is a problem with the data set or that the model which defines the line is not a good model to completely explain the connection between the two variables ( $x$  and  $y$ ). The slope and  $y$ -intercept values are specifically chosen so that the rms value is as small as possible.

You may find rms values labeled as either “rms”, “rms  $\sigma$ ”, or as just “ $\sigma$ ” in various science classes; we’ll use the rms term here, to avoid confusing rms values for  $(x,y)$  data with the spread of  $y$  values around a single measurement (which we called  $\sigma$  when introducing histograms in §1.2.1). These deviations are very similar quantities, albeit for two types of data, and  $\sigma$  is a fine general term for them. The primary difference is that a standard deviation ( $\sigma$ ) for a repeated measurement compares each measurement of  $y$  to the mean (average) value  $\mu$ , while an rms deviation compares each measurement of  $y$  to the *predicted*  $y$ -value (such as  $mx + b$ ) generated by a fit to paired  $(x,y)$  data.

4. In Figure 1.7, the rms value for the line in the left panel is 5.7, while the value for the right panel is 10.5. Does the lower rms value belong to the line which passes most closely through the points? ( Yes / No ) What would it mean if a data set had an rms value of exactly zero? Does this seem very likely to happen? (2 points)

We have examined rms deviation values for a simple linear fit through a set of data points, but we don’t need to limit our usage of this quantity to linear fits (straight lines). We can fit any function that we choose to reproduce  $y$  as a function of  $x$ , and then calculate the offsets between the  $y$  values and whatever shape of line our chosen model defines.

In Figure 1.8, we fit a straight line ( $y = mx$ ) and then a curved line ( $y^2 = mx$ , or  $y = m\sqrt{x}$ ) to a single set of data points. (For simplicity, we will just assume that the  $y$ -intercept is equal to zero in each case, so that all lines pass through the origin.) Which of these two functions produces a line which best matches the data points? We can often decide which function is a better match to the data by eye, just by examining the offsets in  $y$ . In a case like this, even a cursory examination shows that the offset lines are larger on the right than on the left, so we know before considering the rms values which model is better.

When comparing the two rms values, be sure to take their ratio (divide them) rather than subtracting them from each other. The fact that one is larger than the other by +1.71 is significant because both rms values are less than three, but would be much less significant if both values were larger than, for example, 1,000 (say, 1000.79 versus 1002.50). However, the fact that one is a factor of 3.16 times larger than the other is significant in any case.

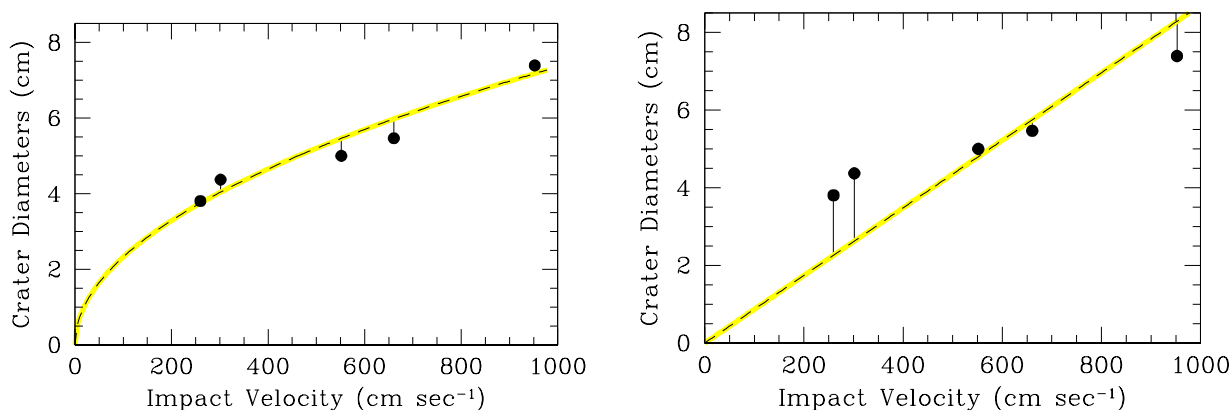


Figure 1.8: Two plots showing the relationship between the impact velocity of a small rock ( $x$ ), and the size of the resulting crater ( $y$ ). The points are fit with the function  $y = m\sqrt{x}$  on the left panel, and  $y = mx$  on the right panel. Because the points are less offset in  $y$  from the yellow model line in the plot on the left (rms = 0.79) than on the right (rms = 2.50), we observe that this function is a better match to the relationship between  $x$  and  $y$ .

## 1.5 Trends, or Relationships, Between Variables

We often seek to understand the relationship between two measured quantities, in order to better understand the underlying behavior of the Universe. How does the angle between the Earth, Moon, and Sun relate to the apparent lunar phase? Are the brightest stars in the sky intrinsically bright, or just very near to us? Why do the accretion disks surrounding some black holes give off high energy x-rays? Many such questions can be explored by comparing the behavior of two variables, to see whether or not they are dependent on each other and if so, how they relate.

Be sure to answer fully each of the five questions in this section.

### 1.5.1 Learning about $xy$ (or Scatter) Plots

If we are studying two variables, we often label them  $x$  and  $y$ . A plot which shows the value of  $y$  for each value of  $x$  is then called an  $xy$  plot, or a scatter plot. When working with such data, make sure that you know the definition for each measured quantity, and the units, and check whether there are associated errors.

Consider how representative the data set may be. If you wanted to study the typical ages of human beings, how might your results differ if you only examined people found at a pre-school, or only those who live in a retirement home? In each of these cases, you would be drawing your data from biased samples (samples which do not represent the entire population of humans on this planet), and so your results would be biased as well. Because we typically sample a small subset of the entire set of objects under study, we need to be very careful to select representative objects.



We are going to plot the values of two measured quantities against each other and then examine their relationship. We will analyze June temperatures for various locations as a function of the Sun's peak altitude (how high it appears in the sky at noon). This exercise will get you thinking about how seasonal variations across the surface of this planet, and to help you to become comfortable making *xy* plots.

### 1.5.2 Plotting and Analyzing your *xy* Data

Begin by returning to the GEAS lab exercise web page (see the URL on page 2 in §1.1.2), and click on the link labeled “Seasonal table.” This links to a page of data. A set of comments at the top of the page describes the listed quantities, and the three columns of numbers listed below represent the peak altitude of the Sun in the sky, in units of geometric degrees (column 1), June temperatures, in degrees (column 2), and associated errors, in degrees (column 3) for a set of northern and southern hemisphere locations across the globe.

Keep in mind that the peak altitudes for the Sun are measured in geometric degrees, where an altitude of zero means that the Sun never rose above the horizon at that location and an altitude of 90° means that Sun lay directly overhead at noon. In contrast, the temperatures are measured in terms of heat, or energy, in units of degrees Fahrenheit (where water freezes at 32° F and boils at 212° F).

Because the temperatures were recorded in June, the solar altitudes reflect the fact that the northern hemisphere of the Earth was tilted over by 23° toward the Sun, while the southern hemisphere tilted away from it. Residents of Mazatlan, Mexico, located 23° north of the Equator, thus saw the Sun pass directly overhead at local noon, and from all locations north of Mazatlan, the Sun was 23° higher at noon than it was three months earlier, when the North Pole was neither tipped toward nor away from the Sun. We will not attempt to explore the complete causes of the seasons here on Earth today. Instead, we will ask ourselves one simple question – are June temperatures at various locations on Earth tied to the observed peak altitudes of the Sun from those locations?

Read through the table, and make sure that it makes sense to you. Then copy the three columns of data (by highlighting the rows in the file which contain them and typing Control-C or Command-C) and return to the GEAS web page. Click on the link for “Web application #2 (plotting tool)” to return to the tool which you used previously to create histograms. Now, you are going to use it to plot and analyze the relationship between solar altitude and June temperature.

As before, type your last name and access code into the two text boxes at the top of the form. Re-read through all of the information listed on the web page, paying special attention to the special options for *xy* plots.

Begin by selecting “Y versus X plot, with errors in Y” from the pull-down list of three plotting options. Next type in a title for your plot, and labels for the *x*- and *y*-axes, keeping each label under 40 characters in length. These labels should clearly identify the contents of the

plot that you are creating. You might type a title of “Northern Summer Temperatures”, and use  $x$ - and  $y$ -axes labels of “Peak Solar Altitude (degrees)” and “Temperature (degrees).” Be careful not to reverse the  $x$ - and  $y$ -axes labels – the  $x$ -axis identifies the quantity being plotted along the horizontal axis (from size to side), and the  $y$ -axis the quantity being plotted along the vertical axis (from the bottom to the top of the plot).

If you selected the option “invert” for either axis, it would run from the highest to the lowest values, rather than from the lowest to the highest (the distribution of points would be reversed, from right to left for  $x$ , or from top to bottom for  $y$ ). You do not need to do this now. Go ahead and select “show linear fit to data,” to cause a line to be drawn fitting the relationship between  $x$  and  $y$  values. Do not select “force fit through origin,” however, as this would force the line to pass through the origin (where both  $x$  and  $y$  are zero).

Scroll down until you see the text box labeled “numbers to plot”, and copy your three columns of data into the box, in the order ( $x$ ,  $y$ ,  $e$ ), or altitude, temperature, and associated error. Do not add any commas, parentheses, or other extra characters – just place each set of three measurements on a separate line. Once you have completed these steps, click on the button labeled “Create Plot” and your plot should appear in a separate browser window.

Examine the plot carefully, and make sure the the correct quantities are plotted on each axis, that the error bars are present, and that both axis labels and the title are correct. Someone reading your plot should be able to identify the quantities shown, and their units, from the title and axis labels alone. If there are any mistakes, even small misspellings, hit the “back” button on your browser, fix them, re-enter your access code, and recreate the plot.

Now examine the trend shown between  $x$  and  $y$  (Sun’s altitude and peak temperature).

1. Does  $y$  increase, or decrease, as  $x$  increases? At what altitude do the peak temperatures occur? What do you conclude about a possible connection between maximum solar altitude and the peak local temperature? (2 points)

The plot will be labeled with a slope and a  $y$ -intercept value.

2. If another survey of different locations derived a slope of  $0.786 \pm 0.055$  for this relationship, would your results be consistent? Why, or why not? (2 points)

3. What is the physical meaning of the  $y$ -intercept for these data (what does it tell us about the relationship between the Sun's maximum altitude and the local temperature)? (2 points)

4. The plot is also labeled with a correlation coefficient,  $R$ . If  $R$  is near to 1, then  $x$  and  $y$  are positively correlated ( $y$  increases with  $x$ ). If  $R$  lies between  $-0.5$  and  $0.5$ ,  $x$  and  $y$  are fairly independent of each other (you cannot predict  $y$  from  $x$ ). If  $R$  is near to  $-1$ ,  $x$  and  $y$  are negatively correlated ( $y$  decreases as  $x$  increases). Is there a strong relationship between  $x$  and  $y$  here? ( Yes / No ) (2 points)

5. Save a copy of your plot, and include it in your lab report here. Include a figure caption describing the primary conclusions that one can draw by examining the plot. If you are unsure what to write, imagine that you showed it to a friend and explained what it meant to you. That explanation can form the bulk of your figure caption. (2 points)

## 1.6 Final (Post-Lab) Questions

1. Imagine that you measure parallax angles for nearby stars and find a mean value of  $0.40 \pm 0.04$  arcseconds, while someone else finds a value of  $0.41 \pm 0.08$  arcseconds in a similar survey. Would you say that their technique was less precise than yours, with a larger measurement error? ( Yes / No )

If a third scientist found a value of  $0.80 \pm 0.04$  arcseconds is the discrepancy more likely caused by systematic error or by measurement error?

( Systematic Error / Measurement Error ) (3 points)

2. You want to study the population of dogs in your neighborhood. Since you often take an evening walk, you collect data by walking around the block every evening for a month and taking note whenever a dog barks at you through a fence. Is your survey sample biased in any way? If so, how? (3 points)

3. People often confuse “causation” (where a change in one variables *causes* a change in another) with “correlation” (where two variables vary together in a predictable fashion). Gravity causes dropped objects to fall to the ground, but purchasing flood insurance does not make your property more likely to flood. Identify an example of causation and an example of correlation, and describe the variables involved. (You get an extra pat on the back if your variables are astronomical in nature.) (3 points)

## 1.7 Summary

After reviewing this lab's goals (see §1.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. Describe briefly the set up and implementation of your own data-taking experiment, your analysis of the resultant data, and your overall conclusions. (25 points)

Be sure to cover the following points.

- Describe what you have learned about the way to design and implement an experiment.
- Describe the primary quantities that you will use to evaluate fits to data (see §1.4), and perhaps suggest an appropriate use for each (an application based on a particular data set).

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

## 1.8 Extra Credit

Conduct an additional survey of two related quantities of interest to you, plot the results, and derive a mathematical relationship between them. You may obtain data by either direct measurement (such as measuring the height of a bean sprout in your garden every day for a week), querying a relevant population (asking all your Facebook friends to tell you their heights and shoe sizes), or extracting data from tables (comparing the periods and peak luminosities for Cepheid variable stars in astronomy articles). Describe your experiment clearly and succinctly. Note any biases in the sample population which may limit the impact of your results.

Choose two quantities that you think are connected, so that your plot contains a clear trend (ideally a linear one that can be fit to determine a slope and a  $y$ -intercept). This is a chance for you to demonstrate how to collect and analyze data efficiently, and to display your creativity with an intriguing data set. (5 points)



# Lab 2

## Observing the Sky

### 2.1 Introduction

Early humans were fascinated by the order and harmony they observed in the celestial realm, the regular, predictable motions of Sun, Moon, planets, and “fixed” stars (organized into patterns called constellations). They watched stars rise in the east, their height (or altitude) above the horizon steadily increasing with each passing hour of the night. After they reached a high point, at culmination, they were observed to slowly set in the west. Other stars never rose or set, but rotated once a day around a fixed point in the sky (the celestial pole).

The ancients recognized seven objects that moved slowly with respect to the fixed stars. These were the Sun, the Moon, and five bright planets (from the Greek word for “wandering star”). The Sun and Moon moved steadily eastward through a band of twelve constellations located above the Earth’s equator called the zodiac. As people spent a lot of time outdoors without artificial lighting, changes in the appearance of the Moon were obvious to them. They tracked the Moon’s phase (fractional illumination) from night to night, as it grew from new to full and back to new each month (or “moonth”).

The Sun moves more slowly through the sky. Its journey once around the zodiac takes a full year, and its changing position heralds seasonal changes. The ancient Babylonians, Egyptians, Mayans, Chinese and other civilizations designed calendars based on their observations of the Sun and Moon, and carefully watched the skies. In New Mexico, ancient pueblo peoples used celestial patterns to determine when to plant crops in spring, when to prepare irrigation ditches for summer rains, and how to orient buildings to maximize solar heating in winter. Buildings and roads constructed between the 10<sup>th</sup> and 12<sup>th</sup> centuries in the Chaco Canyon region (between present-day Albuquerque and Farmington) follow geometric patterns and orientations related to lunar and solar cycles based on astronomical knowledge that must have been gained over generations.

Celestial studies contributed to the development of science, if only through the realization that there is order in the natural world. Scientists attempt to make sense of observations of natural phenomena. Their conceptual models enable predictions of future events, leading in time to the development of useful tools and technologies.

Consider models of the night sky. For a long time, people held to a geocentric model in which all celestial objects moved in circles around a stationary Earth. This idea was rejected only a few hundred years ago, as we began to appreciate that “things are not always as they appear.” Despite what our eyes tell us, few celestial objects actually circle the Earth. Their apparent daily motions from east to west are caused by the Earth spinning around once on its axis each day in the opposite direction. The Sun’s steady advance eastward (with respect to the stars) is not due to its circling the Earth once a year, but rather to the Earth revolving westward around the Sun, taking a year to make a complete circuit.

This more sophisticated understanding allowed humans to construct more accurate calendars, and to use the stars to navigate reliably across the Earth’s surface. The ancient Polynesians used the stars to steer small vessels for thousands of miles to the small island we call Hawaii. By the 19<sup>th</sup> century, ship navigators were able to establish latitudes and longitudes using a sextant, a handheld instrument for measuring angles, and a marine chronometer, a clock for keeping time precisely at sea. Modern global positioning system (GPS) technology is even more reliable and precise.

In this lab, you will use a simple sextant to determine your latitude (how far you lie above the equator) by measuring the altitude of the North Star. You’ll also find it another way, as one might have needed to do when at sea if clouds to the north obscured the North Star: you’ll measure the altitude of a first quarter Moon appearing due south around sunset.

The Moon is the one celestial object that actually does circle the Earth. You’ll track its changing appearance and estimate the fraction of the visible lunar disk that is illuminated by sunlight at various times over an eight-day period. During this time, you’ll also use a sextant to measure the angle between the direction to the Moon and the direction to the Sun. After considering the rate at which the Moon moves, you’ll plot its fractional illumination versus time and look for a mathematical correlation.

### **2.1.1 Goals**

The primary goals of this laboratory exercise are to understand the monthly orbit of the Moon, relate its changing appearance and location in the sky to its phase, illumination and elongation angle, and to visualize the positions of the Moon, Earth, and Sun throughout this cycle. We will learn about the daily and yearly motions of the Earth and Moon, and how they affect the appearance of our night and day skies, and the passage of the Sun, Moon, and stars through them. We will also develop our skills at reading figures, conducting observations, recording data accurately, and analyzing the results.



## 2.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed here.

<http://astronomy.nmsu.edu/geas/labs/labs.html>

You will need the following items to perform a simple experiment:

- several sextants (provided on pages 67 through 71 )
- two pieces of 10.5 inch by 6.5 inch thick cardboard
- the cardboard tube from the center of a roll of paper towels or toilet paper
- a needle, a pin, and two 9-inch pieces of brightly colored thread
- a 2<sup>1</sup>/<sub>2</sub> foot piece of string
- a pair of scissors, and a roll of tape
- a ruler, yardstick, or measuring tape
- a straight drinking straw
- a paper clip
- a pencil or pen
- a calculator (or calculator app)
- a flashlight (or flashlight app; with a red filter or red cellophane cover, if possible)
- a friendly assistant (helpful when making observations)

Your assistant could be an adult or an older child, and needs no special knowledge of astronomy. S/he can assist you in measuring angles on a protractor, and holding your measuring apparatus still, at a particular angle while you use it.

You will also need a computer with an internet connection, to analyze the data you collect from your experiment.

## 2.1.3 Primary Tasks

This lab is built on one major observing activity (tracking the position and appearance of the Moon over an eight-day period), and two short sky observations that can be completed in one sitting. Most observations will be done during the day, and you will have quite a bit of flexibility in scheduling them. One will take place near to sunset, and one in the evening.

Make sure that you have a safe place to observe the sky outdoors. A flat location with some free space around it will help to ensure that no buildings block your view of the Moon or Sun in the sky. When you work at dusk or in the dark, make certain that any cars which might come along can see you clearly.

Because we cannot schedule the rising and setting of the Sun or Moon for our convenience, you may need to begin your observations for this lab exercise a couple of days early, or to end them a day or two late (before or after the beginning or the end of the two-week period

associated with this exercise). Be sure that you understand which days you will be observing the Moon, and discuss any questions that you have with your instructors well ahead of time.

It is critical that you are ready to observe the Moon at dusk three days after the new moon phase, and near sunset when the first quarter moon culminates 3 or 4 days later. Make a note of the dates of these two events as soon as you begin this exercise, so that you are prepared. Your other seven daily observations of the Moon can then occur with some flexibility as long as they are completed over the next ten evenings, by the full moon.

### 2.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. The 29 questions for this exercise (including the final questions), are worth 2 points each, for a total of 58 points. The data table in Section 2.2.3 (§2.2.3) is worth 12 points. Note that §2.5 contains 5 extra credit points.

Table 2.1: Breakdown of Points

Activity	Moon	First Quarter	North Star	Questions	Summary
Section	§2.2.3	§2.2.5	§2.2.6	§2.3	§2.4
Page	42	51	58	62	63
Points	30	18	14	8	30

### 2.1.5 Timeline

*You may need to make your first observations of the elongation and illumination of the Moon a few days before Week 1 officially begins, as we cannot alter the timing of the lunar month. Check with your instructor ahead of time to determine if this is the case.*

Week 1: Read §2.1–§2.2, begin activities in §2.2.3, §2.2.5, and §2.2.6, and begin final (Post-Lab) questions in §2.3. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report template, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Complete observations, finish final (Post-Lab) questions in §2.3, write lab summary, and submit completed lab report.

## 2.2 Observing the Sky

Pay close attention to the figures in this section, and use your visualization skills. You will learn the most from the figures if you first examine them, reading the captions and making

sure to understand each aspect, then read the relevant accompanying text, and then review the figures to make sure that you understand them. If you are confused by a drawing, try recreating it piece by piece for yourself in your notes, adding each component as you determine its purpose. There is a real skill to reading figures accurately, one that we are developing in this course.

### 2.2.1 The Position of the Sun in the Sky over a Year

The Sun is the brightest celestial object that we can observe. Its position above or below the horizon determines whether it is daytime or nighttime here on Earth. We see no other stars during the day, because the Sun outshines them all. Before discussing the apparent motion of the Sun over the course of a day and throughout the year, let's define a few terms.

The Sun lies below the horizon at night – so the ground (the Earth) blocks our view of it. The horizon forms a circle around us, where the ground and sky appear to meet. Imagine yourself at the center of this circle, looking eastward. As you turn your head to the right and look south, you will shift through one-fourth of the circle, or  $90^\circ$ . The angle between east and south is thus  $90^\circ$ . If you continue turning your head rightward until you've seen the horizon in all directions, you will have rotated once around a complete circle – through  $360^\circ$ . Now imagine moving your head up and down rather than to the side. As you direct your eyes up from the horizon to an imaginary point directly overhead (the zenith), your view will again move through one-fourth of a circle, or another  $90^\circ$ .

We can depict the apparent motion of celestial objects by tracing their changing positions on the “celestial sphere,” an imaginary dome which extends across the entire sky, pictured in Figure 2.1. Place yourself in the position of the observer on the left, drawn in blue at the center of this celestial sphere. Imagine you are looking eastward toward the horizon on the first day of Spring (March 21, for the northern hemisphere). At sunrise, the Sun will appear on the horizon directly east of you. It rises every morning in the east and sets hours later in the west. On the first day of Spring, however, it rises exactly due east of the observer and sets exactly due west, twelve hours later. As the figure illustrates, it will reach its highest elevation (altitude) above the horizon at local noon. When an object stops rising through the sky and begins to set, we say that it is “culminating” (or transiting).

Using Figure 2.1, orient yourself due south and imagine raising your eyes up from the horizon toward the zenith (directly overhead), and then continuing to bend backwards (careful!), tracing a line all the way across the sky to the northern horizon. You will have traced a half-circle across the sky, one starting due south of you, passing overhead, and then descending to the north. This arc is called your “meridian” (a line of constant longitude). As celestial objects culminate, they intersect the observer's meridian. An object's altitude at culmination is the angle between the direction to the southern (or northern, if nearer) horizon (marked “S” and “N”), and the direction to the object (labeled culmination). Note that the largest altitude possible is  $90^\circ$  (for an object lying directly overhead, at the zenith.)

The Sun culminates every day at local noon. From the continental United States, it will

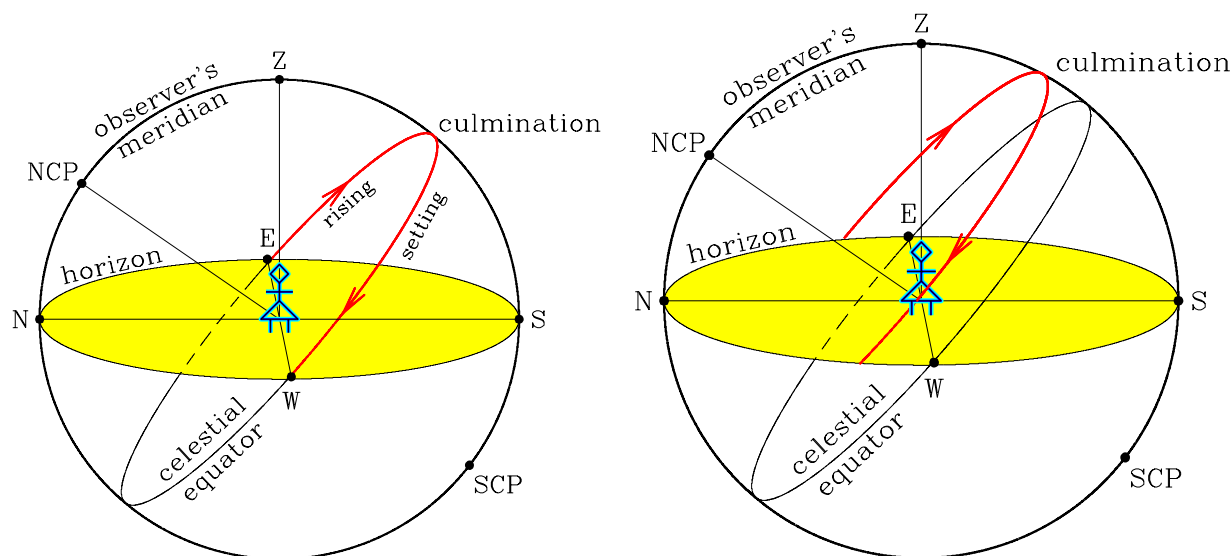


Figure 2.1: This figure identifies landmarks used when observing, both in an observer's frame of reference (their local horizon, stretching off to the north and south, and east and west, and the sky above them), and along the celestial sphere that surrounds the Earth. The red line indicates the path of the Sun through the sky over the course of a day on the Spring Equinox (March 21, on the left), where the Earth is tipped neither toward nor away from the Sun, and three months later on the Summer Solstice (June 21, on the right), where the Earth's North Pole is tipped over by  $23^\circ$  toward the Sun and the Sun thus appears to rise and set in the north, and to culminate higher in the sky.

always lie to the south (it appears in the north when viewed from much of the southern hemisphere). Its altitude at culmination will depend upon the time of year, or how much the Earth's North Pole is currently tipped over toward (or away from) the Sun.

Consider the Sun's position on March 21, when the Earth is tipped neither toward nor away from the Sun. With the Earth effectively upright, the Sun will be located in the same plane as the Earth's equator, along the celestial equator (the projection of the Earth's equator onto the celestial sphere). Its path through the sky on that day is shown in red on the left-hand side of Figure 2.1.

From March 21 to June 21, the Sun will shift steadily north of the celestial equator. As the Earth's North Pole tips over toward the Sun, the Sun will appear higher and higher in the sky when viewed from the continental United States. Its altitude at culmination will increase with each passing day. The right-hand side of Figure 2.1 shows its path (drawn again in red) through the sky on June 21, appearing north of the celestial equator at all times. For the next three months the North Pole will slowly tip back away from the Sun, causing the Sun to descend back to the celestial equator. On December 21, the first day of northern winter, it will lie  $23^\circ$  south of the celestial equator, culminating with its lowest altitude of the year.

## 2.2.2 The Position of the Moon in the Sky over a Month

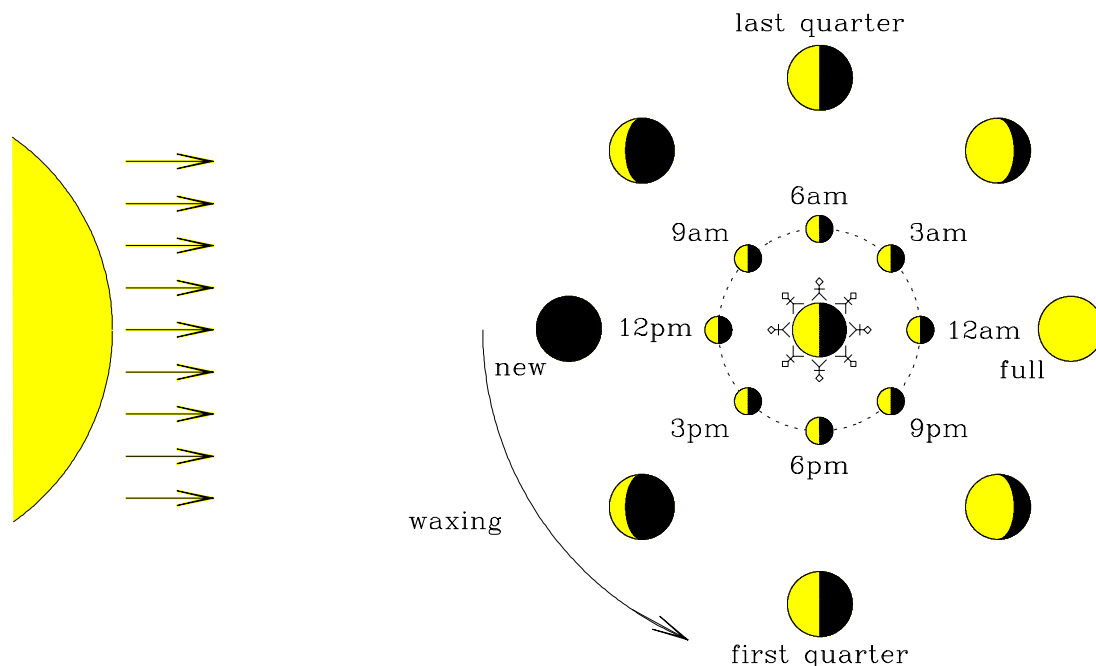


Figure 2.2: This figure illustrates how the light from the Sun (left) always illuminates the half of the Earth and the Moon that face toward it, and how the Moon's orbit around the Earth then makes it appear to pass through a sequence of phases (partially illuminated states) every lunar month. The small people on the Earth's surface are spaced out to cover eight locations around the equator, and the small moons indicate the movement of the Moon around the Earth every 29.5 days. The larger moon sketches surrounding the Earth and Moon indicate *the appearance of the Moon as viewed from Earth* at these times. The Moon begins in the new phase, dark in the sky and culminating around noon, and then its perceived illumination waxes (increases) for two weeks as the Moon moves around the Earth through the waxing crescent, first quarter, and waxing gibbous phases, until it appears on the opposite side of the Earth than the Sun, as a “full” moon, culminating round midnight. It then takes another two weeks to swing back around in front of the Earth, now appearing to wane (decrease) in illumination each night.

Having examined the yearly apparent movement of the Sun through the sky, let us now turn our attention to the Moon. To understand its movement and changing appearance, we will need to take into account the relative positions of the Moon, Earth, and Sun.

Just as the Earth takes a year to orbit once around the Sun, the Moon takes a month (29.5 days) to orbit once around the Earth. Figure 2.2 shows the Moon in eight evenly spaced positions around the Earth; it orbits the Earth in a counter-clockwise direction when viewed from above the North Pole. The inner ring of Moons, and the Earth that they surround, are all illuminated on the side that faces the Sun, and dark on the side which is cast in shadow. Eight stick figures below them represent observers looking up at the sky; in each case, *the phase of the Moon is determined by the fraction of the illuminated portion of the*

*Moon which can be seen from Earth.* The outer ring of Moons shows its appearance, or phase. Walk through each of the eight observer's positions in turn. Your impression of how much each of them can see of the illuminated portion of the Moon above them should match the phase shown along the outer, larger ring of Moons.

The Sun defines the time of day: it rises in the east at dawn, ascends high in the sky at local noon (when it casts the shortest shadows), and then sets at dusk in the west. We can similarly tell what time of the month it is from the Moon.

At local noon, the Sun culminates high in the sky. If the Moon appears near to it, the illuminated portion will face away from the Earth, toward the Sun. We call this a “new” moon, and it is very difficult to even find in the sky. Over the next few days of the month the Moon slowly orbits around the Earth, shifting away from the Sun. As a small portion of its illuminated side begins to peek into view from Earth, it takes on the shape of a thin, waxing (increasing) crescent. This crescent grows with time, over a week. The Sun, Earth, and Moon now make up a right angle, and so we see a half-illuminated lunar disk from Earth (overhead as the Sun sets at dusk). We call this the “first quarter” moon, because one-fourth of the lunar month has passed since the new phase. As the Moon continues to move around the Earth, more and more of the illuminated portion comes into view. The Moon is more than half-illuminated, and so we call it a “gibbous” (or bulging) moon. After two full weeks have passed, the Sun and Moon will lie on opposite sides of the Earth. The Moon will rise as the Sun sets, and its entire disk will be illuminated (a “full” moon).

The Moon now comes around to the other side of the Earth, and over the next two weeks dwindles from the full back to the new phase. It spends almost a week in the waning gibbous phase, each night rising later in the evening and becoming less illuminated. After three weeks, the Moon again makes a right angle with the Earth and Sun, and we call it the “last quarter” moon. Over the fourth, final week of the month, the Moon slowly sneaks around back in front of the Sun, becoming harder to see by eye and taking on a thin crescent appearance again.

If we know the phase of the Moon, we know roughly how many days have passed since the last new phase. We can predict when it will rise, pass overhead, and set, as long as we know the angle created by the Sun, Earth, and Moon (the lunar elongation angle).

### **2.2.3 The Elongation and Illumination of the Moon**

Our primary observing project is to measure the elongation of the Moon, the angle formed by the Moon, Earth, and Sun. We will begin by constructing a simple sextant, to measure the angle between two positions on the sky. True sextants are mechanical devices made of metal and glass, combining the properties of a small telescope with the ability to measure small angles precisely. Our devices will be much simpler, but we'll be able to construct them quickly out of common household materials.

Cut out the sextant template found on page 67, being careful to form straight edges and cut

along the *outer dotted line* on the page. You will have a large rectangle with a protractor drawn on it, with a small tab attached to the lower-left corner. Fold the lowest 20% of the tab up by 90°, along the dashed line. Then fold the rest of the tab to the right by 90°, making a sharp crease so that the “lollipop” shape drawn on the tab stands upright. You should have formed a small, three-sided square corner on the lower-left corner of your sextant. Tape the raised edges into place, so that this corner is stable (and as straight as possible). Tape the whole sextant securely to a piece of cardboard now, so that it will stay flat and be rigid.

Thread a needle and stick it through the center of the sextant (the dot centered between the “0” and the “180”). Pull just enough thread through that you can tape it securely to the back of the cardboard, and do so. Pull the other end of the thread tight, and tape it loosely to the edge of the sextant (beyond the ring of numbers) so that it will stay out of the way for now. Take the pin and stick it through the center of the sextant, leaving just enough of it sticking up to form a second lollipop shape, with a 1/3-inch base. (You want the head of the pin and the center of the lollipop symbol on the tab to both be at the same height above the protractor, roughly 1/3 of an inch.) Tape over the sharp end of the pin with a small piece of tape, so that it cannot scratch anyone.

Your sextant is now ready for use! You are going to use it to find the angle between a line pointing to the Sun and a line pointing to the Moon on the sky. You want to align the Sun to fall at zero degrees along the edge of the sextant, and then place the sextant in the plane that contains both the Sun and the Moon, and read off the angle of the Moon. You will place the sextant so that the Sun lies along the zero degree position, but **do not look directly at the Sun**. Instead, observe the shadow the pin casts – line it up so that the shadow of the pin head appears in the center of the lollipop mark on the raised tab.

Once you have the sextant pointing at the Sun, rotate the protractor so that the Moon lies along it as well, at some angle between 0° and 180°. Then adjust your position so that the pin shadow is again centered within the lollipop symbol. (You may have to go back and forth a few times, but with practice this will become easier to do.) With the Sun and the Moon in place, adjust the thread to run from the pin to the Moon. As the Sun lies at zero degrees, the thread will lie along the angle between the Sun and the Moon!

If the Moon lies quite close to the Sun (in the crescent phase), you will find it helpful to have an assistant. S/he can look over your shoulder and check that the shadow of the pin stays in place as you focus on moving the thread to mark the angle to the Moon, or vice versa.

### **Measuring the crescent Moon elongation without an assistant**

If you cannot scrounge up an assistant at dusk during the crescent moon phase or keep your eye on both the pin shadow and the Moon simultaneously, we have an alternative plan. (The sextant works very well for the first quarter through full phases, but can be tricky to use when the Moon is close to the Sun.) This technique will be simpler, but less accurate. Wait until near sunset (when the Sun will be less bright), and then estimate the angle between the Sun and the crescent moon by measuring it with the knuckles of your clenched hand, held outstretched at arm’s length.



First determine the distance between the setting Sun and the moon in units of the width of the knuckles (all four fingers) as your fist is clenched at arm's length. You might find that you can place your fist 2.6 times in the gap, for example. Measure the gap three times.

Next, convert the width of your knuckles into an angle on the sky (see Figure 2.3). First measure  $r$ , the width of your fist, with a ruler. Then measure  $d$ , the distance between your eyes and your outstretched fist held at shoulder height. (You can stretch a piece of string out from your fist to your eyes, and then measure the length of the string.) Divide  $r$  by  $d$ , and find this ratio in the  $r/d$  ratios listed in Table 2.2. Read off the value for the angle  $\alpha$  to the *left* (not right) of the  $r/d$  value closest to your value. This number is the width of your outstretched fist, in units of degrees, and should be roughly eight or ten degrees.

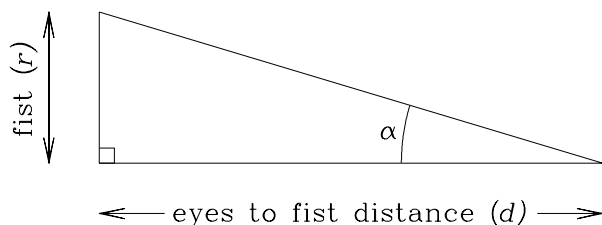


Figure 2.3: You can form a right triangle with your fist at arm's length, and then determine the angular width of your knuckles. The height of the triangle  $r$  is equal to the width of your fist, and its side  $d$  is equal to the distance between your fist and your eyes (roughly the length of your outstretched arm). When you estimate the size of an angle on the sky with your fist, each fist-width is equal to  $\alpha$  degrees.

To determine  $\alpha$ , we formed a right triangle that is as long as your arm and as tall as your fist. The ratio of these values determines a unique angle that corresponds to the angular width of your fist on the sky. If you found that your knuckles were  $10^\circ$  wide, for example, then you should be able to walk your arms through a right angle by placing each fist in turn on top of the other, and counting out nine fists, from the horizon to the zenith (overhead).

Table 2.2: Height-to-Side Ratios ( $r/d$ ) for Right Triangles

$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$
4.0	0.0699	6.0	0.1051	8.0	0.1405	10.0	0.1763	12.0	0.2126
4.5	0.0787	6.5	0.1139	8.5	0.1495	10.5	0.1853	12.5	0.2217
5.0	0.0875	7.0	0.1228	9.0	0.1584	11.0	0.1944	13.0	0.2309
5.5	0.0963	7.5	0.1317	9.5	0.1673	11.5	0.2035	13.5	0.2401

Now multiply your three values for the number of fist-widths between the Moon and the Sun by  $\alpha$ , and you will have converted your angular measurements from fists into degrees!

If you use this technique for observing the crescent moon, record your values for  $r$ ,  $d$ , and  $\alpha$  below so that your instructor can check your work. Don't forget to include units.

$r =$  \_\_\_\_\_,  $d =$  \_\_\_\_\_,  $\alpha =$  \_\_\_\_\_.



We will be measuring the Moon–Earth–Sun (MES) angle over an eight-day period between the new and full moon phases, and observing the apparent phase and fractional illumination of the moon as well. You may work whenever both the Sun and the Moon are above the horizon. Use your lunar phase wheel to work out good observing times that work best with your schedule. We recommend waiting until three days after the new moon to begin, so that enough of the Moon is illuminated to make it clearly visible in the sky.

**As you make all lunar observations, be careful to never stare at the Sun!** Doing so would cause headaches, and could damage your eyes permanently.

Close to the new moon, the Moon and Sun will be close together, both above the horizon for most of the day. As the lunar month progresses, however, the Moon will lag behind the Sun more and more, until by the full moon it will be just rising as the Sun sets. By this point, your daily observing window will have narrowed to sunset. In addition, within several days of the new moon you will find it easiest to observe at sunset, because the Sun, lying so close to the Moon, will be less bright at dusk. Make sets of three independent observations at a time, once a day if possible, starting three days after the new moon. If you have to skip a day here and there due to cloudy weather or a busy schedule, that is understandable.

Because the Moon takes 29.5 days to orbit once around the Earth, it travels more than ten degrees every 24 hours. You could take observations twice over the course of a day and expect to see a small change over a few hours, if your measurements were accurate enough.

Record your MES measurements in Table 2.3. Each day, record the date and the time of day (to the nearest minute) in the first two columns, and then place your three measurements of the MES angle in the fourth through sixth columns (to the nearest degree). Record the time of the middle observation, as this should be a reasonable “average” time.

Don’t worry if your three measurements differ by more than a few degrees on some days. Though you should try to be as careful as possible, realize that the primary purpose of taking three measurements is to estimate how difficult it is to use the sextant apparatus accurately.

The third column stores the number of days between your observations and the nearest new moon, to the nearest thousandth of a day (0.001 days is roughly one minute). Look up the time of the nearest new moon on the GEAS project lab exercise web page (see the URL on page 37 in §2.1.2). New moon times are listed there in a table, accessed from the link labeled “Table #1 (lunar phase dates)”. Add the date and time to the title of your table, and below.

1. The nearest new moon is on \_\_\_\_\_, at \_\_\_\_\_.

#### Example 2.1

Let’s practice finding the amount of time between an observation and the time of the nearest new moon. Imagine that you made your first observation on April 6, 2011 at 7:20 pm. We would inspect the table of lunar phases, and see that the nearest new moon date to April 6 is a few days earlier, on April 3 at 8:32 am.

We first count the number of full days between April 3 and April 6 – 3 days. We then calculate the number of hours between 8:32 am and 7:20 pm: 3:28 hours between 8:32 am and noon plus 7:20 hours in the afternoon, or 10 hours and 48 minutes. Three days plus 10 hours and 48 minutes sums to  $3 + 10/24 + 48/1440 = 3.450$  days, as there are 24 hours in a day and 1440 ( $24 \times 60$ ) minutes in a day. We enter this value in the third column of Table 2.3.

We next combine our three measurements of the MES angle, and find the mean value and standard deviation. Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled “Web application #1 (plotting tool).” Scroll down until you see the text box labeled “numbers to plot”, and enter your three MES measurements into the box. Do not add any commas, parentheses, or other extra characters – just type the three numbers. The mean (average) value of your three measurements will appear above the box, followed by the standard deviation. Record these values in the seventh column of Table 2.3.

As you make your observations, compare the appearance of the Moon with the images in Figure 2.4. Estimate the fraction of the lunar disk which is illuminated, by taking into account the values listed for each image on the figure, and place this value in the eighth table column. (This should be a fraction, a number between 0 and 1.) In the final column, make a note as to the phase of the Moon (thin crescent, just past first quarter, ...). Your table row should now be complete!

Once you have filled Table 2.3 with with eight sets of measurements, you will be ready to analyze the results. Your first task is to explore the relationship between the observed elongation of the Moon and the amount of time since the new moon. Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled “Web application #1 (plotting tool).” Select “Y versus X plot, with errors in Y” from the pull-down menu, to plot the MES angle and associated errors ( $\sigma$ ) against time ( $t$ ).

Place  $t$  on the x-axis (the horizontal axis, running from left to right) and the MES angle on the y-axis (the vertical axis, running up and down), so when you enter your data under “numbers to plot” on the form, place  $t$  first on each line, followed by the average MES angle and then  $\sigma$ , the standard deviation measured for each set of three averaged MES angle values. Give your plot a title – something like “Lunar Elongation Data” is fine. Label the x-axis “Time (days)”, and label the y-axis “MES Angle (degrees)” or something similar.

Trace the points on the plot by eye, and ask yourself whether a straight line is a good fit to the data (do the points follow a straight line?). The correlation coefficient  $R$  should be within a few percent of 100%, indicating a strong positive correlation (as  $t$  gets larger, the MES angle increases as well).

2. Be sure to save a JPG- or PNG-format copy of the plot to your computer, to include in your lab report.

Let’s now inspect the slope and  $y$ -intercept for our line fit. Remember that the slope ( $m$ ) is defined as the change in  $y$  divided by the change in  $x$ , and the  $y$ -intercept ( $b$ ) is the  $y$  value for which  $x$  is zero.

Table 2.3: Lunar Observations near to New Moon on \_\_\_\_\_

Date	Time <sup>1</sup>	$\Delta$ Time <sup>2</sup> (days)	Moon–Earth–Sun Angle <sup>3</sup> (°)			$\mu \pm \sigma$ (°)	Illum. <sup>4</sup> (%)	Phase <sup>5</sup>

(12 points)

<sup>1</sup>Time of day of the observations to the nearest minute (such as 6:52 pm).

<sup>2</sup>Time since or till the nearest new moon phase in units of days, to the nearest thousandth of a day (such as 2.456 days).

<sup>3</sup>Elongation angle formed by the Moon, Earth, and Sun. Measure this angle three times in a row each day, and then enter the average value and standard deviation as  $\mu \pm \sigma$ .

<sup>4</sup>The fractional illumination of the Moon, estimated by comparison to the pictures in Figure 2.4.

<sup>5</sup>The approximate phase of the Moon (such as waxing thin crescent, or waning gibbous).

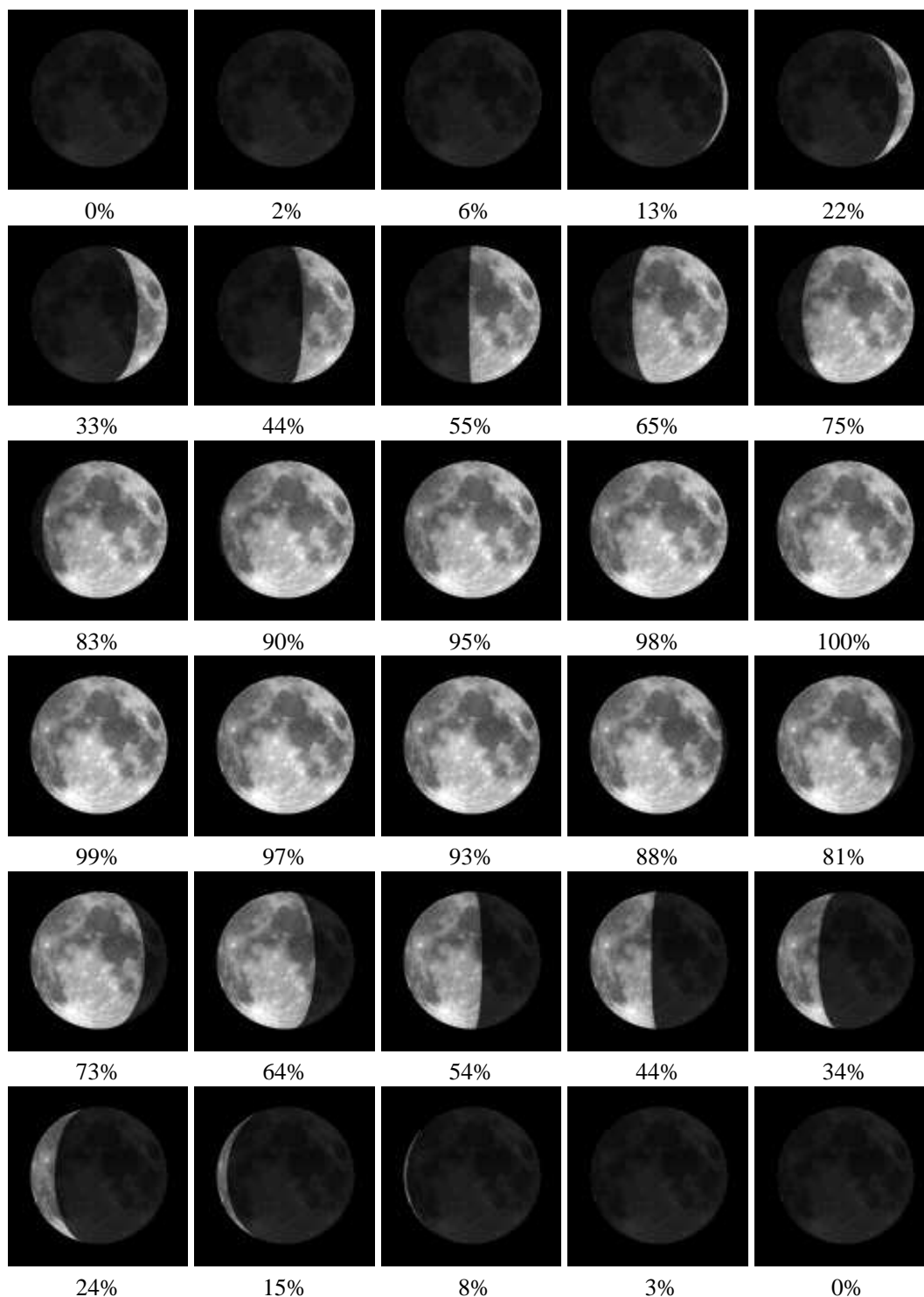


Figure 2.4: This set of images connects the phase of the Moon with its fractional illumination (listed below each image), over the course of a lunar month. When observing, either select an image that matches the appearance of your Moon or interpolate (choose an intermediate stage between two images). Make a note of the associated illumination (or average the values for the two images that surround yours in appearance).

3. For this plot, the units of the slope are \_\_\_\_\_,  
and the units of the  $y$ -intercept are \_\_\_\_\_.
4. It is important to be able to connect this plot to its physical basis. What is the meaning of the slope and  $y$ -intercept, in terms of the connection between the Sun and the Moon? (How do they help us to explain the movement of the Moon around the Earth?)
5. In a more “perfect” (read “boring”) solar system, the Moon’s orbit would be a perfect circle around the Earth, lying exactly within the plane of the ecliptic (the plane in which the Earth orbited around the Sun), and all measurements would be free of error. The Moon would still take 29.5 days to orbit once around the Earth. In this case, what exact values of  $m$  and  $b$  would you expect to measure?
6. You have plotted points covering a substantial fraction of the first half of the lunar month, and fit a straight line to them. If you took data for another two weeks, describe what a plot of all of the data would look like (over the entire month).

We will next plot the fractional illumination of the Moon as a function of the time since the new moon. Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled “Web application #2 (illumination versus time modeling tool).” Place  $t$  on the x-axis again, and fractional illumination on the y-axis, so when you enter your data under “numbers to plot”, place  $t$  first on each line, followed by the illumination estimate. Give your plot a title, and label the  $x$  and  $y$  axes. Then click on the button labeled “Create Plot” and a plot of your data should appear in a separate browser window.

The relationship between the illumination and time can be fit well with a sine function, an up-and-down wave pattern which varies between 0 and 1 over each month. If you have calculated your times since the new moon phase correctly, the green line on the plot should pass near to or through the origin (0, 0).

7. Be sure to save a JPG- or PNG-format copy of this plot as well, to include in your lab report.

8. (a) Based on this plot, the illuminated fraction of the Moon 27.5 days after a new Moon

is equal to the fraction \_\_\_\_\_ days after a new Moon.

(b) On the tenth day of the lunar month, the illuminated portion of the Moon is ( more / less ) than five times that of the second day.

9. Based on this plot and your understanding of the geometry of the Sun, Earth, Moon system, what illumination values would you expect for the new, first quarter, full, and last quarter moon? Explain your answer, describing the physical positions of these three bodies and the patterns of illumination and shadow in each case.

## 2.2.4 Rescheduling Observations due to Cloudy Weather

If you have very cloudy weather for both weeks of this exercise, you may need to swap the order in which you complete exercises and observe the Moon in the second half of the month. (If this were to happen, your instructor would discuss the new schedule with you.) In this case, use the second sextant (found on page 69). During the second half of the lunar month, the Moon will rise ahead of the Sun, and proceed ahead of it across the sky. During the waning gibbous phase, you will catch the Moon setting as the Sun rises at dawn. Over the

next two weeks, the Moon will set later and later, and you will be able to observe later and later in the morning, until by the waning crescent moon phase the Moon will have almost rejoined the Sun in the sky.

You will construct this sextant according to the same directions, and again measure the angle formed by the Moon, Earth, and Sun. Calculate the amount of time between your observation and the nearest new moon date that follows (roughly two weeks for the waning gibbous phase, and a few days for the waning crescent phase).

### 2.2.5 Measuring the Moon's Altitude, Finding Our Latitude

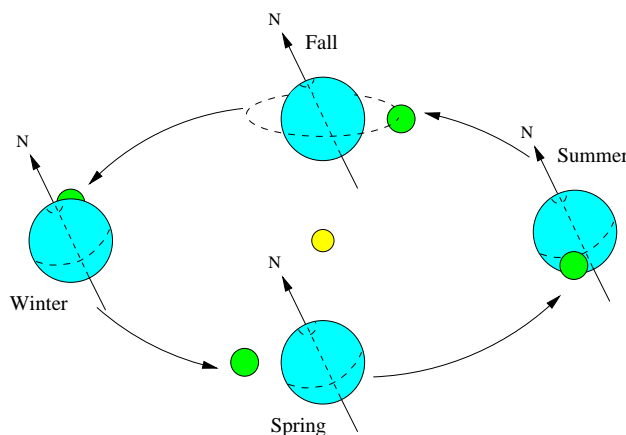


Figure 2.5: This figure is centered around the Sun (yellow globe), and shows the positions of the Earth (light blue globes) and the first quarter phase Moon (green globes) during each of the four cardinal seasons. The Earth is tilted over by  $23^\circ$ , indicated here by showing the tilt of the rotational axis running through the North (N) and South Poles. A small dashed circle indicates the position of the northern polar cap, and a longer dashed arc traces the Earth's equator. The northern hemisphere of the Earth experiences a warm Summer when the North Pole tips it over toward the Sun, and a cold Winter six months later when it is tipped away from the Sun. During the transitional Fall and Spring seasons, the Earth is not tipped toward or away from the Sun. Because the Sun, Earth and first quarter Moon form a right angle, the Earth-Sun “tip pattern” is shifted by three months for the Earth-Moon system. (If the Earth is currently tipped toward the first quarter Moon, as shown on the bottom, then three months later it will tip toward the Sun, as shown to the right.)

We can measure the altitude of the Moon, its height above the horizon, very easily. Like the Sun, the Moon rises in the east, ascends through the sky, then descends, and finally sets in the west. We will determine its maximum altitude, by observing it *culminate*, when it transits, or passes as close as it can to directly overhead. (If you are located more than  $29^\circ$  north of the equator, then the culminating Moon will always pass south of you.)

This will lead us to estimate our own latitude, how far north or south of the equator we are located. The basic idea is quite simple. Because the Moon orbits around the Earth and stays within  $29^\circ$  of the equator, if we are on the equator, the Moon will pass close to overhead. If

we shift our position up to the North Pole, however, the Moon will stay close to the horizon at all times, never rising very high in the sky. By charting its maximum altitude above the horizon, we can determine if we are nearer to the equator or to the poles.

The Moon orbits the Earth in the same plane as the Earth orbits the Sun, rather than staying above the Earth's equator! Because the Earth is tipped over by  $23^\circ$ , the altitude of the Moon will thus appear to shift north and south by  $\pm 23^\circ$  over the course of a month, plus an additional  $\pm 5^\circ$  of wobble as it dips in and out of the ecliptic (the plane in which the Earth orbits around the Sun).

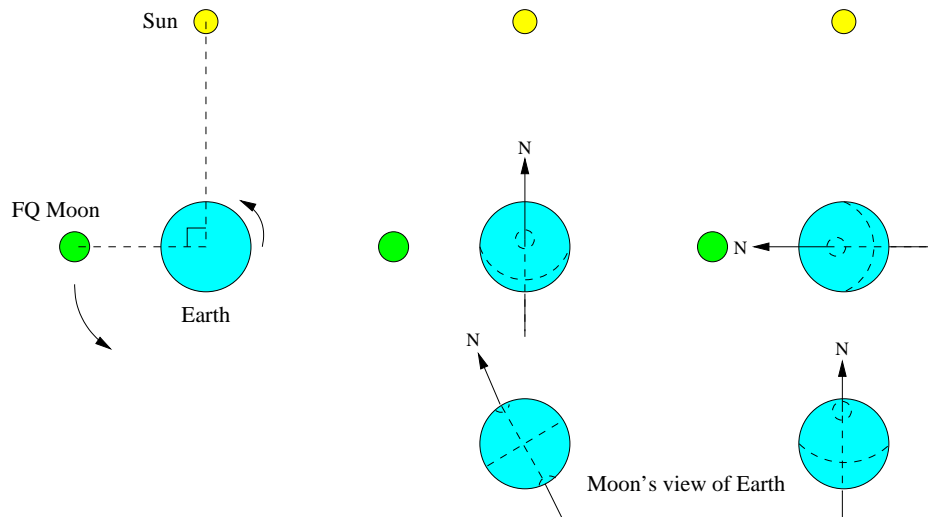


Figure 2.6: This three-panel figure displays the right angle formed by the Sun, Earth, and first quarter Moon (left). The two right panels show the Earth tipped toward the Sun during Summer (middle panel), and tipped toward the first quarter Moon three months earlier (right panel). The additional Moon images in the two right panels show *how the Earth looks when viewed from the Moon at these times*. In the middle panel the Moon sees the Earth's equator as a tilted line, with both northern and southern polar caps peeking over the edge of the globe. In the right panel the Earth has tipped over toward the Moon, so the entire northern polar cap is visible and the South Pole is hidden from view. The Moon is now orbiting somewhat north of the Earth's equator, above the northern hemisphere.

Let's review the movements of the Moon and the Earth around the Sun over a year, to understand where the Moon will appear in the sky. Figure 2.5 shows the path of the Earth around the Sun, with the northern hemisphere seasons labeled. As the Earth orbits around the Sun, the North Pole tips alternatively toward, and then away, from the Sun. This causes seasonal variations in temperature due to changes in the length of the day (the amount of time during which the Sun lies above the horizon) and the height of the Sun in the sky. The higher the Sun gets, the more directly, and intensely, the sunlight is experienced below.

As the Earth makes its yearly orbit, the Moon is zipping around the Earth once per lunar month. Over the course of each month the Moon makes a complete circle around the Earth, so at one point the Earth will be tipped toward it, and then two weeks later it will be tipped



away from it. Because the phase of the Moon depends upon the position of the Moon relative to the Sun, the Earth will tip toward the Moon during different lunar phases during each of the four seasons.

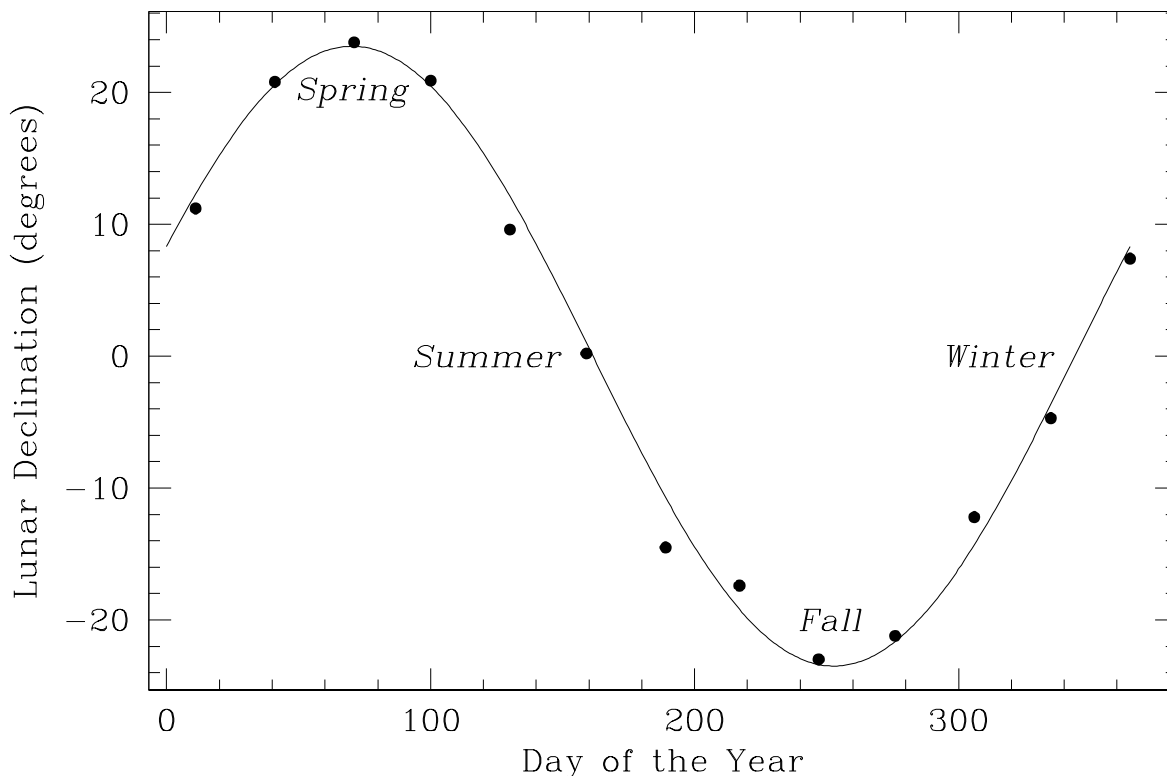


Figure 2.7: This plot shows the variation in the declination of the first quarter Moon (how far it lies north or south of the equator) over the course of a year. It can be fit well by a  $\pm 23^\circ$  sine wave, though a secondary trend pushes most of the points slightly off of the curve. We can see that in the Spring the Moon has a large positive declination, and so it lies north of the equator. This is because the Earth has tipped its North Pole over *toward* the first quarter moon; as the North Pole tips over, the equator dips below the Moon. Three months later, in Summer, the Earth is tipped over toward the Sun, and so the Moon has a declination of roughly zero (because the Earth is no longer tipped over toward it). The pattern continues into the Fall, where the Earth tips *away* from the first quarter moon and the Moon lies south of the equator and takes on a negative declination. In Winter, the Earth is again upright relative to the first quarter moon, and so the Moon has a declination near zero again. Because the Sun, Earth, and first quarter moon form a right angle, the Earth's rotational axis is either tipped toward or away from the Sun (causing Summer and Winter) and the Moon lies on the equator with a declination of roughly zero, or the Earth is tipped toward or away from the Moon rather than the Sun (during the neutral Spring and Fall seasons), causing the Moon to appear north or south of the equator.

1. During which season is the North Pole tipped over toward the first quarter moon? During which season is it tipped away from the first quarter moon?

2. Explain the physical cause of the shape of the curve shown in Figure 2.7, justifying both the general form and the fact that the points do not lie exactly on the curve.

We're going to make a short observation of the first quarter moon, and measure its altitude above the horizon as it culminates. This observations needs to be made at a particular time, so check the data listed at the GEAS project lab exercise web page. First quarter times are listed there in a table, accessed from the link labeled "Table #1 (lunar phase dates)". To the right of the times of the first quarter moons for the year, we have listed the times at which those first quarter moons culminate (attain their highest elevation in the sky). This is the time of your observation. The next column in the table lists the declination of the Moon at this time (lunar *dec*), or how far it lies to the north (+) or south (−) of the equator.

3. (a) The nearest first quarter moon culmination is on \_\_\_\_\_, at \_\_\_\_\_.  
(b) The lunar declination (*dec*) at this time is \_\_\_\_\_ degrees, which is ( north / south ) of the equator.

Our goal is to measure the altitude of the culminating first quarter moon, how far (at what angle) it lies above the horizon. We begin by constructing another simple sextant.

Cut out the sextant found on page 71, a large rectangle with a protractor drawn on it. Tape the sextant securely to a piece of cardboard so that it will stay flat. Thread the needle and stick it through the center of the sextant. Pull just enough thread through that you can tape it securely to the back of the sextant, and do so. Attach a paper clip or other fairly light object to the other end of the thread, so that gravity will pull it down along a straight, vertical line (without a weight, a light breeze could shift it easily).

Take a straw and tape it carefully along the line between the two "90" labels, making every attempt to keep it straight along this line. This is your sighting tube.

Your altitude-finding sextant is now ready for use! You are going to use it to find the angle of the Moon above the horizon. First verify that if you stare at the horizon through the straw the thread points to zero degrees, and if you stare straight up the thread points to  $90^\circ$ . As you now tilt the straw to point up at the Moon, the sextant will tilt with you, and the vertical thread will indicate the tilt angle. Peer through the straw, and place the Moon at its center. Have an assistant read off the angle marked by the thread as you do so. This is the altitude of the Moon, the angle at which it lies above the horizon.

Schedule your observations to take place with a few minutes of the listed culmination time, to increase your accuracy. If the Moon is still rising in the east, or has begun to descend to the west, then you will systematically underestimate its culmination altitude.

Take three independent observations so that you are less sensitive to small random shifts in the equipment as you read the angle to the nearest degree. Use the GEAS plotting tool to average the three measurements and find the standard deviation.

4. The three observed values for the lunar altitude are \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_ degrees.

5. The altitude of the culminating first quarter moon was \_\_\_\_\_  $\pm$  \_\_\_\_\_ degrees.

Once you are done with your third observation, turn around and face in the opposite direction (due north). Take a note of the direction and any local landmarks, as you will need to find due north for your next observation (of the North Star). You might even draw an arrow pointing north on the ground with a piece of chalk.

You have measured the altitude of the culminating Moon, how far it lies above the southern horizon. We will combine this number with the declination of the Moon, how far it lies above or below the equator, to estimate our latitude (how far we are located above the equator).

We will assume that you are located at least  $29^\circ$  above the equator as we do this (though not so far north that the Moon lies below the horizon), so that the Moon always culminates *south* of your position. (If you are at a lower latitude, work with your instructor to understand how to modify this procedure for your location, where the Moon could appear to the north.)

In Figure 2.8, we explain the definition of an observer's latitude (*lat*) and the declination (*dec*) of the Moon. We chart positions on the surface of the Earth according to two coordinates, latitude and longitude. Latitude tells you how far north or south you are from the equator, while longitude tells you how far east or west you are from the Royal Observatory in Greenwich, England. A positive latitude lies in the northern hemisphere, while a negative value indicates a location south of the equator. We define the declination of an astronomical object very similarly to latitude; declination tells you how far north or south an object lies of the plane containing the equator.

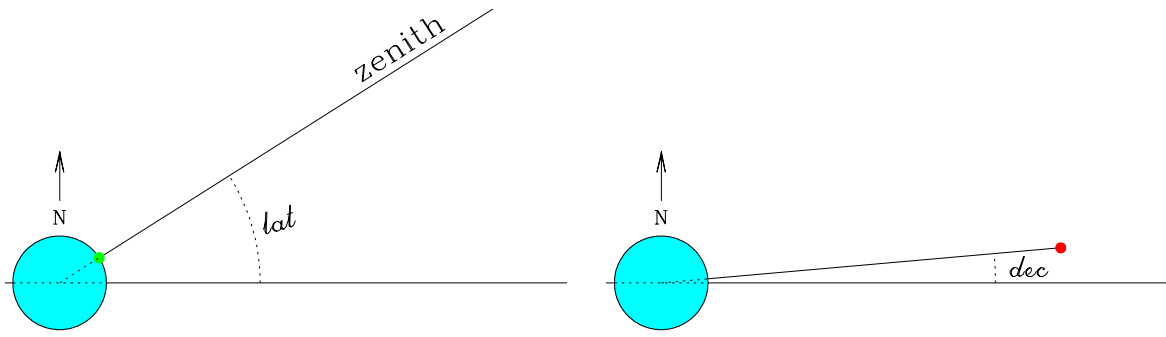


Figure 2.8: In this figure, the large light blue globe represents the Earth, with the North Pole facing upward, the South Pole facing downward, and the equator running horizontally across the page. On the left, we define the latitude ( $lat$ ) of the observer (marked with a green dot) as the angle between the equator and the observer's position. The zenith points up into the sky above the observer's head. On the right, we illustrate that the declination ( $dec$ ) of the Moon (red dot) is its angular position above the equator. Both angles are measured upward from the equator.

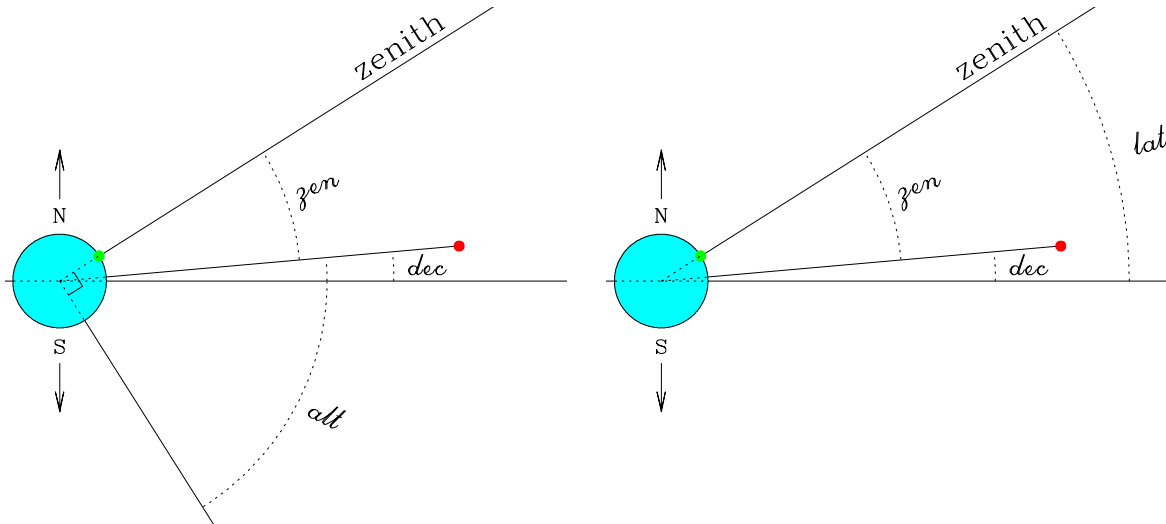


Figure 2.9: In this figure, as the previous one, the light blue globe represents the Earth, with the North Pole facing upward, the South Pole facing downward, and the equator running horizontally across the page. On the left, we again illustrate the declination of the Moon (red dot). We add to this figure the zenith distance ( $zen$ ) and the altitude ( $alt$ ). The altitude of the Moon is the angle it makes with the observer's horizon, which sums to  $90^\circ$  when combined with the zenith distance ( $zen$ ). On the right, we see that we can define the latitude ( $lat$ ) as the sum of the zenith ( $zen$ ) and the lunar declination ( $dec$ ).

In Figure 2.9, we add the definitions of the altitude ( $alt$ ) and the zenith distance ( $zen$ ) of an object such as the Moon. While the declination of an object is the same for all observers, the altitude of an object in the sky is a local measurement. (The Moon can lie almost directly overhead in Florida at the same time that an observer in Hawaii will see it rising above the eastern horizon.) Altitude is tied to the observer's location; the closer an object lies to

directly overhead (near to the zenith), the larger its altitude must be. An object rising on the horizon has an altitude of zero, while an object directly overhead has an altitude of  $90^\circ$ . If you put one arm out in front of you and point at the horizon, and point the other arm directly overhead, your arms will form a right angle ( $90^\circ$ ). As the altitude of an object is the angle between it and the horizon and the zenith distance is the angle between it and the zenith, the sum of an object's altitude and zenith distance is always equal to  $90^\circ$ .

$$zen = 90^\circ - alt$$

For simplicity, you may assume that the standard deviation  $\sigma$  of the zenith distance and latitude are the same as that estimated for the lunar altitude. (If you measured the Moon's altitude to within  $2^\circ$ , you can estimate your zenith and latitude to the same precision.)

6. The zenith distance of the culminating first quarter moon was \_\_\_\_\_  $\pm$  \_\_\_\_\_ degrees.

In the second panel of Figure 2.9, we can combine these concepts to deduce a relationship between the zenith distance of the Moon and its declination, for an observer at a given latitude.

$$lat = zen + dec$$

7. The latitude of my observing location, \_\_\_\_\_,  
is \_\_\_\_\_  $\pm$  \_\_\_\_\_ degrees.

Now look up the latitude of your observing location, and compare this value to your estimate.

8. The tabulated latitude of my observing location is \_\_\_\_\_ degrees.

9. By how many  $\sigma$  do the two latitude values differ? If the difference is more than  $2\sigma$ , discuss possible sources of error in your measurements that could account for the difference.

## 2.2.6 Measuring the North Star's Altitude, Finding Our Latitude

We can estimate our latitude another way, without needing to look up any numbers. We will take advantage of a lucky feature of the northern hemisphere, one that has saved countless sailors and land-based travelers from losing their way at night. (If you live in the southern hemisphere, speak to your instructor about an alternate activity or a multi-day field trip.)

We know that the Earth spins around in place once a day. Its rotational axis is an imaginary line which extends above the South Pole down through the Earth and out above the North Pole. If you continue on up into the sky high above the North Pole (along what we call the North Celestial Pole, or the NCP), you will eventually run into a star – Polaris, or the North Star! (By chance, there is no star visible from Earth in the patch of sky above the South Pole along the SCP, which is why penguins make such poor navigators and should not be entrusted with the maps on long journeys.)

Because the North Star lies almost directly above the North Pole, and the Earth rotates once a day around its rotational axis, the northern sky appears to rotate counter-clockwise around the North Star, completing a full circle once every 24 hours. This star neither rises nor sets (nor do its nearest neighbors)! Figure 2.10 shows the orientation of the constellations surrounding it, to help you find the North Star on any night of the year.

You are going to find the North Star and measure its altitude above the horizon. This will lead directly to an estimate of your local latitude. Refer to Figure 2.10 to find the North Star in the early evening after dark. Work away from street lights, house lights, and other sources of light pollution, but make sure that your location is safe. If you have never gone sky-watching at night, you might ask a family member or good friend who has done so to join you. The North Star is so easy to find, however, that you should not worry if you end up looking for it on your own.

Try to perform this observation with a new or crescent phase Moon; if you wait until the first quarter or gibbous Moon, the moonlight will make the sky quite bright, and Polaris will fade into the background and become harder to find.

Give your eyes at least five (ideally twenty) minutes to adjust to the dark once you go outside; as they do so, fainter stars will slowly appear. Face in the opposite direction than you stood when viewing the Moon culminating due south. Review Figure 2.10 and examine the sky. Look first for the Big Dipper, the brightest constellation near to the North Star. The stars of the Little Dipper are fainter, so don't worry if they are less visible – the North Star is the brightest star in the Little Dipper, so if you find one star in it, you have succeeded.

Once you have identified the North Star, attempt to measure its altitude. This will be somewhat more challenging than your earlier work measuring the altitude of the Moon, because the stars are so much fainter than the Moon. If you have good eyesight and are observing in a location with a very dark sky (with no street lamps or porch lights), try to sight it through the straw of the sighting tube on your sextant. If this proves to be too difficult, do not despair. Replace the straw with the larger tube from the center of a roll of

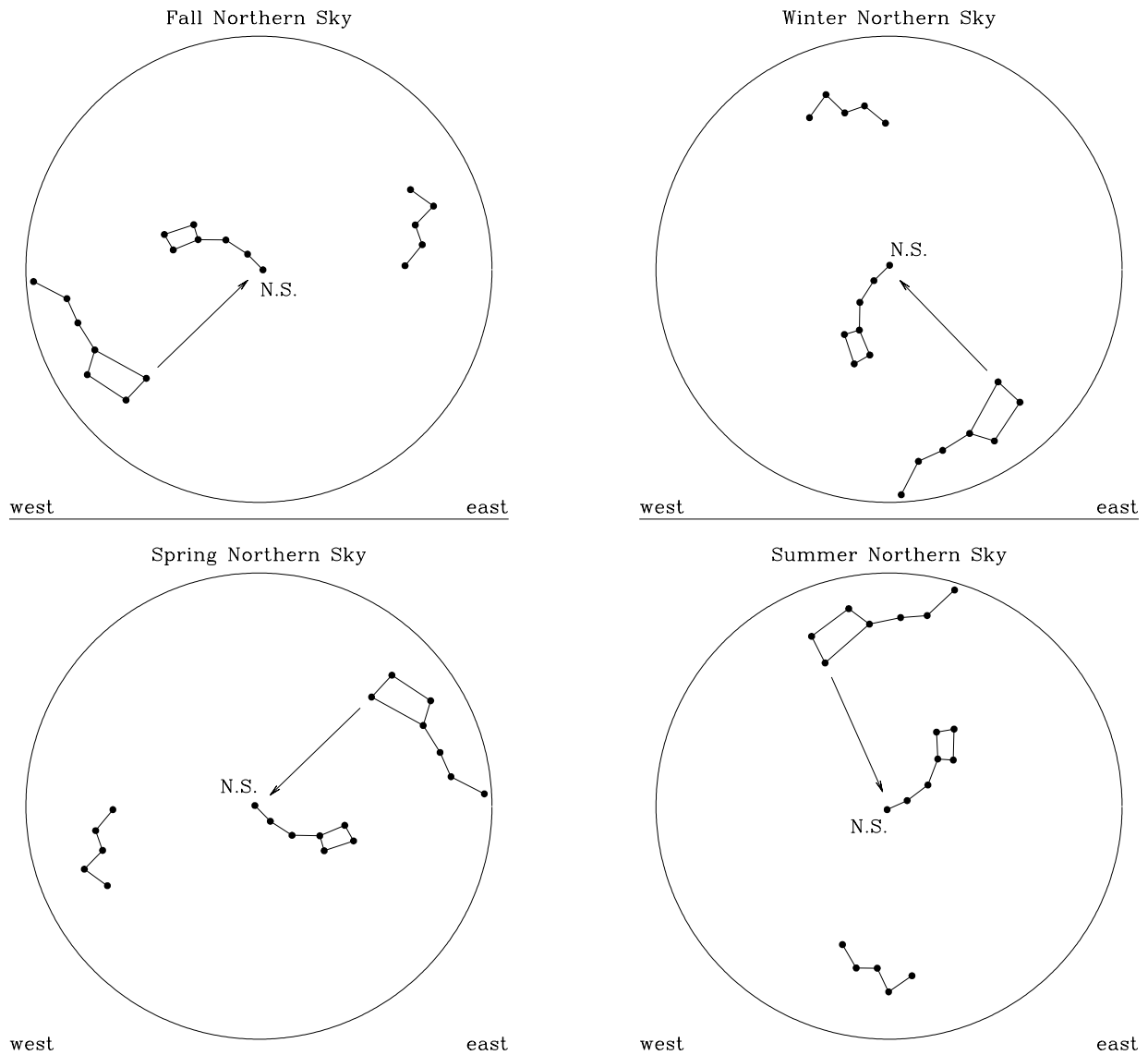


Figure 2.10: This set of images shows the brightest constellations surrounding the North Star (Polaris, labeled N.S.), throughout the year in the early evening. The classic technique for finding the North Star is to face northward, identify the bowl of the Big Dipper in Ursae Major, and then follow the line formed by the two stars which make up the lip due North (see arrows on plots). This will lead you to Polaris, a star in the handle of the Little Dipper (Ursae Minor) which happens to lie above the Earth’s rotational axis and so forms a handy reference for navigation. When the Big Dipper is low on the horizon, it may be helpful to orient yourself by first finding the characteristic “W” shape of Cassiopeia, on the other side of the North Star from the Big Dipper. Over the course of each night these constellations will rotate counter-clockwise, mapping out a full circle in the northern sky around Polaris once every 24 hours.

paper towel or toilet paper (attaching it along the same line connecting the 0- and 180-degree labels), and try to center the star in the larger field. If this too proves to be difficult, then either hold the sextant in your hand, placing the 0–180 degree line firmly between and along your index and middle fingers, or tape the paper lightly to your arm. Now point your index finger toward the North Star.

With each of these techniques, you will have pointed the 0–180 degree line toward the North Star. Now ask your assistant to read off the altitude measurement from the sextant. (Tell them not to shine their flashlight beam into your eyes as they do so.) If you have the sextant in your hand, sit down so that your assistant can easily reach the sextant. (You may want to add a red filter to the flashlight if you have one, or to wrap a small piece of red cellophane around the lit end. This will help your eyes to stay dark-adapted.)

Make your measurement three times, taking care to drop your arm down to your side between measurements so that you re-find the North Star each time. If you have to sight the star along your arm rather than finding it through a sighting tube you will probably end up with a larger standard deviation for your three measurements, but that's an acceptable compromise.

Jot the numbers down to the nearest degree, and then combine them to form a mean value and standard deviation.

1. The three measurements of the North Star altitude are \_\_\_\_\_, \_\_\_\_\_, and \_\_\_\_\_ degrees.
2. The altitude of the North Star was \_\_\_\_\_  $\pm$  \_\_\_\_\_ degrees.

How can we deduce our latitude from the observed altitude of the North Star? Examine the two examples shown in Figure 2.11, where the observer is located near to the equator and then at a higher northern latitude. Keep in mind the the angle between the observer's zenith and horizon is always  $90^\circ$ , as is the angle between the Earth's equator and North Pole.

3. Bearing in mind that the North Star is located high in the sky above the North Pole, at what altitude (how far above the horizon) would it appear if viewed from the North Pole, with a latitude of  $90^\circ$ ? (*Remember that the angle between the horizon and the zenith, directly overhead, is  $90^\circ$ .*)

4. At what altitude (how far above the horizon) would the North Star appear if viewed from the equator, with a latitude of  $0^\circ$ ?



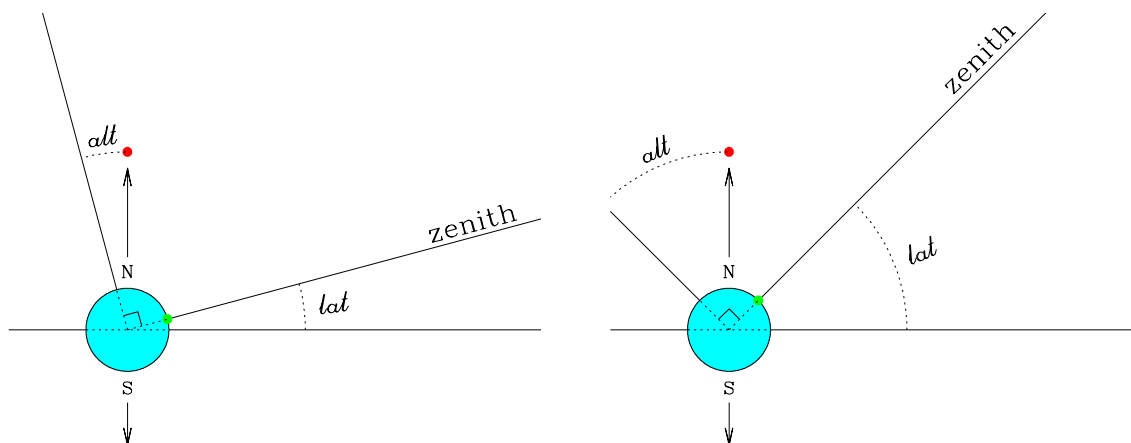


Figure 2.11: In this figure, as in previous ones, the light blue globe represents the Earth, with the North Pole facing upward and the equator running horizontally across the page. We mark the altitude of the North Star (red dot), and the latitude of an observer (green dot), for two observers who lie near to (left) and far from (right) the equator. What is the relationship between the altitude of the North Star and the latitude of the observer?

5. Explain the simple connection between the altitude of the North Star and an observer's latitude.

6. The latitude of my observing location is \_\_\_\_\_  $\pm$  \_\_\_\_\_ degrees.

7. By how many  $\sigma$  do the latitude estimate based on the North Star and the tabulated value you looked up earlier (see §2.2.5) differ? If the difference is more than  $2\sigma$ , discuss possible sources of error in your measurements that could account for the difference.

## 2.3 Final (Post-Lab) Questions

1. Fill in the following 12 blanks with the words sunrise, noon, sunset, or midnight.

The new moon rises at (roughly) \_\_\_\_\_, culminates at \_\_\_\_\_, and sets at \_\_\_\_\_.

The first quarter moon rises at \_\_\_\_\_, culminates at \_\_\_\_\_, and sets at \_\_\_\_\_.

The full moon rises at \_\_\_\_\_, culminates at \_\_\_\_\_, and sets at \_\_\_\_\_.

The last quarter moon rises at \_\_\_\_\_, culminates at \_\_\_\_\_, and sets at \_\_\_\_\_.

2. Describe one way in which you could introduce a systematic error into the measurement of your latitude, while estimating the altitude of the culminating first quarter moon.

3. For an observer at a latitude of  $+32.3^\circ$  (north of the equator), what should a first quarter moon's altitude be at culmination on June 21? Show your work.

4. If the culminating first quarter moon has a declination of  $-23^\circ$  in the Fall (appears  $23^\circ$  south of the equator), what will the declination be of the last quarter moon two weeks later?

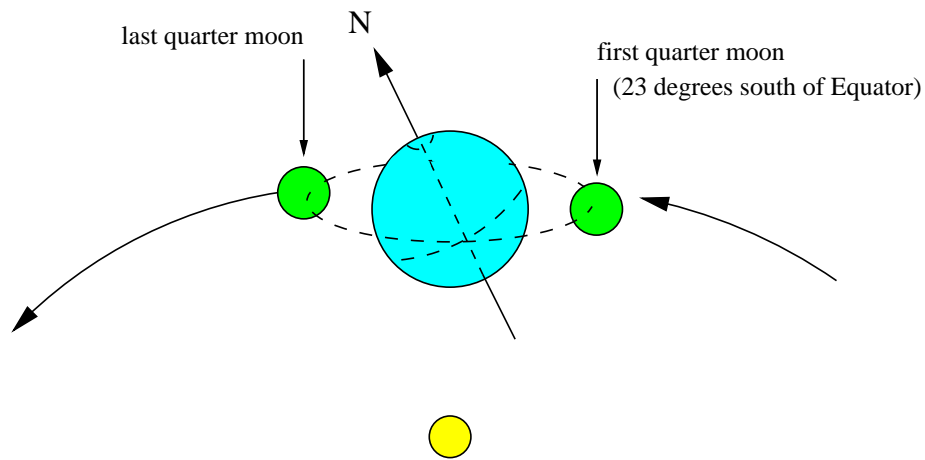


Figure 2.12: This figure shows the Sun (yellow globe) and Earth (light blue globe), and the Moon in its first and last quarter phases (green globes) during the Fall season. The Earth is tilted over by  $23^\circ$  to the left, as shown by the tilt of the rotational axis running through the North (N) and South Poles. A small dashed circle indicates the position of the northern polar cap, and a longer dashed arc traces the Earth's equator. The orbital path of the Moon is also drawn, as a dashed ellipse.

## 2.4 Summary

After reviewing this lab's goals (see §2.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. Describe your observations of the sky, your analysis of the resultant data, and your overall conclusions. (30 points)

Be sure to cover the following points.

- Describe the movement of the Moon over the course of a lunar month, and explain how its appearance in the sky changes accordingly.
- Tell us what you have learned about techniques for measuring angles on the sky, and how to observe basic properties (location, size, appearance) of the Sun, Moon, and stars in the sky.
- Report on any observational, measurement-related, and/or logistical challenges that you faced while conducting this lab, and explain how you dealt with them. *(If you came up with a clever idea to solve a problem, we'll pass it on to students in future classes.)*

- Explain the relationship between an observer's latitude and observables like the altitude of the culminating first quarter moon and the North Star, and tell us why it was important that travelers determine their latitudes in the past.
- If you thought of any particular ideas or visualization tricks that helped you to understand the movement of the stars or Moon through the sky that aided you in completing this exercise, please tell us about them.

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

## 2.5 Extra Credit

Estimate the radius of the Earth, by peering around the edge of the horizon at sunset or sunrise.

Figure 2.13 illustrates two positions for an observer watching the Sun set at dusk, first lying down and seeing it set at ground-level ( $R$ ), and then standing and watching it appear to set again from an elevation of  $R + h$ , where  $h$  is the height of the observer. The Earth rotates counterclockwise through a small angle  $\alpha$  between the mini-sunsets.

You can time the interval  $\Delta t$  between the two sunsets, and then compare this time to the full 24 hours (or  $24 \times 60 \times 60 = 86,400$  seconds) needed for a complete rotation of the Earth.

$$\frac{\Delta t}{24 \text{ hours}} = \frac{\Delta t}{86,400 \text{ seconds}} = \frac{\alpha}{360^\circ}$$

so if  $\Delta t$  is measured in units of seconds and  $\alpha$  is measured in degrees,

$$\alpha = \Delta t \times \frac{360}{86,400} = \frac{\Delta t}{240}.$$

Your time interval  $\Delta t$  should lie somewhere between 2 and 20 seconds, and be measured to the nearest tenth of a second.

Once we have measured  $\Delta t$  and  $h$  (your height), we can combine them to form an estimate of the radius of the Earth. There is a right triangle formed in Figure 2.13, with width  $R$  and hypotenuse (longest side)  $R + h$ . The ratio of the width to the hypotenuse for angle  $\alpha$  is called the cosine of  $\alpha$ , and can be calculated by scientific calculators. If we know  $\alpha$  and  $h$ , we can thus deduce a value for  $R$ .

$$\cos(\alpha) \equiv \frac{R}{R + h}$$

and so

$$R = R \cos(\alpha) + h \cos(\alpha)$$

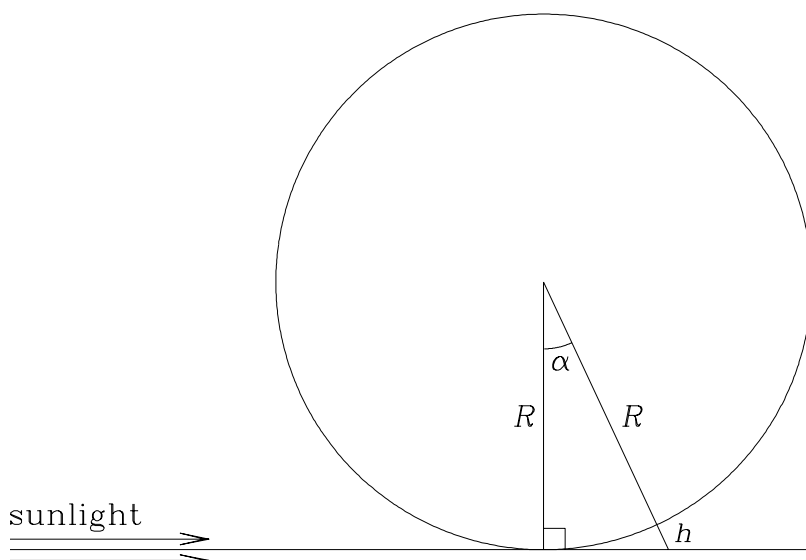


Figure 2.13: You can see two sunsets for the price of one, by first lying down and watching the Sun set below the horizon and then quickly leaping up and watching it set again (your increase in height will allow you to see “around” the Earth a bit further. At sunrise, you can also double your view by first watching the Sun rise while standing, and then lying down and seeing it rise again. This figure indicates the viewing geometry at sunset, with the Earth’s radius labeled as  $R$ , the observer’s height as  $h$ , and the angular rotation of the Earth between mini-sunsets as  $\alpha$  (magnified for clarity). Note that the Earth’s rotates in a counter-clockwise fashion in this figure.

and

$$R[1 - \cos(\alpha)] = h \cos(\alpha)$$

and solving for  $R$ ,

$$R = h \left[ \frac{\cos(\alpha)}{1 - \cos(\alpha)} \right].$$

Check that your calculator is set up to receive angles in units of degrees, by calculating  $\cos(60^\circ)$ , which is equal to  $1/2$ . (If you calculate a value of 0.9998, you need to switch your calculator units from radians to degrees.)

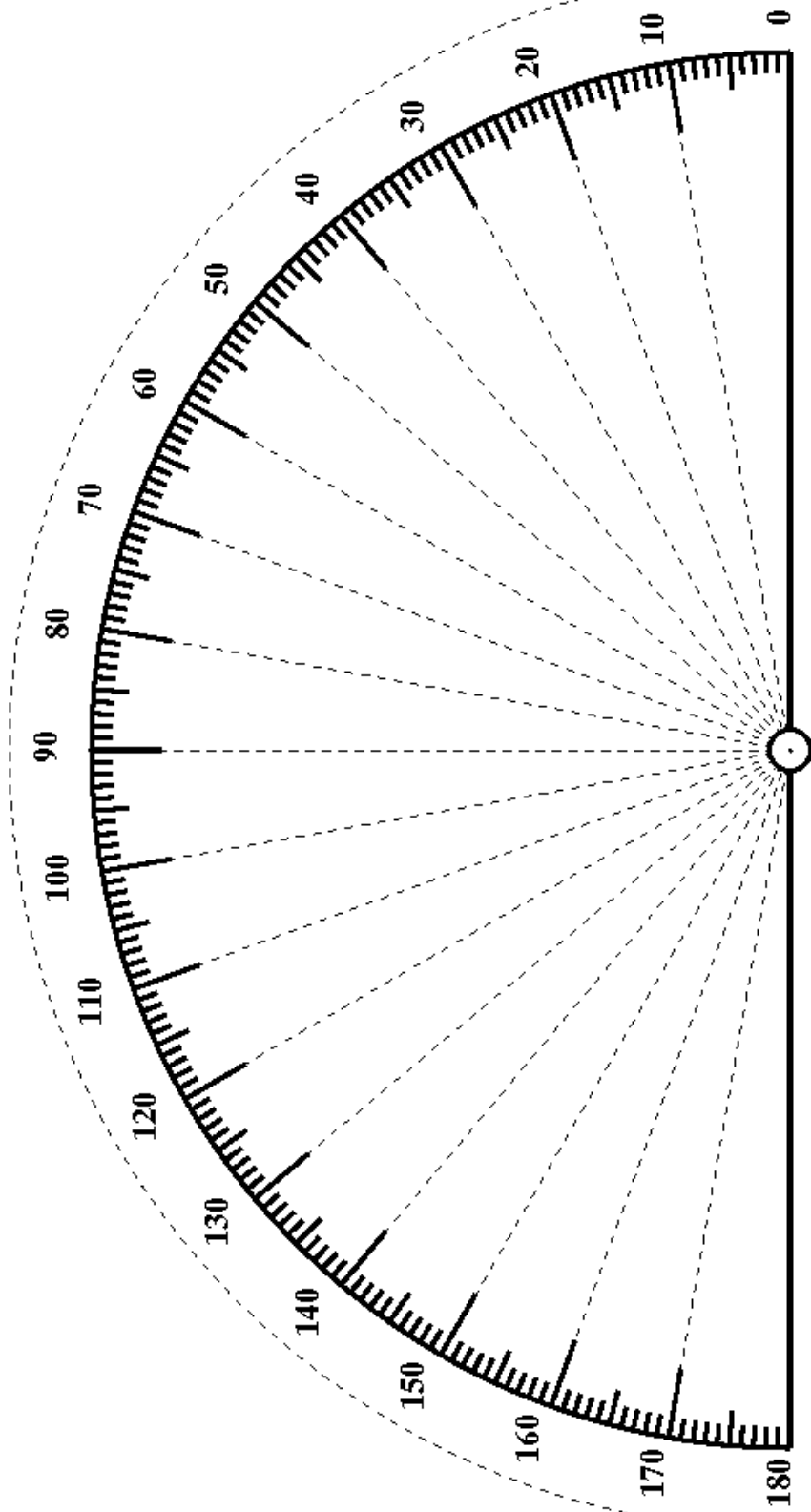
Now recall that  $\alpha = \frac{\Delta t}{240}$ , and calculate a value for  $R$ .

The Earth’s radius has been measured to be  $6,371 \pm 8$  kilometers. How does your home-grown estimate compare?



Cut along outer dotted line.

Sextant (new to full Moon)

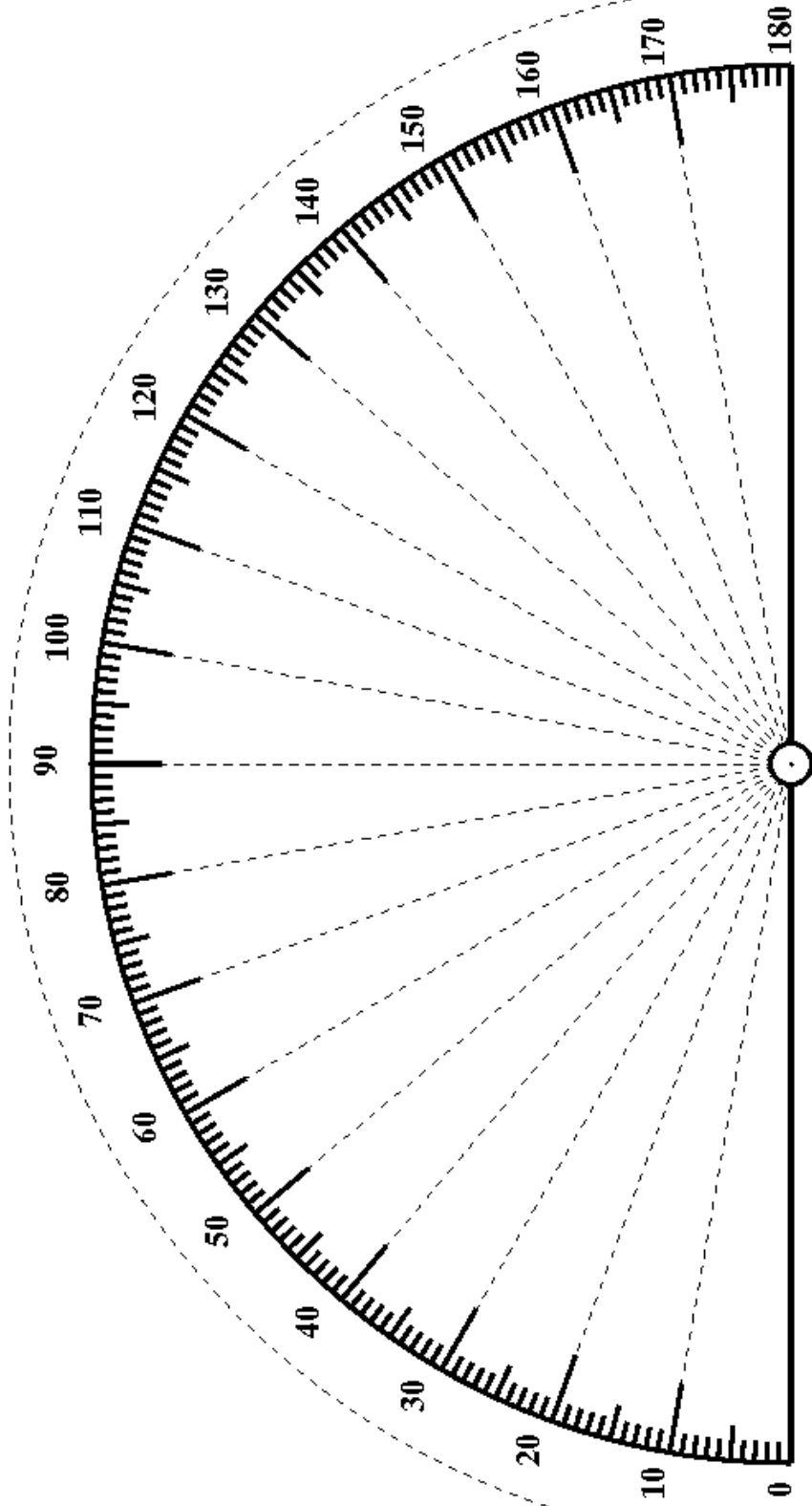






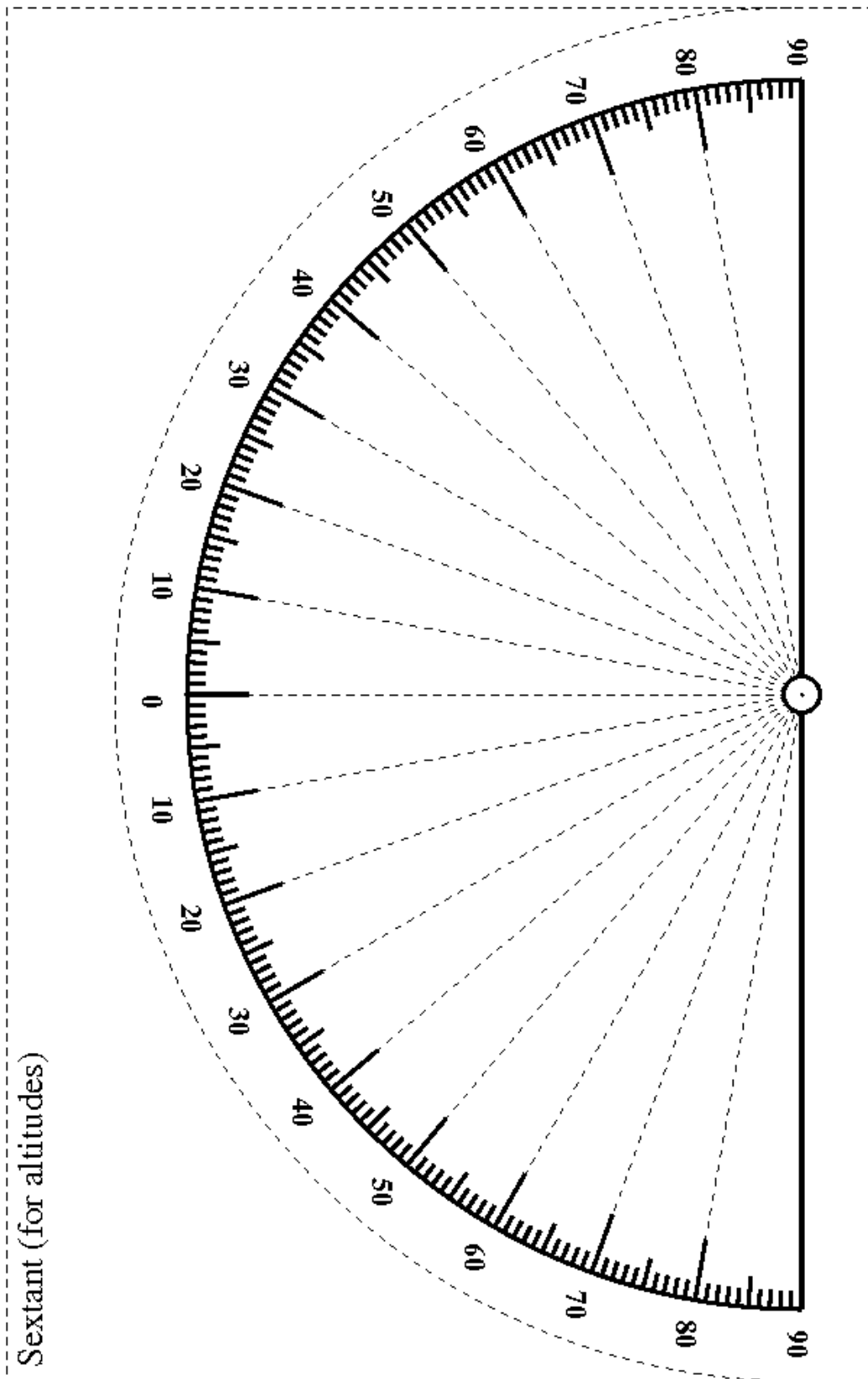
Cut along outer dotted line.

Sextant (full to new Moon)





Cut along outer dotted line.





## Lab 3

# Cratering and the Lunar Surface

### 3.1 Introduction

Even a child can look up at the Moon and distinguish between light and dark regions (which are often used to construct the face of a “Man in the Moon”). We refer to the light-colored areas as lunar highlands, and the dark-colored areas as lunar seas, or maria. Telescope images (see Figure 3.1) illustrate the dramatic contrast between the brighter, rugged, heavily cratered lunar highland terrain and the smooth, darker, flatter lunar maria, which are actually ancient lava flows. While we refer to the lava flows as ancient, we think that the mare (the singular form of maria) are generally younger than the highlands.



Figure 3.1: Telescope and space craft images of (a) the cratered surface of Moon, (b) the craggy lunar highlands, and (c) the smooth lunar mare.

How can we date surface features on the Moon? This exercise will acquaint you with that process, building on geological studies of the Earth. It will also help you to better understand cratering, an important process which shapes the appearance of many planetary surfaces.

We can study the Moon to learn more about the history of both the Moon and the Earth. Geological forces continually shape and reshape the surface of the active Earth. The Moon, lacking internal heat and thus geologically dead, has changed very little over the last three billion years. Its current appearance was formed in the early days of the solar system, a violent era in which the inner regions were littered with huge chunks of rocky planetesimals. This was material left over from the accretion process which drove the formation of the planets some 4.5 billion years ago. Impact craters resulting from frequent collisions with abundant space debris have mostly been obliterated on Earth, by plate tectonic-related activity and erosive forces. On the Moon, however, they have been preserved. By chronicling impacts, and accurately dating craters across the lunar surface, we can construct a record of the changing patterns of rocky debris in the inner part of the solar system. This is of interest for many reasons, and is an important step in estimating the probability that life could occur in various locations throughout the solar system.

The surface of the young Earth was kept in a hot, molten or semi-molten state, as space debris bombarded it over the first half billion years of its existence. The kinetic energy (energy of motion) of each rocky collision was transformed into thermal energy (heat). As the rate of collisions slowed, the planetary surface cooled and solidified, and conditions capable of supporting life slowly came into being. We think this happened around four billion years ago (four thousand million years). The earliest signs of terrestrial life date from 3.8 billion years ago, suggesting that life formed quickly once it became possible.

Sixty-five million years ago, a rare collision between the Earth and a huge rock ended the reign of the dinosaurs. The 180 kilometer (110 mile) diameter Chicxulub Crater in the Yucatan peninsula of Mexico is the “smoking gun” for this apocalyptic event. What size projectile (a huge rock, a larger meteoroid, a small asteroid, or even a comet) produced this crater? After experimenting with creating craters in this lab and using a program to simulate the effects of Earth impacts, you will better understand how such size estimates are made.

Could the Earth suffer a future collision, with catastrophic consequences for our species? We can help answer this question by observing the lunar surface, and establishing the current cratering rate. As reported in 2010 in *Nature* magazine, researchers studying NASA’s Lunar Reconnaissance Orbiter images have established that at least five new lunar craters have appeared in the past 40 years.

### **3.1.1 Goals**

The primary goals of this laboratory exercise are to analyze the appearance of the lunar surface and recognize how studying cratering can increase our understanding of the Earth’s impact history. We will examine the key factors affecting the appearance of impact craters and ejecta, investigate the relationship between the velocity of a projectile and the crater it forms, and identify factors that determine the environmental consequences of impacts here on Earth.

### 3.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed here.

<http://astronomy.nmsu.edu/geas/labs/html/home.shtml>

You will need the following items to perform your experiment:

- a cooking pan, paint tray, or box (a container at least 10"  $\times$  12" across, and 2" deep)
- two quarts of clean, dry, loose sand (lunar base surface material)
- two cups of salt, sugar, or colored sand (lunar topping material)
- several dense round projectiles (such as  $\frac{3}{8}$ " ball bearings or marbles)
- a cup or large spoon, small brush, and sieve to help in dressing your lunar surface
- a magnet (useful but not essential), to use in extracting your projectiles from the sand
- a small plastic container (not essential), to use to toss projectiles up to an assistant
- a page or two of newspaper, to keep your work area clean
- a 3"-8" ruler
- an 8-foot or longer tape measure

You will also need a computer with an internet connection, to analyze the data you collect from your cratering experiment. An assistant would be helpful for conducting the experiment in Section 3.3.1, but you can also work alone.

### 3.1.3 Primary Tasks

This lab is built around three activities: 1) studying the lunar surface and age-dating features, 2) creating your own craters (through experimentation), and 3) determining a crater size, projectile velocity relationship

### 3.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that Section 3.7 (§3.7) contains 5 extra credit points.

Table 3.1: Breakdown of Points

Activity	Crater Ages	Crater Experiment	Crater Sizes	Questions	Summary
Section	§3.2.1	§3.3.1	§3.4.1	§3.5	§3.6
Page	78	83	90	92	95
Points	19	16	15	15	35

### 3.1.5 Timeline

Week 1: Read §3.1–§3.4, complete activities in §3.2.1 and §3.3.1, and begin final (Post-Lab) questions 1–7 in §3.5. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report template, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Complete activity in §3.4.1, finish final (Post-Lab) questions in §3.5, write lab summary, and submit completed lab report.

## 3.2 Relative Ages on the Moon

When it first formed, the Moon's interior was so hot that the upper mantle was molten (in a liquid state). Frequent impacts cracked the lunar crust repeatedly, and so this molten rock (or lava) flowed freely onto the surface. As the Moon cooled over hundreds of millions of years, however, large projectiles impacting the surface triggered flows less and less often. We think that the era of impact-triggered, widespread lunar lava flows that produced the lunar mare ended around three billion years ago. How was this age determined?

There are two primary techniques used for dating rock samples. Radioactive dating (measuring the amounts of various elemental isotopes which decay at different rates) can produce solid, absolute ages. This technique has been used on lunar rocks collected from a few select regions by Apollo astronauts, and provides the foundation for estimating the duration and placement of many events in the Moon's evolutionary history. As most of the lunar surface remains unvisited and unsampled by humans, this chronology has been filled in using relative dating methods. These techniques can establish which of two rock samples is older, or determine which of two events happened first, but they cannot generate absolute times for events unless combined with sufficiently detailed, independent, absolute measurements for similar samples.

Large impacts are key events in lunar history. Consider how an impacting meteoroid creates a crater. The force of the high-speed impact obliterates the projectile and displaces part of the Moon's surface, pushing the edges of the resulting crater high above the surrounding rock. At the same time, displaced material shoots outward from the crater, creating rays of ejecta (seen as radial bright streaks centered on the crater). The most recent such event occurred some 110 million year ago, and produced the 85 kilometer (53 mile) wide crater Tycho, easily seen near the bottom of Figure 3.1a. Note the bright rays emanating outward from it in all directions, some extending over 1500 kilometers in length. Close-up views of the crater show a prominent central peak, produced when molten excavated material from the blast flowed back towards the center of the excavation and formed a mountain peak.

The Law of Cross Cutting Relations is the key principle behind relative dating of craters. It states that any geological feature (such as a rock structure or a fault) that cuts across



another geological feature must be younger. A crater in a lunar mare region must thus be younger than the mare – because the mare had to be present before a meteoroid could hit it, producing a crater that cut through it. If we see a crater sitting atop the rim of another, partially obliterating it, the superimposed crater must be younger.

### Example 3.1

In the image at the right (Figure 3.2), smaller crater Thebit A appears in the upper left-hand corner, on top of and interrupting the rim of larger crater Thebit. Common sense tells us that Thebit must have formed before the rock that created Thebit A could crash into and destroy part of its rim.



Figure 3.2:  
Crater Thebit

Relative ages for features over an entire planetary surface can be obtained by comparing the number of impact craters in different areas. The longer a surface has been exposed to meteoric bombardment, the more impact craters it should have acquired. Older surfaces thus have accumulated more impact craters than younger ones.

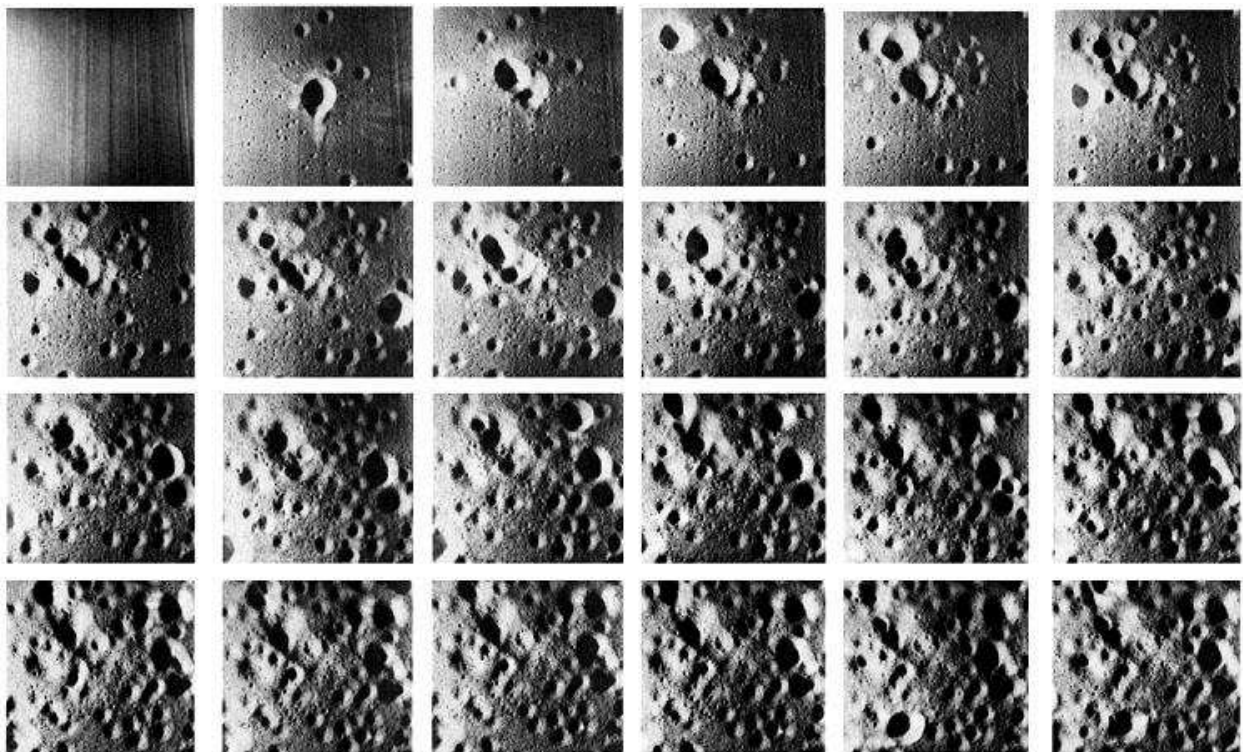


Figure 3.3: A series of time step images, starting in the upper left-hand corner and proceeding across each of four rows, in turn, models the changes in appearance for a smooth surface exposed to a steady rain of small particles of varied sizes.

### Example 3.2

The preceding image (Figure 3.3) shows the results of cratering experiments conducted in a laboratory here on Earth. The first frame (upper left-hand corner) shows an initially smooth surface, one which is then exposed to a steady bombardment of small particles of various sizes. The time-lapse sequence of images, with time increasing from left to right and then

top to bottom, illustrates the sequential degradation of the surface. Note how as the surface gets “older” it presents a steadily increasing number of craters. The “age” associated with each image could be estimated by counting the number of superimposed craters atop the surface.

Crater counts can be used to establish relative ages for surfaces. If an average cratering rate (the number of new craters created per unit area per year) is known, counts can be used to estimate ages for surface features. The basic idea is that older surfaces are more heavily cratered, so age is proportional to the surface density of craters. Complicating factors include an inability to correct for numerous secondary craters produced by secondary projectiles thrown up by a primary projectile, and lava (or water) flows that can obliterate craters. Given the Moon’s minimal surface activity of late, it is safe to assume that once widespread lunar lava flows ended, few or no craters were erased from the record.

### 3.2.1 Activity: The Lunar Surface, Craters, and Relative Dating

Our introduction to the lunar surface continues with an examination of the regions visited by the crews of the Apollo 12 and Apollo 16 manned missions. We will examine two Lunar Orbiter survey photos, shown at low-resolution in Figure 3.4.

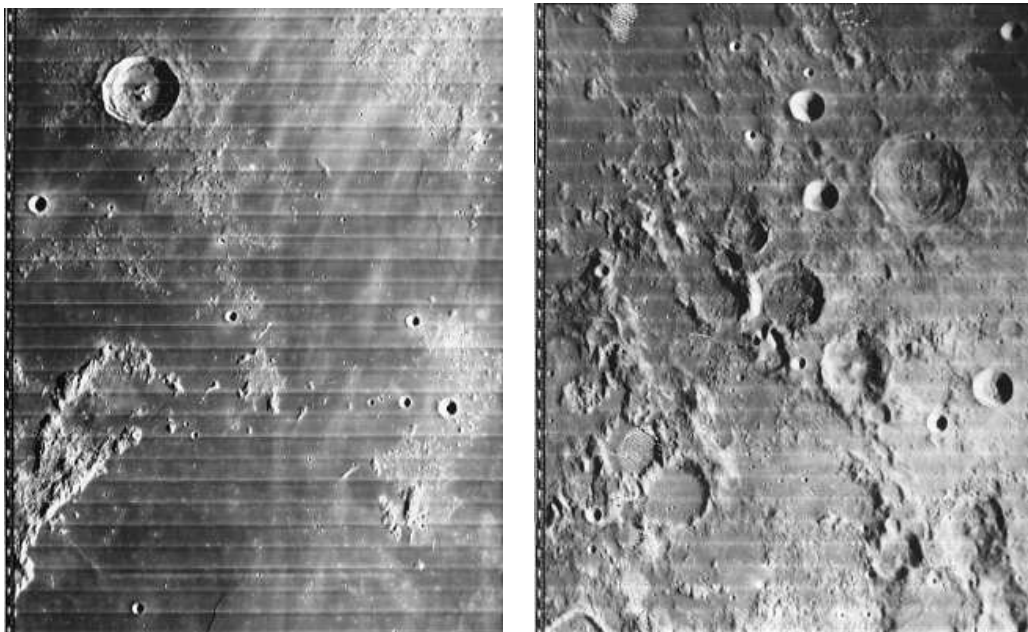


Figure 3.4: Low-resolution Lunar Observer images of the landings sites for the Apollo 12 (left) and the Apollo 16 (right) missions, taken from a space craft orbiting the Moon.

1. It is important that you view higher-resolution versions of the two images from the GEAS project lab exercise web page (see the URL on page 75 in §3.1.2). Click on the labeled and unlabeled versions of the images on the website to bring up higher-resolution images, zooming in to study detailed features. Do not use the lower-resolution images shown in the

manual, as you would lose too much detail.

Complete the following nine questions by circling the appropriate answer. (9 points total)

(a) Which landing site better represents the lunar mare region? (Apollo 12 / Apollo 16)

(b) Which landing site better represents the lunar highlands region? (Apollo 12 / Apollo 16)

(c) Which of the following craters lacks a central peak? (Lansberg / Eppinger / Taylor)

(d) Which of the following names on the Apollo 12 site image denotes a mountain chain?

(Mare Insularum / Montes Rhiphaeus / Luna 5)

(e) Which of the following craters on the Apollo 16 site image no longer has an intact, well-defined circular rim?

(Alfragenus / Theon Senior / Theon Junior / Zollner)

(f) Which lunar mission landed in an area crossed by a prominent lunar ray?

(Apollo 12 / Apollo 16)

(g) Which landing site appears to have a greater density of craters? (Apollo 12 / Apollo 16)

(h) If Crater Eppinger (at the bottom of the Apollo 12 landing site image) has a diameter of six kilometers, what is the diameter of the smallest crater shown in this image?

(0.1 kilometers / 1 kilometer / 2 kilometers / 3 kilometers)

(i) The largest crater identified by name on the Apollo 16 image has a diameter of:

(30 kilometers / 40 kilometers / 50 kilometers / 100 kilometers)

2. Next consider a region at the lunar highland–mare interface, shown in Figure 3.5. Note that parts of the rims of craters Fracastorius and Beaumont appear to have been destroyed by lava flows associated with the Sea of Nectar (Mare Nectaris). There is a bright streak (a lunar ray) to the lower-left of crater Rosse, illuminated when observed at full moon with the Sun overhead (as shown in the lower-left corner of Figure 3.5).

Consider four events:

(a) the lava flow associated with the impact that created the Mare Nectaris

(b) the impact event that created crater Rosse, sitting in the Mare Nectaris

(c) lunar highland crater production (of which Fracastorius and Beaumont are examples)

(d) a lunar ray (associated with crater Tycho, not shown), which is observed to pass over and rest on Rosse.

Order these four events in time, from earliest to latest: \_\_\_\_\_

Explain why you ordered them as you did. (4 points)

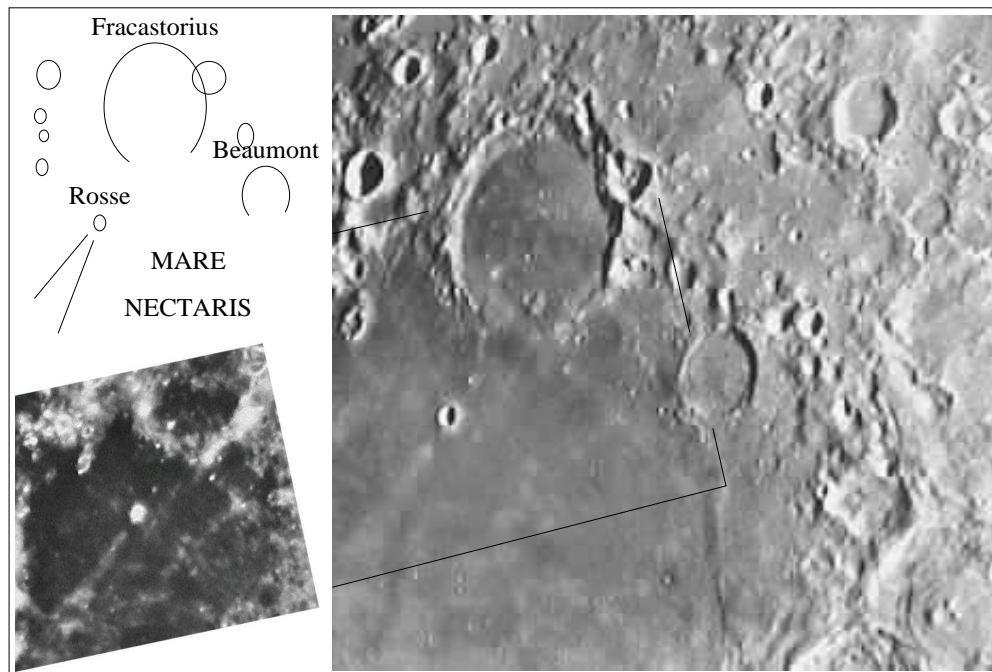


Figure 3.5: The region surrounding the large craters Fracastorius and Beaumont (right), identified by name (upper-left), and a second image with the Sun directly overhead, (lower-left) centered around crater Rosse in the heart of the Mare Nectaris (marked at right).

3. Now build two plots, showing the density of new craters found superimposed atop large craters which formed at various times in the distant past. Access the first plotting tool listed for this lab exercise from the GEAS project lab exercise web page (see the URL on page 75 in §3.1.2).

(a) In your first plot, track the large crater age versus the surface density of later, superim-



posed craters for the first four of the large craters listed in Table 3.2 (those less than 150 million years old). Be sure to give your plot a title – something like “Crater Properties” is fine. Label the x-axis “Surface Density (per  $10^6 \text{ km}^2$ )”, and label the y-axis “Age (Myr)” to indicate units of millions of years. Select “yes” for both “show linear fit to data” and “force fit through origin,” to add a line predicting the age of a crater covered by new craters to a certain density, and to force the line to go through the origin. (This is equivalent to saying that a brand-new crater should have no craters superimposed on top of it when it first forms, just like a freshly-washed counter-top should show no fingerprints.)

Table 3.2: Crater Properties

Entry	Crater Name	Density (#/10 <sup>6</sup> km <sup>2</sup> )	Age (Myr)
1	Cone (Apollo 14)	20	26
2	North Ray (Apollo 16)	50	50
3	Necho	67	80
4	Tycho	90	109
5	Aristarchus	270	450
6	Copernicus	1,200	850

(b) Use the best-fit line on the plot to estimate the age of the crater Giordano Bruno, a young, virtually unblemished crater which lies slightly on the far side of the Moon. Recent studies have reported a surface density of 3.2 craters per million square kilometers on the continuous ejecta surrounding the rim of the Bruno crater. Use the relation between age and density derived for the four objects in your plot to estimate how long ago this crater formed, given the observed surface density of additional craters.

To do this, use the relation

$$\text{Age} = m \times \text{Surface Density}$$

where the slope  $m$  of the relation is taken from the fit to the data in your plot. (For example, if the slope of your plot was 2, then the derived age would be  $2 \times 3.2 = 6.4$  million years.)

Derived age: \_\_\_\_\_ (Myr). (1 point)

To check your answer, add a new data point to the input data for your plot, with the crater density and the age for the Bruno crater. When you recreate the plot, the slope should not change appreciably, and the new point (the one closest to the origin, with an x-coordinate of 3.2) should appear right on top of the fitted line.

(c) In the 1970s, astronomer Jack Hartung proposed that the Bruno crater formed quite recently, on June 18, 1178. He hypothesized that five monks from Canterbury observed the impact, based on a written account of their testimony.

*“Now there was a bright new [thin crescent] moon, and as usual in that phase its horns were tilted toward the east and suddenly the upper horn split in two. From the midpoint of this*

*division a flaming torch sprang up, spewing out, over a considerable distance, fire, hot coals, and sparks. Meanwhile the body of the moon, which was below, writhed ... like a wounded snake."*

Does your age estimate support or counter Hartung's hypothesis? (Support / Counter)

Explain your answer. (2 points)

(d) In your second plot, add in the two oldest craters (Aristarchus and Copernicus) listed in Table 3.2 to the data to be plotted. Once again, fit a line through the data points and through the origin. Compare the slope of the line in the first plot to that of the second plot.

Based on the distribution of all six data points, and the best-fit lines, has the lunar cratering rate changed significantly over the last 850 million years? Explain your answer. (3 points)

Be sure to include both plots in your lab report.

*Congratulations, you have completed the first of this lab's three activities. You may want to answer Post-Lab questions 1 through 3 on page 92 at this time.*

## 3.3 Cratering Experiments

Astronomers are rarely presented with opportunities to perform active experiments, being constrained to observe whatever the Universe happens to send our way. We can, however, better understand the process of planetary and lunar cratering through direct experimentation. By launching projectiles of various masses at various speeds, and angles of impact, towards a variety of hard and soft surfaces, we can simulate a wide range of potential impact events.

In this activity, you'll investigate how the velocity of a projectile is related to the size of the craters it creates. To control the projectile speed as much as possible, we will drop each one from a set height, calculating its speed at impact by computing the kinetic energy (energy of motion) it gains as it falls.

These velocities will be much smaller than the tens of kilometers per second ( $\text{km s}^{-1}$ ) at which space debris typically crash into planets or satellites. Our craters will thus be smaller (whew!), as less energy will be released upon impact. Note that space debris typically travel fast enough to explode on impact, a phenomenon which tends to produce circular, symmetric craters, but we will not be able to reproduce this exciting behavior.

### 3.3.1 Activity: Forging Craters

Start by listing the factors you think might define a crater's size and appearance (the shape, the presence or absence of central peaks, ejecta, rays, etc.). (1 point)

#### Experimental Procedure

1. Begin by spreading out a page of newspaper on the ground or floor, to collect any scattered materials. Choose a location that is sheltered (or indoors) if the weather is very hot or windy, or if it might rain, and make sure that you have at least seven feet (and ideally thirteen feet) of accessible space above the paper so that you can release projectiles from a variety of heights above it easily. Prepare a simulated "lunar test surface" by filling an appropriate container with your base material. You could use a large cooking pan, a paint tray, a clean litter box, or even a sturdy cardboard box, one with dimensions of at least 10 by 12 inches. Fill the bottom inch of the container with clean, dry, uncaked sand, or a similar substance. Smooth the surface carefully, tapping the sides of the container gently to help the material to settle evenly and form a flat, even surface.
2. Sprinkle a fine layer of appropriate lunar topping material on top of the base layer. The topping should differ distinctly in color and in appearance from the base. You might try

Table 3.3: Crater Diameter Measurements I

<i>Drop from height of 25.4 cm (10 inches), for impact velocity of <math>223 \text{ cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								
<i>Drop from height of 50.8 cm (20 inches), for impact velocity of <math>316 \text{ cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								
<i>Drop from height of 76.2 cm (30 inches), for impact velocity of <math>386 \text{ cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								
<i>Drop from height of 101.6 cm (40 inches), for impact velocity of <math>446 \text{ cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								

(6 points)



Table 3.4: Crater Diameter Measurements II

<i>Drop from height of 152.4 cm (60 inches), for impact velocity of <math>547 \text{ cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								
<i>Drop from height of 203.2 cm (80 inches), for impact velocity of <math>631 \text{ cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								
<i>Drop from height of 304.8 cm (120 inches), for impact velocity of <math>773 \text{ cm s}^{-1}</math>, or _____ cm for velocity of _____ <math>\text{cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								
<i>Drop from height of 406.4 cm (160 inches), for impact velocity of <math>892 \text{ cm s}^{-1}</math>, or _____ cm for velocity of _____ <math>\text{cm s}^{-1}</math>.</i>								
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	$\text{Avg}_5 \pm \sigma$
Diameter (cm)								
Notes								

(6 points)

table salt, bath salts, colored sand, or any inexpensive powder with a similar consistency that is handy. Be sure to cover the base layer completely, and to a uniform depth, with the topping.

3. You will be dropping a projectile repeatedly into the container, measuring the resultant crater diameter, noting anything unusual (such as distinctly non-circular craters), and resurfacing the materials in the container between sets of trials. In order to obtain useful data, it is important that you consistently prepare the surface in the same manner, and measure crater diameters the same way. If a crater is non-circular, measure the longest dimension; if its rim is terraced, measure from the outer terrace on one side to the outer terrace on the other side. Use a small ruler to measure crater diameters to the nearest half of a millimeter or two-hundredths of an inch, and a longer tape measure to determine the height above the surface from which projectiles will be dropped.

4. Begin the data collection process by dropping the projectile from a height of 10" (25.4 cm) onto the prepared surface. (If you measure the height from the ground level, or the bottom of your container, make sure to account for the roughly one-inch height of your lunar surface!) Examine the crater *before* removing the projectile from the sand, as its removal may damage the delicate crater pattern. Measure the diameter of the resulting crater and record it under "Trial 1" in Table 3.3, in units of centimeters. Measure the length to the nearest half of a millimeter (0.05 cm), so write "3.25 cm," for example, rather than just "3 cm." (If you do not have a metric ruler, one marked with centimeters and millimeters, recall that one inch is equal to 2.54 centimeters and translate your measurements from fractions of inches accordingly.) Take notes about crater appearance as appropriate (when you notice interesting or unusual patterns).

5. Perform seven trials from this height, recording diameters and notes in Table 3.3 each time. If your container is large enough, you may be able to fit in an entire set of seven trials in one go by dropping the projectile onto various areas. If your projectile ever forms a crater which overlaps with an existing structure, discard that trial and drop the projectile again. Whenever the surface fills up with craters, remove the top layer of material, smooth away any craters in the base, and re-apply a new, smooth top layer.

6. Examine the set of diameter measurements, and discard (set aside) the largest and the smallest of the seven values. (This is called removing the outliers, and eliminates the points most likely to have been affected by the wind or errors in procedure or measurement.) Then average the remaining five values and record an error estimate based on their spread in value, by creating a histogram of the values in the first plotting tool listed for this lab exercise on the GEAS project lab exercise web page (see the URL on page 75 in §3.1.2). Place these values in the final column of Table 3.3.

7. Next increase the height from which the projectile is dropped to 20" (50.8 cm), and perform seven more trials, recording your observations in the next two lines of Table 3.3. When your measurements are completed, discard the largest and smallest values and average the remaining five together, estimating the spread, or scatter ( $\sigma$ ), again as well.

8. Repeat this process at heights of 30" (76.2 cm), 40" (101.6 cm), 60" (152.4 cm), and 80" (203.2 cm), entering the data into Tables 3.3 and 3.4. Each completed data table is worth up to 6 points.

9. There are two remaining lines in Table 3.4. If you are working near a tall staircase and can **safely** drop a projectile from higher than seven feet, then go ahead; ideal heights are 120" (10 feet) and 160" (13 feet, 4"). However, if you do not have a safe, secure way to drop a projectile from a larger height then leave these two lines blank or work at lower heights. **Do not use a step ladder or climb anything but a stable staircase – your safety is much more important than these final two sets of data points.**

You may want to enlist a helper (and a few identical projectiles) for this part of the experiment, to save you time by dropping the projectiles while you measure the crater diameters below. If you do so, bring along a 35-mm film cannister or another small plastic container, so that you can easily toss the projectiles back up to your assistant after each cratering event. **Be careful that no projectiles are dropped on anyone's head!**

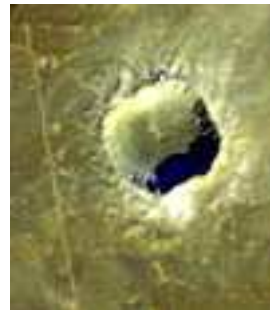
Discuss your findings in the space below. In particular, comment on observed ray patterns and lengths, any non-circular craters and central peaks seen, and anything else of interest. (3 points)

*Congratulations, you have completed the second of this lab's three activities. You may want to answer Post-Lab questions 4 and 5 starting on page 92 at this time.*

## 3.4 Predicting the Sizes of Impact Craters

It is quite difficult to study impact craters on the surface of the Earth, as our warm planet is geologically active and undergoing substantial amounts of surface evolution (e.g., lava and water flows, glaciation, erosion, continental drift). You may, however, already be familiar with one of the best-known impact craters in our vicinity, shown in Figure 3.6.

Figure 3.6: The Meteor Crater, in northern Arizona, is one of the best-preserved craters known on Earth. It formed some 50,000 years ago, when a 40-meter diameter chunk from an asteroid entered the atmosphere at a speed of 20 kilometers per second and struck the Earth at a  $45^\circ$  angle. The projectile massed around three-hundred-million kilograms, and contained a substantial amount of iron (with a density around eight grams per cubic centimeter, or 8,000 kilograms per cubic meter). The resulting crater spans more than a kilometer, and is more than 170 meters (550 feet) deep.



Consider the details of this event that are relevant to the cratering experiments that you have conducted. The most important properties of a projectile are its mass,  $m$ , and impact velocity,  $v$ . They determine how much energy of motion (kinetic energy) the moving projectile possesses. As you might expect, the kinetic energy of an object scales with its mass, and with its velocity.

We can write this mathematically as follows. The kinetic energy of a projectile, KE, is equal to one-half times the product of its mass  $m$  and the square of its velocity  $v$ :

$$\text{KE} = \frac{1}{2} m v^2.$$

Because the kinetic energy increases linearly with the mass but goes as the square of the velocity, the velocity is the more “important” term in the equation. (A relatively small change in velocity can produce a large increase, or decrease, in kinetic energy.) If the mass of a projectile doubles, its kinetic energy goes up by only a factor of two. If the velocity doubles, however, its kinetic energy increases by a factor of  $2^2$ , or four.

### Example 3.3

Let’s calculate the kinetic energy at impact of the projectile that caused Meteor Crater, with a velocity of 20 kilometers per second ( $20,000 \text{ m sec}^{-1}$ ).

$$\text{KE} = \frac{1}{2} m v^2 = \frac{1}{2} \left( 2.7 \times 10^8 \text{ kg} \right) \left( 20,000 \frac{\text{m}}{\text{sec}} \right)^2 = 5.4 \times 10^{16} \text{ joules, or 13 megatons of TNT.}$$

In §3.3.1, you worked with a projectile of fixed mass  $m$ , varied the impact velocity  $v$  by dropping the projectile from different heights, and measured the resulting crater diameters. How are the projectile velocity  $v$  and the crater diameter  $D$  related? We might hypothesize that they are simply related directly – the greater the velocity  $v$ , the greater the crater diameter  $D$ , so that if  $v$  doubles, then so does  $D$ . We can write this mathematically as

$$D \propto v, \quad \text{meaning } D \text{ is proportional to } v$$

and we will call this model #1.

We can improve this model by realizing that the kinetic energy of the projectile can do various things upon impact. It could be used to deform surface material, lifting it up and out while forming a crater, it could heat the surface, and it could also create seismic waves. If most of the energy goes into the first task, however, then we can argue that  $v$  and  $D$  should relate to each other in a different way.

Let's assume that material is scooped out of the ground in a vaguely spherical shape, like a scoop of earth. The volume of the material lifted will then be proportional to the cube of the radius of this sphere, as the volume of a sphere is  $\frac{4\pi}{3} R^3$  for a sphere of radius  $R$ , or to the cube of its diameter (as the diameter is simply twice the radius). This sphere's diameter, and the crater diameter  $D$  and depth  $y$ , will all scale (or change) together in size. The amount of energy used to lift the material is equal to the weight of the material times the height  $y$  that it is lifted. As weight scales with volume, and as the crater height scales with its diameter (larger craters are also deeper), the energy used to lift the material will scale as  $D^3 \times D$ , or as  $D^4$ .

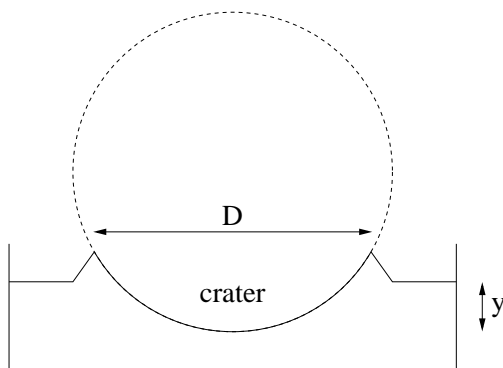


Figure 3.7: A simple model, illustrating the connection between the diameter of a sphere of material scooped out to form a crater, and the diameter  $D$  and depth  $y$  of the crater.

As we already know that the energy is proportional to the square of the projectile velocity, we can now say that  $D^4 \propto v^2$ , or  $D^2 \propto v$ . Let's write that again for emphasis:

$$D^2 \propto v, \quad \text{meaning } D \text{ squared is proportional to } v$$

and we will call this model #2.

We can take the square-root of each side of this equation if we wish, and we will then see that

$$D \propto v^{\frac{1}{2}}, \quad \text{meaning } D \text{ is proportional to the square-root of } v.$$

Let's also test a third model, where rather than  $D^2$  varying with  $v$ ,  $v^2$  varies with  $D$ . In this case,

$$D \propto v^2, \quad \text{meaning } D \text{ is proportional to the square of } v (v \times v).$$

We now have three models for the behavior of  $D$  versus  $v$ , predicting the size of a crater produced by a projectile traveling at a certain velocity. The models make different predictions, so by studying the data we have collected regarding crater sizes for projectiles dropped from various heights, and thus impacting at different speeds, we can test the models against each other.

#### Example 3.4

Simulations tell us that if the projectile that created Meteor Crater fell straight down at 15.3 kilometers per second when it hit sedimentary (soft) rock, it would have created a crater 1.6 kilometers in diameter. At twice that speed (30.6 kilometers per second), it would have forged a 2.17 kilometer crater. The ratio of the two crater diameters is  $\frac{2.17}{1.6}$ , or 1.36. This lies closer to the prediction of model #2 (a factor of two change in projectile velocity should produce a factor of  $\sqrt{2}$ , or 1.414 in crater size), than that of model #1 (which predicts a change of a factor of two in size) or model #3 (which predicts a change of a factor of four in size).

If  $D \propto v$ , then  $D_1/D_2 = v_1/v_2 = 2$ .

If  $D^2 \propto v$ , then  $D \propto v^{\frac{1}{2}}$ , and  $D_1/D_2 = \sqrt{v_1/v_2} = \sqrt{2} = 1.414$ .

If  $D \propto v^2$ , then  $D_1/D_2 = (v_1/v_2)^2 = 2^2 = 4$ .

### 3.4.1 Activity: What Determines How Big an Impact Crater Is?

Before we test the crater diameter data recorded in Tables 3.3 and 3.4 against our suite of models, let's examine the relationship between the height from which we dropped our projectiles and their resulting speed at impact on the surface. The impact velocity of a projectile of a certain mass is a simple function of the height  $h$  from which it is dropped and the strength of the surface gravity  $g$ . The velocity  $v$ , derived from Isaac Newton's laws of motion, is

$$v = \sqrt{2gh}.$$

If you completed the last two lines of Table 3.4 by dropping your projectile from different heights than 120" and 160", make sure to update the height and velocity values listed in the table to reflect your changes. For a height  $h$  measured in units of centimeters, use a value of 980 centimeters per squared second for  $g$ , to calculate a velocity  $v$  in units of centimeters per second. (Check your calculations by first recalculating the velocities for the 120" and 160" cases and verifying that you reproduce the values listed in the table.)

1. Plot the crater diameters, including associated errors, against velocity for the data you entered into Tables 3.3 and 3.4, using the second plotting tool listed for this lab exercise from the GEAS project lab exercise web page (see the URL on page 75 in §3.1.2). This application will plot crater diameter as a function of impact velocity, and then attempt to fit each of the three models for the behavior of  $D$  versus  $v$ .

Place  $v$  on the x-axis (the horizontal axis, running from left to right) and  $D$  on the y-axis (the vertical axis, running up and down), so when you enter your data under “Numbers to plot” on the form, place  $v$  first on each line, followed by  $D$  and then  $\sigma$ , the error (or spread) measured in each  $D$  value. Give your plot a title – something like “Crater Experiment Data” is fine. Label the x-axis “Impact Velocity (cm per second)”, and label the y-axis “Crater Diameter (cm).” Trace the distribution of the points on the plot by eye, and select the model which best fits the data. Examine the rms deviations ( $\sigma_1$ ) through ( $\sigma_3$ ), as the best fit should have the lowest  $\sigma$  value. Be sure to save the plot as well, to include in your lab report. (4 points)

The best-fitting model is \_\_\_\_\_. (1 point)

This means that the relation between  $D$  and  $v$  is \_\_\_\_\_. (1 point)

Is this the result you expected? Why, or why not? (2 points)

2. Use the Earth Impact Effects program found on the GEAS project lab exercise web page (see the URL on page 75 in §3.1.2) to answer the following questions.

(a) Consider two hypothetical impact events occurring in the uninhabited area between Deming and Columbus, in a sedimentary rock region some 50 miles from Las Cruces, NM.

Impact A: A 100-meter icy comet traveling at  $50 \text{ km s}^{-1}$  strikes at a  $45^\circ$  angle.

Impact B: A 40-meter iron-dominated rock traveling  $20 \text{ km s}^{-1}$  strikes at a  $45^\circ$  angle.

Answer each of the six question below by circling A or B. (1/2 point each, 3 points total)

Which impact would:

- i) be caused by the object carrying the most kinetic energy? ( A / B )
  - ii) produce the largest crater? ( A / B )
  - iii) be the most unusual (the rarest)? ( A / B )
  - iv) sound the loudest in Las Cruces? ( A / B )
  - v) produce the largest earthquake in Las Cruces? ( A / B )
  - vi) disturb the air most in Las Cruces (consider winds, and changes in pressure)? ( A / B )
- (b) Click on “impact examples,” (just below “Enter Impact Parameters”) and compare the size of the projectile that created Meteor Crater to the one that ended the reign of the dinosaurs and created Chicxulub Crater. (1 point)

(c) Chicxulub Crater has a diameter of 113 miles. What size would it be under the following three circumstances?

- i) The projectile landed on sedimentary rock rather than in 100-meter deep water.

Diameter = \_\_\_\_\_ miles. (1 point)

- ii) The projectile was made of iron rather than rock.

Diameter = \_\_\_\_\_ miles. (1 point)

- iii) The projectile landed on sedimentary rock *and* was made of iron.

Diameter = \_\_\_\_\_ miles. (1 point)

### 3.5 Final (Post-Lab) Questions

1. A careful examination of the lunar surface reveals that most lunar craters (a) come in widely assorted shapes, (b) are very oval shaped (elongated), or (c) are circular. (Circle one.) (1 point)



2. Is it easier to obtain relative or absolute ages for lunar surface features? Why? (Which can be estimated from images, and which require an analysis of actual rock samples?) (2 points)

3. Suppose current lunar cratering rates were found to be much higher than those averaged over the last 100 million years. Would this be a cause for concern? Why, or why not? (2 points)

4. How did the craters that you created differ from lunar craters? Were your initial guesses about which factors would determine the sizes and appearances of your craters confirmed, or denied? (3 points)

5. Nearly all lunar craters are circular because the projectiles that create them (a) travel almost straight down through the atmosphere, or (b) have high impact velocities, producing explosions on impact. (Circle one.) (1 point)

6. For a projectile of a given mass, what factors besides its impact velocity determine the resulting crater diameter? (3 points)

7. Suppose that the huge meteoroid that created the Chicxulub crater was ten million times more massive than the much smaller object that forged the Meteor Crater in Arizona, but they were both traveling at  $20 \text{ km s}^{-1}$  on impact.

(a) Compute the kinetic energy, in units of megatons of TNT, associated with the Chicxulub progenitor object just before the moment of impact, by using the information derived in Example 3.3 (on page 88). Kinetic energy is proportional to the projectile mass times the square of its velocity, so if this projectile traveled at the same speed as the Meteor Crater projectile and was ten million times more massive, then its kinetic energy is also ten million times larger. (2 points)

(b) Compare this amount of energy with the most powerful man-made explosive device ever detonated, a hydrogen bomb yielding 50 megatons of TNT.

The Chicxulub impact event was \_\_\_\_\_ times more powerful. (1 point)

### 3.6 Summary

After reviewing this lab's goals (see §3.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. Describe the set-up and implementation of your cratering experiment in detail, your analysis of the resultant data, and your overall scientific conclusions. (35 points)

Be sure to cover the following points.

- Describe the appearance of the lunar surface, identify the primary drivers for its surface

evolution, and explain the scientific basis for our conclusions regarding the relative and absolute ages of features.

- What primary factors determine the size and appearance of impact craters?
- How can studying the cratering process on the Moon increase our understanding of the Earth's impact history, and help us to estimate the likelihood of future catastrophic events?
- Describe the set-up and execution of your cratering experiment. What types of materials did you use to form the base and surface layers? What type of projectile did you use, and of what size? What were the most challenging aspects of the experimental procedure? How might you change your procedure if you repeated the experiment now, knowing what you now know?
- Include the two plots of crater age versus feature density that you made as part of the activity in §3.2.1, and the plot of crater diameter versus projectile impact velocity for the data you collected in §3.3.1.
- Discuss which of the three proposed models for the behavior of crater diameter  $D$  versus impact velocity  $v$  was best supported by your experimental data.

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

### 3.7 Extra Credit

An 350-meter asteroid named Apophis (after a serpent from Egyptian myth) has an orbit that crosses that of the Earth, and so it might collide with us at some time in the future. Orbital crossings will occur in 2029, 2036, and 2068, but the possibility of an actual impact event is computed to be a few percent at most.

If Apophis were to hit the Earth, how large a crater would be created? You may assume that it travels at a speed of  $20 \text{ km s}^{-1}$ , and lands in sedimentary rock at an angle of  $45^\circ$ . We think that it is composed of nickel and iron, with an overall density of  $8000 \text{ kg m}^{-3}$ . What would you expect the environmental consequences of such an event to be? (5 points)

## Lab 5

# Parallax Measurements and Determining Distances

### 5.1 Introduction

How do we know how far away stars and galaxies are from us? Determining the distances to these distant objects is one of the most difficult tasks that astronomers face. Since we cannot simply pull out a very long ruler to make a few measurements, we have to use other methods.

Inside the solar system, astronomers can bounce a radar signal off of a planet, asteroid or comet to directly measure its distance. How does this work? A radar signal is an electromagnetic wave (a beam of light), so it always travels at the same speed, the speed of light. Since we know how fast the signal travels, we just measure how long it takes to go out and to return to determine the object's distance.

Some stars, however, are located hundreds, thousands or even tens of thousands of “light-years” away from Earth. A light-year is the distance that light travels in a single year (about 9.5 trillion kilometers). To bounce a radar signal off of a star that is 100 light-years away, we would have to wait 200 years to get a signal back (remember the signal has to go out, bounce off the target, and come back). Obviously, radar is not a feasible method for determining how far away the stars are.

In fact, there is one, and only one, direct method to measure the distance to a star: the “parallax” method. Parallax is the angle by which something *appears* to move across the sky when an observer looking at that object changes position. By observing the size of this angle and knowing how far the observer has moved, one can determine the distance to the object. Today you will experiment with parallax, and develop an appreciation for the small

angles that astronomers must measure to determine the distances to stars.

To get the basic idea, perform the following simple experiment. Hold your thumb out in front of you at arm's length and look at it with your right eye closed and your left eye open. Now close your left eye and open your right one. See how your thumb appeared to move to the left? Keep staring at your thumb, and change eyes several times. You should see your thumb appear to move back and forth, relative to the background. Of course, your thumb is not moving. Your *vantage point* is moving, and so your thumb *appears* to move. That's the parallax effect!

How does this work for stars? Instead of switching from eye to eye, we shift the position of our entire planet! We observe a star once, and then wait six months to observe it again. In six months, the Earth will have revolved half-way around the Sun. This shift of two A.U. (twice the distance between the Earth and the Sun) is equivalent to the distance between your two eyes. Just as your thumb will appear to shift position relative to background objects when viewed from one eye and then the other, over six months a nearby star will appear to shift position in the sky relative to very distant stars.

### 5.1.1 Goals

The primary goals of this laboratory exercise are to understand the theory and practice of using parallax to find the distances to nearby stars, and to use it to measure the distance to objects for yourself.

### 5.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed below.

<http://astronomy.nmsu.edu/geas/labs/labs.html>

You will need the following items to perform your parallax experiment:

- a protractor (provided on page 123)
- a 30-foot long tape measure, or a shorter tape measure (or a yardstick) and at least 15 feet of non-stretchy string (not yarn)
- a thin object and a tall object with vertical sides, such as a drinking straw and a full soft drink can, or a chopstick and a soup can
- a piece of cardboard, 30 inches by 6 inches
- a pair of scissors, and a roll of tape
- a needle, and 18 inches of brightly colored thread
- 2 paper clips
- 3 coins (use quarters, or even heavier objects, if windy)

- a pencil or pen
- a calculator (or calculator app)
- 2 large cardboard boxes, chairs, or stools (to create 2 flat surfaces a couple of feet off the ground; recommended but not required)
- a clamp, to secure your measuring device to a flat surface (helpful, but not required)
- a friendly assistant (helpful, but not required)

Your assistant could be an adult or an older child, and needs no special knowledge of astronomy. S/he can assist you in measuring the distances between objects with a tape measure, shifting the position of an out-of-reach object on your request, and holding your measuring apparatus still while you use it.

You will also need a computer with an internet connection, to analyze the data you collect from your parallax experiment.

### 5.1.3 Primary Tasks

This lab is built on two activities: 1) a parallax measurement experiment, to be performed in a safe, dry, well-lit space with a view of the horizon (or at least out to a distance of 200 feet), and 2) an application of the parallax technique to stars, which can be recorded either on paper or directly on a computer. Students will complete these two activities, answer a set of final (Post-Lab) questions, write a summary of the laboratory exercise, and create a complete lab exercise report via the online Google Documents system (see <http://docs.google.com>).

The activities within this lab are a combination of field-work and computer-based ones, so you may either read most of this exercise on a computer screen, typing your answers to questions directly within the lab report template at Google Documents, or you may print out the lab exercise, make notes on the paper, and then transfer them into the template when you are done. We strongly recommend that you print out at least pages 98–114 for use in conducting the parallax experiment in §5.2.

### 5.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that Section 5.6 (§5.6) contains 5 extra credit points.

### 5.1.5 Timeline

Week 1: Read §5.1–§5.3, complete activities in §5.2 and §5.3, and begin final (Post-Lab) questions in §5.4. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results

Table 5.1: Breakdown of Points

Activity	Parallax Experiment	Stellar Parallax	Questions	Summary
Section	§5.2	§5.3	§5.4	§5.5
Page	100	114	119	121
Points	46	14	15	25

into your lab report template, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Finish final (Post-Lab) questions in §5.4, write lab summary, and submit completed lab report.

## 5.2 The Parallax Experiment

In this experiment, you will develop a better understanding of parallax by measuring the apparent shift in position of a nearby object relative to the background as you view it from two locations. You will explore the effects of changing the “object distance,” the distance between an object and an “observer” (you), and of changing the “vantage point separation” (the distance between your two viewing locations).

### 5.2.1 Setting up your experiment

Our first step is to create a measuring device for observing angles between various object and landmarks, as shown in Figure 5.5. The more carefully you build your device the more accurate your results will be, so take your time and work carefully. You can put the device together ahead of time, and then conduct your experiment later if this is more convenient than completing the entire experiment all at once.

Find the first protractor provided in your lab manual at the end of this chapter (on page 123) and carefully cut it out. Make sure to follow the dotted lines, so that your edges are straight and form right angles ( $90^\circ$  corners). Trim down a piece of cardboard (such as the side of a large box) to form a rectangle 30 inches wide and 6 inches tall. Cut the edges as perfectly straight as you can make them, again forming right angles.

Place the cardboard frame in front of you on a table, extending 15 inches on either side of you. Now place the protractor on the right end of the cardboard. Tape it securely to the cardboard, making sure that the dotted line forming the edge of the protractor paper is parallel to the long edge of the cardboard. Take your tape measure or yardstick, and measure a distance of two feet (24 inches) to the left of the origin of the protractor. (The origin is the dot in the center of the small circle, at the point where the dashed lines pointing to the numbers 10, 20, and so on up to 170, all meet.) Make sure to align your tape measure precisely with the line on the protractor extending from 180 to 0, and don’t let it tilt. Mark



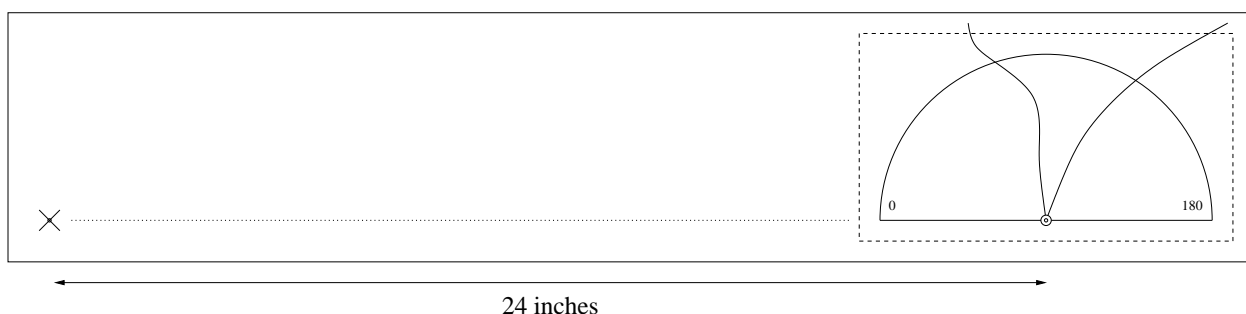


Figure 5.1: The measuring device has a base formed from a piece of cardboard, 30 inches wide and 6 inches high. The protractor is affixed to the right side, and an “X” is marked exactly 24 inches to the left of its origin. This two-foot distance represents the two astronomical units between the Earth’s position around the Sun during January and July. Straightened paper clips are attached perpendicular to the surface at the protractor origin and the “X” mark, to mark sight-lines. A piece of thread is doubled and secured at the protractor origin, to create two threads that can be rotated around the protractor to mark various angles between 0 and 180 degrees.

the point 24 inches to the left with a small “X”.

Next take a threaded needle, and thrust it through the origin of the protractor. Pull just enough thread through the hole that you can cut the needle free, and tape the two ends of thread securely to the back of the cardboard. You should have two pieces of thread coming up through the origin, each long enough to reach past the arc of numbers and trail off of the edge of the protractor paper.

Now take your two paper clips. Unfold one bend in each one, making the resultant long stick as straight as possible. This is very important, so spend a bit of time getting the bend out and straightening any kinks. The remaining portion will have two bends; force the first into a perfect right angle and open up the last one as well. Your paper clip should now have the form of a small flagpole, with a broad “V” shaped base. If you hold the base on a table, the flagpole should point upward. Tape over the tips of the flagpoles with small pieces of tape, so that they do not scratch anything or anyone.

Place the first paper clip on top of the origin of the protractor, and tape it into place. Don’t secure it until you have checked that the flagpole is centered directly over the center of the origin when viewed from all angles around it (you will probably have to touch up your paper clip angles a bit as you do this). This is another critical step, so take your time and do it well.

Place the second paper clip on top of the “X” mark 24 inches to the left of the protractor origin (at the left end of the cardboard). Again take your time and make sure that it is located directly above the mark, and points directly upward.

Our next step is to construct a support for the thin object which you will observe. If you

are using a drinking straw and a soft drink can, tape the straw securely to the side of the can, pointing straight up, so that the top of the straw lies at least 6 inches above the table. If you are using a thin chopstick and a soup can, tape the chopstick securely to the side of the can, pointing straight up, so that the top of the chopstick lies at least 6 inches above the table.

You will be lining up the position of the top of your thin object (the tip of the straw, or the thinner end of the chopstick) with your paper clip markers by eye, so it needs to be held straight, without drooping or falling.

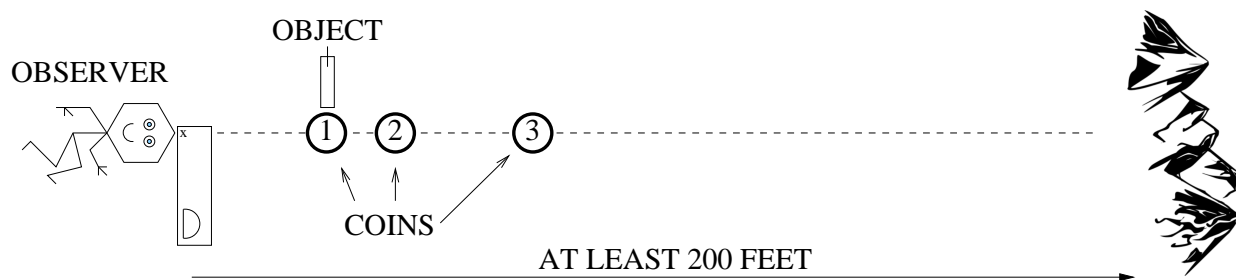


Figure 5.2: Basic layout of the parallax experiment. Lay the coins in a fairly straight line, between 4 and 30 feet away from the measuring device. They are marking the position at which you will place objects for distance estimation, so if you are using existing objects such as street sign or lamp posts, chimneys, or trees, you will not need them.

### 5.2.2 Taking parallax measurements

Complete the following, answering the 10 questions and completing Tables 5.2–5.5. Filling in the three tables correctly is worth 22 points, and the associated questions are worth 20 additional points (2 points per question).

You will need to find a clear, flat fairly-open space (ideally 30 feet long), with a view to the horizon (or to at least 200 feet away), as shown in Figure 5.2. An empty parking lot, a large room with an expansive view of the horizon (look for a ten-foot wide window on the first few floors of a library or a community center, with a view of mountains or a radio tower), or a backyard with a low fence might all be possible places to work. Be sure that your location is safe, well-lit, and not too windy.

You will need a stable support for your measuring device, such as a wall, a table, a stool, or cardboard box. The support should raise it high enough off the ground that you can comfortably sight along both paper clip posts toward features on the horizon, and should keep it level. It is important that your measuring device does not shift position as you work, even slightly. If you have a clamp, use it to secure your measuring device in place; otherwise, tape may help. You are going to use the device to estimate the distance to several objects between you and the horizon (or a landmark at least 200 feet in front of you).

Once you have your measuring apparatus in place, select an initial location at which to place

an object, some five feet or so in front of it. If there are existing objects with clearly-defined vertical lines in front of you, such as a post supporting a street lamp, a chimney, or a radio antenna on a car, feel free to use them. Otherwise, place your foreground object (the can with an attached straw or chopstick) on top of a stool or cardboard box, and place your first coin on the ground next to it at the same distance from the measuring device as the object. Be sure to match the position of the coin with the position of the straw or chopstick, not with the supporting stool or box.

Before committing to a particular position, check that you can sight along both paper clip posts (first one, and then the other) and the object, and have a clear view to the horizon or to a distant landmark. If you need to shift your object (or your measuring device) slightly to the left or right, that is fine. (If you have an assistant, you can sight along the measuring device and direct them to shift the object for you, to save time.)

You are now going to estimate the distance to the object, directly and via the parallax method. First, measure directly by running the tape measure from the object to the center of the line between your two paper clip posts on your measuring device. (Your assistant can help by holding one end of the tape measure for you.) This is the equivalent of measuring the distance between a nearby star and the Sun, if we had a really long, long tape measure. Record this distance, in units of inches, in Table 5.2 under “Position 1.” (If you prefer to use the metric system, then give yourself a pat on the back and replace inches with centimeters throughout this exercise. Be sure to indicate the change of units clearly in Tables 5.2, 5.3, and 5.5, however.) We will use the parallax method to compute this distance independently without leaving our measuring device, and then we will compare our two distance measurements.

Select two more existing objects anywhere between 7 and 30 feet in front of your device, or place two more coins to mark the position at which you will place your foreground object in turn. Record the distance between each object and the center of the measuring device, and record the values in the remaining two columns of Table 5.2. If your tape measure is not long enough to extend all the way from the measuring apparatus to the third object position, use a long piece of string to measure the distance piece by piece.

1. Estimate the distance to the horizon, or to a distant landmark in line with your measuring device and your object at Position 1, by eye. (This can be a very rough estimate.) \_\_\_\_\_ . (2 points)

Table 5.2: Direct Measurements of Distance<sup>a</sup>

<i>Measure distances in inches, to the nearest half-inch.</i>		
Position 1	Position 2	Position 3

(4 points)

<sup>a</sup>Distance from observer and apparatus to three foreground objects.

While it is easy to directly measure these distances for sign posts 20 feet away from us, it is impossible to measure them directly for stars, as they lie many light-years away from Earth. We will use these measurements as “controls,” and compare our parallax-derived distances to them at the end of the experiment.

You will now measure two angles for each of the three object positions, repeating each set of measurements three times and recording the data in Table 5.3. By conducting three trials you will be able to see how much your measurements vary from trial to trial, and get a sense of the measurement errors.

Begin by sitting down behind the left end of your measuring device. Sight down the line formed by the paper clip post and the object at Position 1. Identify a distant landmark which lines up with these two objects, such as a radio tower, the edge of a building, or a feature on a mountain range. Hold the image of that feature in your head!

Now shift over to the right end of our measuring device. Ask your assistant to press down firmly on the middle of the cardboard measuring device, so that you do not accidentally jiggle it. Sight down this second paper clip post, and find the same distant landmark. Because you will have shifted your position by a mere two feet, it will appear at the same position on your horizon. Gently pick up one of the threads attached to the protractor, and line it up with the paper clip post and the distant landmark, taking care not to disturb the measurement device. When you have it in place on top of the protractor, tape it in place gently (or have your assistant do so for you). Use a piece of tape with most of the glue removed (stick it to a spare piece of paper or to your clothes a few times to strip the glue), so that you can place and remove the thread repeatedly without tearing the paper protractor.

Now find the foreground object, and sight down the paper clip post and the object. While the object and the distant landmark lined up together when viewed from the left, by shifting your vantage point you will observe that the foreground object appears to shift to the left across your field of view. Pick up the second thread, and use it to mark the angle on the protractor for the apparent position of the foreground object.

The two pieces of thread indicate the angles at which the landmark and the foreground object appeared to you on the protractor. Record the angle of the landmark as “Angle 1” in the first row of Table 5.3, and the angle of the object as “Angle 2” in the next column. (You will fill in the final four columns later.) There are small lines on the protractor for every degree between 0 and 180; record the angle at which the threads lie to the nearest tenth of a degree. The difference between these angles tells you the angular shift of the foreground object, or the amount by which it appeared to move across the sky, when you shifted your vantage point by two feet. (From the left vantage point, we selected our distant landmark so that Angle 1 and Angle 2 were identical, so we don’t need to record them and calculate their difference.)

Now move on to Positions 2 and 3. Either shift your foreground object to the position of the second (and then the third) coin, or shift your attention to a slightly more distant existing object in your field of view. Again, line up the left paper clip post, the foreground object,

and a distant landmark in a single line. You can use the same landmark if your additional objects also line up with it, or change to another landmark. Then observe through your right vantage point, and record the angular position of the landmark and the foreground object, in the first two columns of the second and third rows of Table 5.3.

Table 5.3: Parallax Measurements

<i>Record angles to the nearest tenth-degree, distances to the nearest half-inch.</i>						
	Angle 1 ( $^{\circ}$ ) <sup>a</sup>	Angle 2 ( $^{\circ}$ ) <sup>b</sup>	$2\alpha$ ( $^{\circ}$ ) <sup>c</sup>	$\alpha$ ( $^{\circ}$ )	$r/d$ <sup>d</sup>	$d$ (in) <sup>e</sup>
<i>Trial 1</i>						
Posn. 1						
Posn. 2						
Posn. 3						
<i>Trial 2</i>						
Posn. 1						
Posn. 2						
Posn. 3						
<i>Trial 3</i>						
Posn. 1						
Posn. 2						
Posn. 3						

(16 points)

<sup>a</sup>Landmark angle; <sup>b</sup>foreground object angle; <sup>c</sup> $2\alpha = \text{Angle 1} - \text{Angle 2}$ ; <sup>d</sup> $r/d$  values from Table 5.4 (page 112); <sup>e</sup> $d = \frac{r}{r/d}$ , where  $2r$  is the vantage point separation (24 inches).

At this point you should have completed the first two columns of the first three rows of Table 5.3. We will now repeat these measurements two more times, in order to measure how

repeatable our results are. The variation in measured angles will give us an estimate of the measurement errors of our technique. As you are measuring angles to the nearest tenth of a degree, there will be variations from trial to trial, and for the most distant objects, the variation could easily be more than a degree. Try not to look at the results of the first trial while conducting the second and third trials, so that you do not bias your data. You want your measurements to be independent of each other. Conduct the second and third trials in turn, and fill in the rest of the first two columns in the table.

**DO NOT perform all three trials of any object position at the same time – that would defeat the purpose of taking three independent measurements.**

If your assistant is curious, you may switch places and let him/her take a set observations. Make sure that you explain the parallax technique fully first, though, so that your assistant understands the purpose of his or her actions and is appropriately careful. Once Table 5.2 and the first two columns of Table 5.3 are complete, your assistant has done his/her part, and is free to go!

2. Estimate the *uncertainty* in your measurement of the object's apparent shift. For example, do you think your recorded measurements could be off by ten degrees? One degree? One tenth of a degree? Compare the measurements made at each position from trial to trial, to help you estimate the reliability of your measurements. (2 points)

### 5.2.3 Dependence of parallax on vantage point separation

Now that we understand how the apparent shift of an object changes as its distance from the observer changes, let's explore what happens when the distance between the vantage points changes.

3. What would happen if the vantage points were farther apart? We separated our vantage points by two feet, to simulate the two astronomical units by which the Earth shifts position over a six-month period. What if we had used a separation of ten feet instead? How would you expect the angular shift of the object (the difference between Angle 1 and Angle 2) to change? (Note that there is no wrong answer to this question. The point is to take a guess, and then to verify or to disprove it.) (2 points)

4. Repeat the experiment with the object at Position 3, but this time estimate the apparent shift from positions separated by four feet (shift the whole measuring device an additional two feet to the right after selecting your landmark on the left). If you are working inside and have a limited view of the horizon, however, don't shift the right vantage point so far that your foreground object moves out of the window and is lost from sight. By how many degrees did the object move using the more widely separated vantage points? (2 points)
5. For an object at a fixed distance, how does the apparent shift change as you observe from more widely separated vantage points? (2 points)

At this point, you should have completed your measurements with the parallax apparatus. You can pack it up, and complete this exercise at a location of your choice.

### 5.2.4 Measuring distances using parallax

We have seen that an object's apparent shift relative to background objects depends both on the distance between the object and the observer and on the separation between the observer's two vantage points. We can now turn this around: if we can measure the apparent shift and the separation of the two vantage points, we should be able to calculate the distance to an object. This is very handy, as it provides a way of measuring distance without actually having to go all the way to an object. Since we cannot travel to the stars, this is an excellent way to measure their distances from us.

We will now see how parallax can be used to determine the distances to the objects in your experiment based only on your measurements of their apparent changes in angular position ("apparent shifts") and the measurement of the separation of your two vantage points (your "baseline").

#### Angular shift

6. The apparent shift of the object is caused by looking at the object from two different vantage points. Qualitatively, what do you see changing from viewpoint to viewpoint? As a foreground object moves farther away from you, does its apparent shift increase or decrease? (2 points)

To check your answer, consider the apparent angular shift for the two objects shown in Figure 5.3.

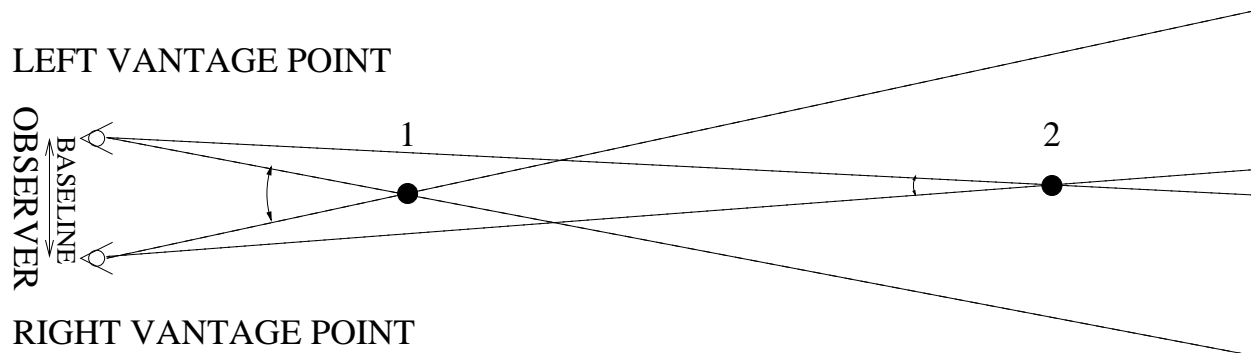


Figure 5.3: Layout of the parallax experiment, comparing the angular shift for foreground objects at two positions.

### Distance between the vantage points (the “baseline”)

We used a baseline of two feet, or 24 inches, in our experiment. Each foot represents the distance between the Earth and the Sun, as over the course of six months the Earth shifts its position around the Sun by two such astronomical units.

### Using parallax measurements to determine the distance to an object

To determine the distance to an object for which you have a parallax measurement, you can construct an imaginary triangle between the two different vantage points and the object, as shown in Figure 5.4.

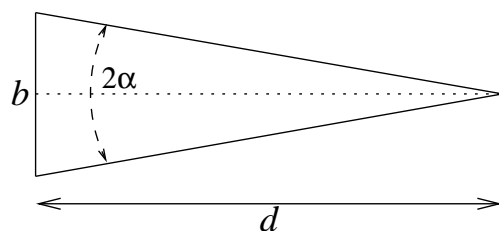
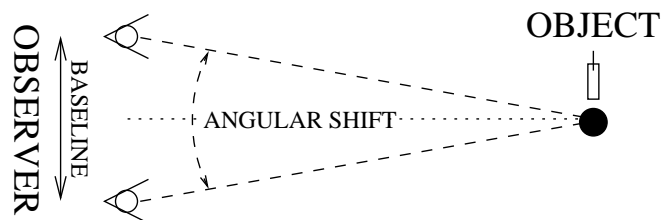
The angular shifts that you have just calculated correspond to the angle  $2\alpha$  on the diagram, and the distance between your vantage points (your “baseline”) corresponds to the distance  $b$ . The distance to the object, which you wish to determine, is  $d$ .

Let’s recast this slightly now, to better match the way in which we observe stars from Earth. The baseline is equivalent to twice the orbital radius of the Earth, so we’ll define “ $b$ ” to be “ $2r$ ” now, as shown in Figure 5.5. We can also separate  $2\alpha$  into two equal angles, each with a value of  $\alpha$ . This figure emphasizes that from the left vantage point, the foreground object and a landmark on the horizon line up with each other, but from the right, they are separated by an angle of  $2\alpha$ .

The  $2\alpha$  angle appears in three places on the diagram, and shows us that a right triangle (a triangle with a right angle) can be formed with height  $r$ , width  $d$ , and angle  $\alpha$ . This triangle



## LEFT VANTAGE POINT



## RIGHT VANTAGE POINT

Figure 5.4: The parallax experiment, with real objects (left) and expressed mathematically (right). Can you find the same triangle on both sides?

holds the key to determining “d.”

Can you see that the difference between Angle 1 and Angle 2, the two angles that you measured in your experiment, is equal to  $2\alpha$ ? Take a good look at Figure 5.5 to ensure that this makes sense to you. Once it does, fill in the third column of Table 5.3 with a value for  $2\alpha$  (Angle 1 – Angle 2). Having done so, divide your values by a factor of two and fill in values for  $\alpha$  in the fourth column.

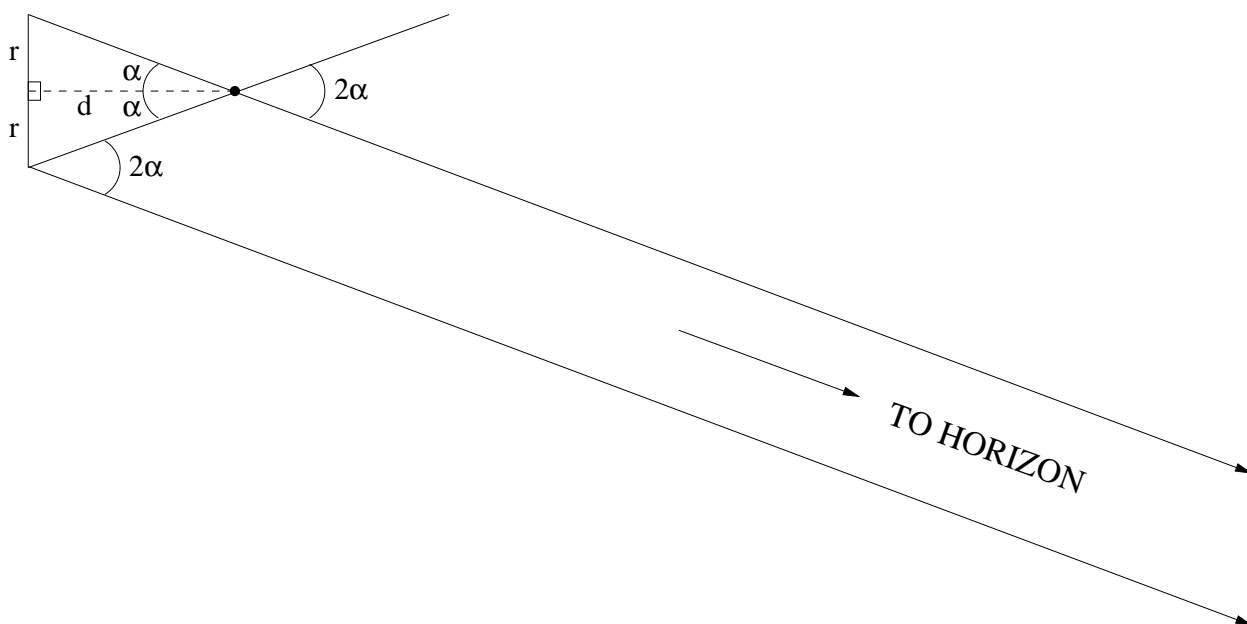


Figure 5.5: Our two vantage points are separated by a distance of  $2r$ , equivalent to twice the Earth’s orbital radius. A landmark on the horizon will appear at the same angle from both vantage points, because it is so far away, but a nearby object a distance  $d$  away will appear to shift position, by an angle of  $2\alpha$ .

People have been studying right triangles for thousands of years, and so we know a fair bit about how they work. One of their most useful properties is the fact that if you know

the ratio of the height to the width of a right triangle, you can determine the values for the interior angles – you know exactly how large  $\alpha$  is. Figure 5.6 illustrates this fact. It shows three different right triangles. Though each triangle has a different height and a different width, because the ratio of the height to the width is the same in each case, the value of  $\alpha$  is also the same for all three triangles.

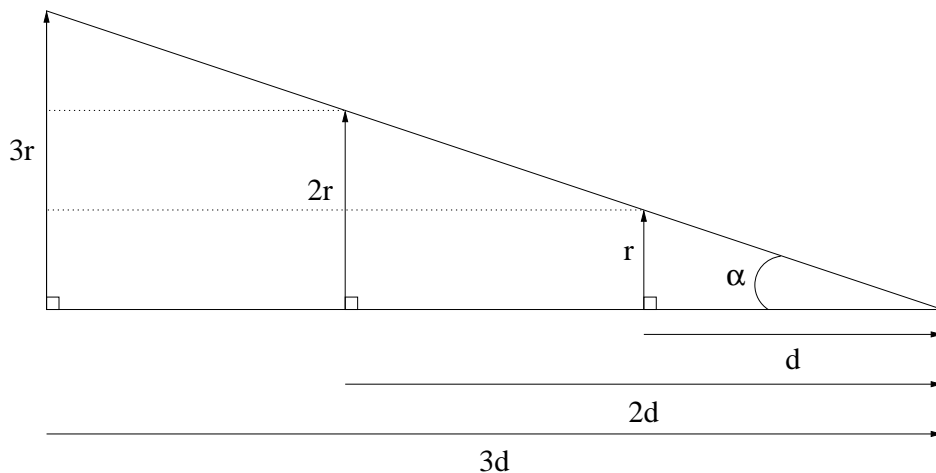


Figure 5.6: The angle  $\alpha$  in a right triangle is determined uniquely by the ratio of its height,  $r$ , to its width  $d$ . We see here that if we double the height and width, or triple them both,  $\alpha$  remains unchanged.

If you have studied trigonometry, you may recognize that this ratio,  $r/d$ , is the tangent of  $\alpha$ . However, you do not need to be familiar with tangents to understand the clear connection between  $r/d$  and  $\alpha$ . We have established that there is a unique relationship between the value of  $r/d$  and the value of  $\alpha$  for a right triangle. Figure 5.7 emphasizes this fact, showing how  $\alpha$  changes value as we vary the value of  $d$  for right triangles of height  $r$ .

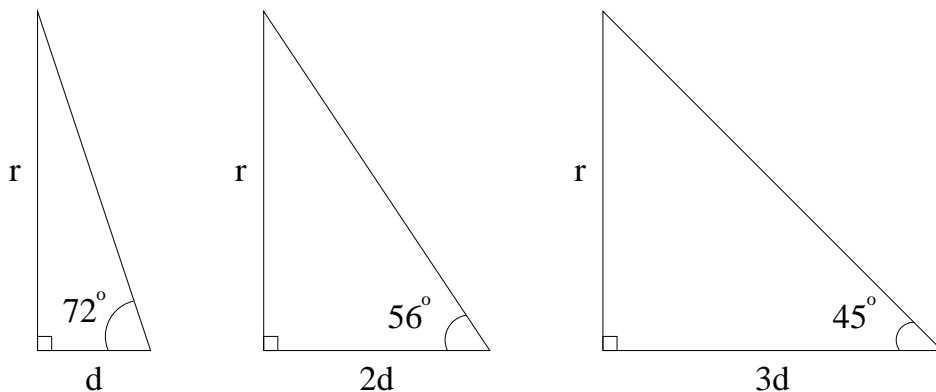


Figure 5.7: If we know the height  $r$  and the width  $d$  of a right triangle, we know the value of its interior angles. We see here that as we double and then triple the width of such a triangle, the labeled angle decreases in response.

In studying our foreground objects, what do we know? We know that we can define a useful right triangle that has a height  $r$  equal to one foot, a width  $d$  equal to the distance to the

object, and that we have determined values for the angle  $\alpha$  to match. From the preceding discussion, we can see that with  $r$  and  $\alpha$  known, we can determine  $d$ !

Table 5.4 provides the last piece of the puzzle. It tabulates values of  $r/d$  for values of  $\alpha$  ranging from zero to 20 degrees. Read this table by combining each set of two columns in order. The first, third, fifth, seventh, and ninth columns list values for  $\alpha$ . Take your first measurement of  $\alpha$ , listed in the fourth column of the first row of Table 5.3, and find it in Table 5.4, reading off the appropriate value of  $r/d$  from the column *to the right*. For example, if  $\alpha = 0.1^\circ$  then  $r/d = 0.0017$ , and if  $\alpha = 20.0^\circ$  then  $r/d = 0.3640$ . If you have a value for  $\alpha$  which lies between two values listed in the table, just average the  $r/d$  values for the nearest two  $\alpha$  values listed. For example, if your value for  $\alpha$  is  $19.95^\circ$  then  $r/d = (0.3620 + 0.3640)/2 = 0.3630$ .

If you are comfortable using a calculator to calculate tangent values you may of course do so instead. Just remember that the values for  $\alpha$  in Table 5.3 are listed in units of degrees, and make sure that your calculator is expecting to work with angles in degrees, not radians. As a check, calculate  $\tan(10)$ . You should get a value of 0.1763.

Look up the values of  $r/d$  now for the nine values of  $\alpha$  you added to Table 5.3, and place them in the fifth column of that table.

Our final step is very straight-forward. We know the values for  $r$  and for  $r/d$ , and we need to solve for  $d$ .

$$d = \frac{r}{r/d} \times d = r \times \frac{d}{r} = \frac{r}{r/d}.$$

Calculate values for  $d$  for your nine rows of Table 5.3, by dividing your value for  $r$ , in feet, by your dimensionless (with no units) values for  $r/d$ . Then multiply your answer by twelve to convert from feet to inches, and fill these values for  $d$  into the final column of Table 5.3.

7. Based on your estimate of the uncertainty in the angular measurements of  $2\alpha$ , estimate the uncertainty in your measurements of the object distances. (Note that there is no wrong answer to this question. The point is to take a guess, and then to verify or to disprove it.) (2 points)

8. Now look at the spread in the three values for each position in the last column of Table 5.3. Is this spread consistent with your estimated uncertainty? (2 points)

Table 5.4: Height-to-Width Ratios ( $r/d$ ) for Right Triangles

$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$	$\alpha$ ( $^\circ$ )	$r/d$
0.1	0.0017	4.1	0.0717	8.1	0.1423	12.1	0.2144	16.1	0.2886
0.2	0.0035	4.2	0.0734	8.2	0.1441	12.2	0.2162	16.2	0.2905
0.3	0.0052	4.3	0.0752	8.3	0.1459	12.3	0.2180	16.3	0.2924
0.4	0.0070	4.4	0.0769	8.4	0.1477	12.4	0.2199	16.4	0.2943
0.5	0.0087	4.5	0.0787	8.5	0.1495	12.5	0.2217	16.5	0.2962
0.6	0.0105	4.6	0.0805	8.6	0.1512	12.6	0.2235	16.6	0.2981
0.7	0.0122	4.7	0.0822	8.7	0.1530	12.7	0.2254	16.7	0.3000
0.8	0.0140	4.8	0.0840	8.8	0.1548	12.8	0.2272	16.8	0.3019
0.9	0.0157	4.9	0.0857	8.9	0.1566	12.9	0.2290	16.9	0.3038
1.0	0.0175	5.0	0.0875	9.0	0.1584	13.0	0.2309	17.0	0.3057
1.1	0.0192	5.1	0.0892	9.1	0.1602	13.1	0.2327	17.1	0.3076
1.2	0.0209	5.2	0.0910	9.2	0.1620	13.2	0.2345	17.2	0.3096
1.3	0.0227	5.3	0.0928	9.3	0.1638	13.3	0.2364	17.3	0.3115
1.4	0.0244	5.4	0.0945	9.4	0.1655	13.4	0.2382	17.4	0.3134
1.5	0.0262	5.5	0.0963	9.5	0.1673	13.5	0.2401	17.5	0.3153
1.6	0.0279	5.6	0.0981	9.6	0.1691	13.6	0.2419	17.6	0.3172
1.7	0.0297	5.7	0.0998	9.7	0.1709	13.7	0.2438	17.7	0.3191
1.8	0.0314	5.8	0.1016	9.8	0.1727	13.8	0.2456	17.8	0.3211
1.9	0.0332	5.9	0.1033	9.9	0.1745	13.9	0.2475	17.9	0.3230
2.0	0.0349	6.0	0.1051	10.0	0.1763	14.0	0.2493	18.0	0.3249
2.1	0.0367	6.1	0.1069	10.1	0.1781	14.1	0.2512	18.1	0.3269
2.2	0.0384	6.2	0.1086	10.2	0.1799	14.2	0.2530	18.2	0.3288
2.3	0.0402	6.3	0.1104	10.3	0.1817	14.3	0.2549	18.3	0.3307
2.4	0.0419	6.4	0.1122	10.4	0.1835	14.4	0.2568	18.4	0.3327
2.5	0.0437	6.5	0.1139	10.5	0.1853	14.5	0.2586	18.5	0.3346
2.6	0.0454	6.6	0.1157	10.6	0.1871	14.6	0.2605	18.6	0.3365
2.7	0.0472	6.7	0.1175	10.7	0.1890	14.7	0.2623	18.7	0.3385
2.8	0.0489	6.8	0.1192	10.8	0.1908	14.8	0.2642	18.8	0.3404
2.9	0.0507	6.9	0.1210	10.9	0.1926	14.9	0.2661	18.9	0.3424
3.0	0.0524	7.0	0.1228	11.0	0.1944	15.0	0.2679	19.0	0.3443
3.1	0.0542	7.1	0.1246	11.1	0.1962	15.1	0.2698	19.1	0.3463
3.2	0.0559	7.2	0.1263	11.2	0.1980	15.2	0.2717	19.2	0.3482
3.3	0.0577	7.3	0.1281	11.3	0.1998	15.3	0.2736	19.3	0.3502
3.4	0.0594	7.4	0.1299	11.4	0.2016	15.4	0.2754	19.4	0.3522
3.5	0.0612	7.5	0.1317	11.5	0.2035	15.5	0.2773	19.5	0.3541
3.6	0.0629	7.6	0.1334	11.6	0.2053	15.6	0.2792	19.6	0.3561
3.7	0.0647	7.7	0.1352	11.7	0.2071	15.7	0.2811	19.7	0.3581
3.8	0.0664	7.8	0.1370	11.8	0.2089	15.8	0.2830	19.8	0.3600
3.9	0.0682	7.9	0.1388	11.9	0.2107	15.9	0.2849	19.9	0.3620
4.0	0.0699	8.0	0.1405	12.0	0.2126	16.0	0.2867	20.0	0.3640

### How good are your parallax-derived distances?

At this point, you are ready to average your distance measurements together, and compute their standard deviations. Access the plotting tool listed for this lab exercise from the GEAS project lab exercise web page (see the URL on page 98 in §5.1.2). Use the plotting tool to create histograms of your distance measurements, entering the three values measured in Trials 1 through 3 for each quantity in turn. You will not need to save the histogram plots. Instead, simply record the averaged values shown for each plot (the “mean value”, or  $\mu$ ), and the associated errors ( $\sigma$ ). In order to see how good your distance measurements are, calculate these averages and errors for each of the distance estimates  $d$  recorded three times in the final column of Table 5.3.

Table 5.5: Comparison of Average Distances

	Direct Distance (from Table 5.2)	Parallax Distance (from Table 5.3)
<i>Measured and parallax-derived distances from observer to object, in inches. For parallax distances record the average values (<math>\mu</math>) of the three trials, and the errors (<math>\sigma</math>) calculated with the plotting tool in this form: <math>nn.n \pm n.n</math>.</i>		
Position 1		
Position 2		
Position 3		

(6 points)

9. Now compare the distances that you calculated for each position using the parallax method to the distances that you measured directly at the beginning of the experiment (in Table 5.2). How well did the parallax technique work? Are the differences between the direct measurements and your parallax-derived measurements within your errors (within  $2\sigma$ )? (2 points)

10. If the differences are larger than  $5\sigma$ , can you think of a reason why your measurements might have some additional error in them? We might call this a “systematic” error, if it is connected to a big approximation in our observational setup. (2 points)

## 5.3 Calculating Astronomical Distances With Parallax

Complete the following section, answering each of the four questions in turn. (Each question is worth either 3 or 5 points.)

### 5.3.1 Distances on Earth and within the Solar System

1. We have just demonstrated how parallax works on a small scale, so now let us move to a larger playing field. Use the information in Table 5.4 to determine the angular shift ( $2\alpha$ , in degrees) for Organ Summit, the highest peak in the Organ Mountains, if you observed it with a baseline  $2r$  of not 2 feet, but 300 feet, from NMSU. Organ Summit is located 12 miles from Las Cruces. (*If you are working from another location, select a mountain, sky scraper, or other landmark at a similar distance to use in place of the Organ Summit.*) There are 5,280 feet in a mile. (3 points)

You should have gotten a small angle!

The smallest angle that the best human eyes can resolve is about 0.02 degrees. Obviously, our eyes (with an internal baseline of only 3 or 4 inches) provide an inadequate baseline for measuring large distances. How could we create a bigger baseline? Surveyors use a “transit,” a small telescope mounted on a protractor, to carefully measure angles to distant objects. By positioning the transit at two different spots separated by exactly 300 feet (and carefully measuring this baseline), they will observe a much larger angular shift. Recall that when you increased the distance between your two vantage points, the angular shift increased. This means that if an observer has a larger baseline, he or she can measure the distances to objects which lie farther away. With a surveying transit’s 300-foot baseline, it is thus fairly easy to measure the distances to faraway trees, mountains, buildings or other large objects here on Earth.

2. What about an object farther out in the solar system? Consider our near neighbor, planet Mars. At its closest approach, Mars comes to within 0.4 A.U. of the Earth. (Remember that an A.U. is the average distance between the Earth and the Sun, or  $1.5 \times 10^8$  kilometers.) At such a large distance we will need an even larger baseline than a transit could provide, so let us assume we have two telescopes in neighboring states, and calculate the ratio  $r/d$  for Mars for a baseline of 2000 kilometers. Can you even find this value in Table 5.4? (3 points)

*You should get a value for  $r/d$  which lies well beyond the bounds of Table 5.4, less than  $5 \times 10^{-6}$  degrees. For the correct value, the equivalent value of  $2\alpha$  is 0.00019 degrees, or 7 arcseconds (where there are 60 arcminutes per degree, and 60 arcseconds per arcminute).*

### 5.3.2 Distances to stars, and the “parsec”

The angular shifts for even our closest neighboring planets are clearly quite small, even with a fairly large baseline. Stars, of course, are *much* farther away. The nearest star is  $1.9 \times 10^{13}$  miles, or  $1.2 \times 10^{18}$  inches, away! At such a tremendous distance, the apparent angular shift is extremely small. When observed through the two vantage points of your two eyes, the angular shift of the nearest star corresponds to the apparent diameter of a human hair seen at the distance of the Sun! This is a truly tiny angle and totally unmeasurable by eye.

Like geological surveyors, we can improve our situation by using two more widely separated vantage points. In order to separate our two observations as far as possible from each other, we will take advantage of the Earth’s motion around the Sun. The Earth’s orbit forms a large circle around the Sun, and so by observing a star from first one position and then waiting six months for the Earth to revolve around to the other side of the Sun, we will achieve a separation of two A.U. (twice the average distance between the Earth and the Sun). This is the distance between our two vantage points, labeled  $b$  in Figure 5.8.

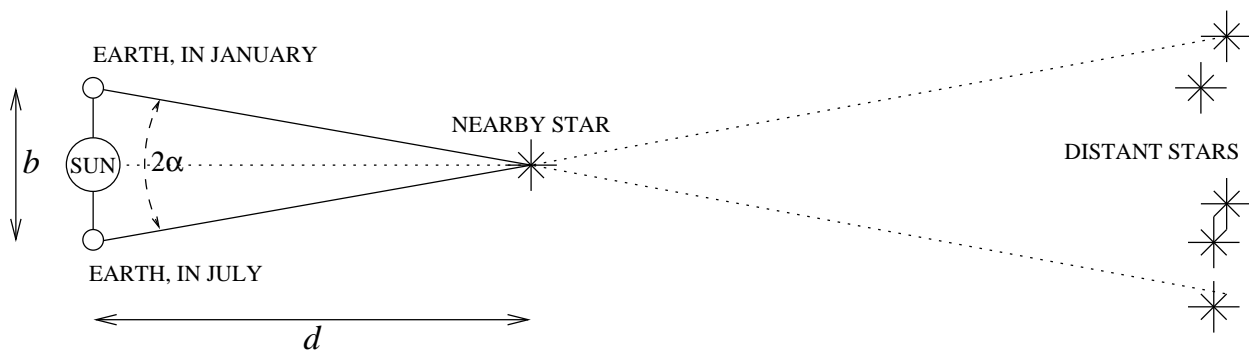


Figure 5.8: The parallax experiment, as done from Earth over a period of six months.

An A.U. (astronomical unit) is equal to  $1.5 \times 10^8$  kilometers, so  $b$  is equal to twice that, or 300 million kilometers. Even though this sounds like a large distance, we find that the apparent angular shift ( $2\alpha$  in Figure 5.5) of even the nearest star is only about 0.00043 degrees, or the width of a quarter coin observed from two miles away. This is unobservable by eye, which is why we cannot directly observe parallax by looking at stars with the naked eye. However, such angles are relatively easy to measure using modern telescopes and instruments. (Note that the ancient Greeks used the lack of observable parallax angles to argue that the Sun rotated around the Earth rather than the reverse, having greatly underestimated the distances to the nearest stars.)

Let us now discuss the idea of angles that are smaller than a degree. Just as a clock ticks out hours, minutes, and seconds, angles on the sky are measured in degrees, arcminutes, and arcseconds. A single degree can be broken into 60 arcminutes, and each arcminute contains 60 arcseconds. An angular shift of 0.02 degrees is thus equal to 1.2 arcminutes, or to 72 arcseconds. Since the angular shift of even the nearest star (Alpha Centauri) is only 0.00042 degrees (1.5 arcseconds), we can see that arcseconds will be a most convenient unit to use when describing them. Astronomers append a double quotation mark (") at the end of the



angle to denote arcseconds, writing  $\alpha = 0.75''$  for the nearest star. When astronomers talk about the “parallax” or “parallax angle” of a star, they mean  $\alpha$ .

### The small angle approximation

If we look closely at the values for  $\alpha$  and  $r/d$  in the first two columns of Table 5.4, we will notice an interesting, useful fact. For small values of  $\alpha$ ,  $\alpha = 57.3 \times r/d$ , and thus  $r/d = \frac{\alpha}{57.3}$ . Please don’t take our word for it – check for yourself.

3. Return to the plotting tool listed for this lab exercise from the GEAS project lab exercise web page (see the URL on page 98 in §5.1.2). Values for  $r/d$  and  $\alpha$  like those in Table 5.4 but extending up to  $80^\circ$  have been listed in a text file for you (called “Table #1,  $\alpha$  and r/d values”). Create a linear plot with  $r/d$  values on the  $x$ -axis and  $\alpha$  values on the  $y$ -axis, and then check the slope of a line fit through the values, for  $\alpha = 0$  to  $80^\circ$  (the complete table) and then after trimming the table to contain only the points for which  $\alpha$  is less than or equal to  $2^\circ$ . Include a copy of both plots in your lab report (making sure to label the axes, and to add titles). Is a straight line a good fit to either set of data (for  $\alpha \leq 80^\circ$ , or just up to  $2^\circ$ )? What is the slope, in the linear region? Are you now comfortable using this approximation to shift between  $\alpha$  and  $r/d$ ? (5 points)

This relationship is known as the *small angle approximation*. Why does it work? For small angles like those listed in the first column, the value of an angle, in radians, is equal to its tangent, or equal to  $r/d$ . It takes 360 degrees to form a full circle, or  $2\pi$  radians (a convenient alternate unit for angles). We can thus divide our values for  $\alpha$  in degrees by  $360/2\pi$ , or 57.3, to convert to radians, and then  $r/d$  is equal to  $\alpha$ . Our expression for  $d$  thus simplifies to

$$d = \frac{57.3 \times r}{\alpha}.$$

For stellar parallaxes,  $r$  is one astronomical unit, so if  $\alpha$  is measured in degrees,

$$d = \frac{57.3 \times 1 \text{ A.U.}}{\alpha},$$

and if  $\alpha$  is measured in arcseconds,

$$d = \frac{57.3 \times 1 \text{ A.U.}}{\alpha/3,600} = \frac{206,265}{\alpha} \text{ A.U.}$$

Astronomers have defined a new unit, the “parsec,” a unit of distance equal to 206,265 astronomical units. The parallax angle expression thus simplifies neatly to

$$d \text{ (pc)} = \frac{1}{\alpha \text{ ("})},$$

where  $d$  is measured in units of parsecs, and  $\alpha$  is measured in units of arcseconds.

The word parsec comes from the phrase “**par**allax **sec**ond.” By definition, an object at a distance of 1 parsec has a parallax of  $1''$ . How far away is a star with parallax angle of  $\alpha = 1''$ , in units of light-years? It is 3.26 light-years from our solar system. (To convert parsecs into light years, you simply multiply by 3.26 light-years per parsec.)

An object at 10 parsecs (32.6 light-years) has a parallax angle of  $0.1''$ , and an object at 100 parsecs has a parallax angle of  $0.01''$ . Remember that the farther away an object is from us, the smaller its parallax angle will be. The nearest star has a parallax of  $\alpha = 0.78''$ , and is thus at a distance of  $1/\alpha = 1/0.78 = 1.3$  parsecs.

You may use the words parsec, kiloparsec, megaparsec and even gigaparsec in astronomy. These names are just shorthand methods of talking about large distances. A kiloparsec is 1,000 parsecs, or 3,260 light-years. A megaparsec is one million parsecs, and a gigaparsec is a whopping one billion parsecs! The parsec may seem like a strange unit at first, but it is ideal for describing the distances between stars within our galaxy.

4. Let’s work through a couple of examples. (3 points)

(a) If a star has a parallax angle of  $\alpha = 0.25''$ , what is its distance in parsecs?

(b) If a star is 5 parsecs away from Earth, what is its parallax angle in arcseconds?

- (c) If a star lies 5 parsecs from Earth, how many light-years away is it?

## 5.4 Final (Post-Lab) Questions

1. How does the parallax angle of an object change as it moves away from us? As we can only measure angles to a certain accuracy, is it easier to measure the distance to a nearby star or to a more distant star? Why? (3 points)
2. Relate the experiment you did in the first part of this lab to the way that parallax is used to measure the distances to nearby stars. Describe the process an astronomer goes through to determine the distance to a star using the parallax method. What did your two vantage points represent in the experiment? (5 points)

3. Imagine that you observe a star field twice, with a six-month gap between your observations, and that you see the two sets of stars shown in Figure 5.9:

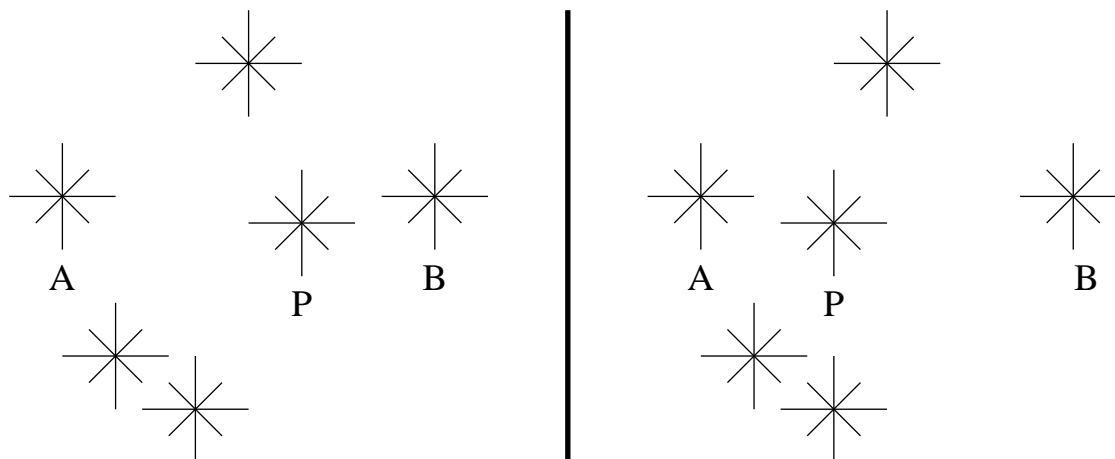


Figure 5.9: A star field, viewed from Earth in January (left) and again in July (right). Which star do you think lies closest to Earth?

The nearby star marked  $P$  appears to move between the two images, because of parallax. Consider the two images to be equivalent to the measurements that we made in our experiment, where each image represents the view of an object relative to a distant object as seen through one of your eyes. None of the stars but  $P$  change position; they correspond to the distant objects in our experiment.

If the angular distance between Stars  $A$  and  $B$  is 0.5 arcseconds then how far away would you estimate that Star  $P$  lies from Earth?

First, estimate how far Star  $P$  has moved between the two images relative to the constant distance between Stars  $A$  and  $B$ . This tells you the apparent angular shift of  $P$  ( $2\alpha$ ). You can then use the parallax equation ( $d = 1/\alpha$ ) to estimate the distance to Star  $P$ . (3 points)

4. Astronomers like Tycho Brahe made careful naked eye observations of stars in the late 1600s, hoping to find evidence of semi-annual parallax shifts for those which were nearby and so weigh in on the growing debate over whether or not the Earth was in motion around the Sun. If the nearest stars (located 1.3 or more parsecs from Earth) were 100 times closer to us, or if the resolving power of the human eye (0.02 degrees) was improved by a factor of 100, could he have observed such shifts? Explain your answer. (4 points)

## 5.5 Summary

Summarize the important concepts discussed in this lab. Include a brief description of the basic principles of parallax and how astronomers use parallax to determine the distances to nearby stars. (25 points)

Be sure to think about and answer the following questions:

- Does the parallax method work for all of the stars we can see in our Galaxy? Why, or why not?
- Why is it so important for astronomers to determine the distances to the stars which they study?

Use complete sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

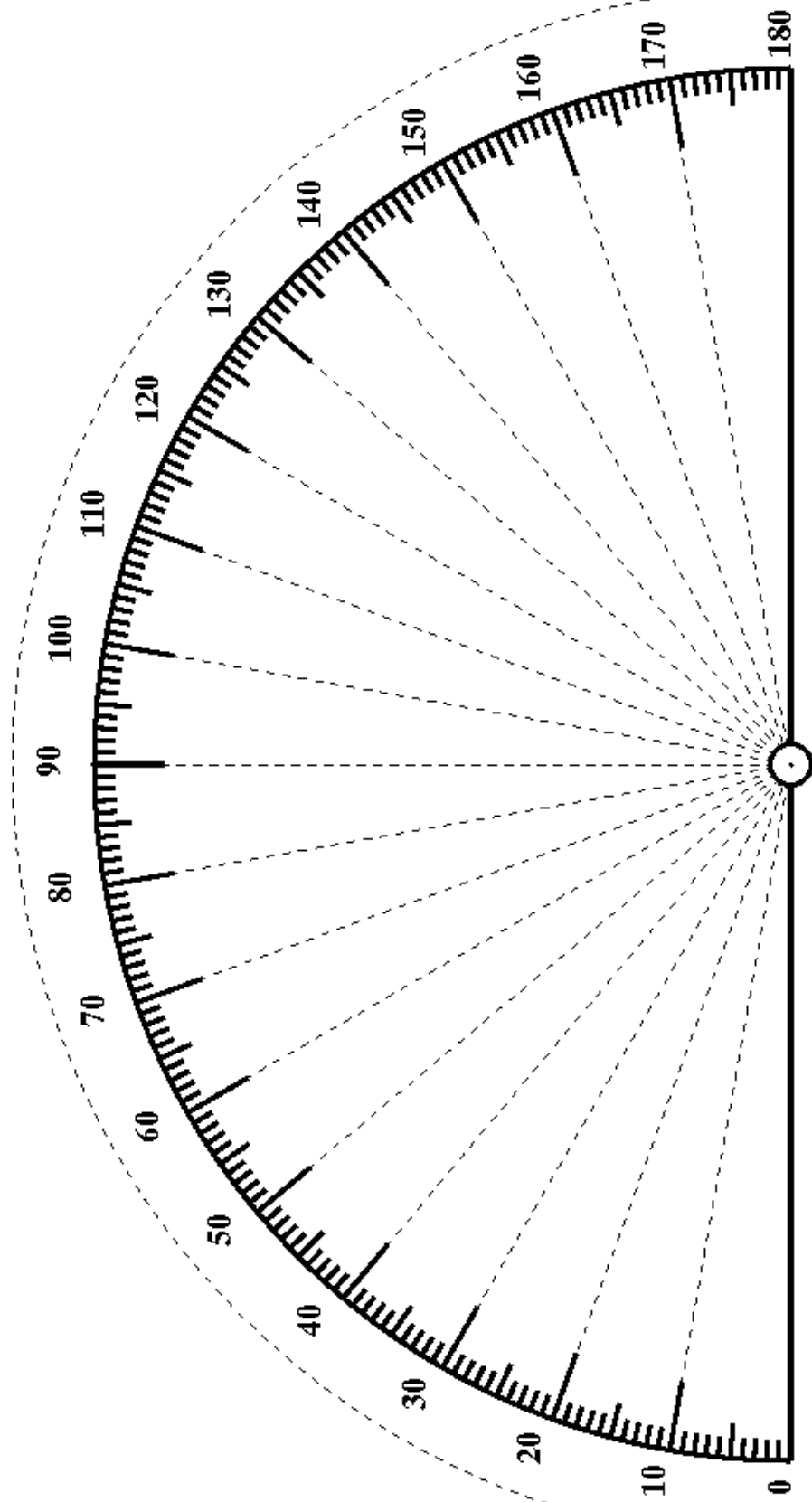
## 5.6 Extra Credit

Use the web to learn about the European Space Agency (ESA) Hipparcos satellite (launched in 1989) and the Gaia space observatory (launched in 2013). What are their mission goals, and how do their scientific instruments work? How accurately are they able to measure parallax angles? How do they compare to the best ground-based parallax measurement programs? Be sure that you understand the units of milliarcseconds (“mas”) and microarcseconds, and can use them in your discussion. (5 points)

Be sure to cite your references, whether they are texts or URLs.

# Protractor

Cut along outer dotted line.







## Lab 6

# The Hertzsprung-Russell Diagram and Stellar Evolution

### 6.1 Introduction

On a clear, dark night, one might see more than two thousand stars. Unlike our distant ancestors, we recognize that each one is a huge ball of hot gas that radiates energy, like our own (very nearby) star, the Sun. Like the Sun, the stars shine by converting hydrogen into helium via nuclear fusion reactions. Unlike the Sun, they are tremendously far away. While the Sun is located a mere 93 million miles from us and seems dazzlingly bright, the next nearest star is roughly 270,000 times farther away (25 trillion miles distant, or 4.3 light years, or 1.3 parsecs) and appears millions of times fainter to us on Earth.

The Sun can be thought of as a nuclear reactor in the sky that we all depend on, planning our days secure in the knowledge that it will continue to send us warmth and life-sustaining energy. But will it always do so? Just as a candle eventually uses up its oil and flickers out, someday the Sun will exhaust its fuel reserves. This is not surprising when you consider that, in converting mass to energy, the Sun turns 600 million tons of hydrogen into 596 million tons of helium with every passing second (losing four million tons in the process). Knowing this, you might justifiably fear for its imminent demise – but fortunately, the Sun has lots of mass!

We refer to the part of a star's lifetime when it's most dependable and stable – when it has plenty of hydrogen fuel to burn at its core – as the Main Sequence phase. It can be unstable and erratic soon after its formation from a collapsing cloud of gas and dust and later in life when it begins to run out of fuel and experience “energy crises,” but not during the Main Sequence phase. How long will the Sun exist in this stable form?

Before this question could be answered, astronomers had to learn to distinguish stable, well-behaved stars from their more erratic, very young or very old neighbors. They did this by plotting two basic observed quantities (brightness and color) against each other, forming a plot called a Hertzsprung-Russell (H-R) Diagram, after the two astronomers who pioneered its usage nearly a hundred years ago. Astronomers realized that the positions of stable, well-behaved stars in the prime part of their lives were restricted to a small part of this diagram, along a narrow band that came to be called the Main Sequence. They also discovered that as one scanned the Main Sequence from one end to the other, the fundamental stellar property that changed along it was stellar mass.

Mass is critically important because it determines how long stars can exist as stable, well-behaved objects. We have learned this by studying H-R diagrams, particularly for large groups of stars (like the Pleiades and M67 clusters featured in this lab exercise). The most massive, luminous stars will spend only a few million years in the Main Sequence phase, while their least massive, faintest cousins may amble along like this for hundreds of billions of years. What about the Sun? We know from geological evidence that the Sun has existed in its current state for 4.5 billion years, and astronomical evidence suggests that it has another five to seven billion years to go in this state. In this lab we will evaluate the observational evidence behind that conclusion, and address the question of whether the Sun is a typical star by comparing it to other stars.

### **6.1.1 Goals**

The primary goals of this laboratory exercise are to understand the significance of the H-R diagram in interpreting stellar evolution, to learn how observations of a star's physical properties define its position on the diagram, and to visualize how it will shift positions as it ages.

### **6.1.2 Materials**

You will need a computer with an internet connection, and a calculator (or calculator app).

All online lab exercise components can be reached from the GEAS project lab URL, listed here.

<http://astronomy.nmsu.edu/geas/labs/html/home.shtml>

### **6.1.3 Primary Tasks**

This lab comprises three activities: 1) an H-R diagram explorer web application, 2) a Pleiades cluster activity, and 3) an M67 H-R diagram web application. Students will complete these three activities, answer a set of final (Post-Lab) questions, write a summary of the laboratory exercise, and create a complete lab exercise report via the online Google Documents system (see <http://docs.google.com>).

All activities within this lab are computer-based, so you may either read this exercise on a computer screen, typing your answers to questions directly within the lab report template at Google Documents, or you may print out the lab exercise, make notes on the paper, and then transfer them into the template when you are done.

### 6.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that Section 6.7 (§6.7) contains 5 extra credit points.

Table 6.1: Breakdown of Points

Activity	H-R Diagram	Pleiades Cluster	M67 Cluster	Questions	Summary
Section	§6.3.1	§6.4.1	§6.4.2	§6.5	§6.6
Page	130	136	140	147	151
Points	19	13	24	19	25

### 6.1.5 Timeline

Week 1: Read §6.1–§6.4, complete activities in §6.3.1 and §6.4.1, and answer questions 1–4 in §6.5. We strongly suggest that you also attempt the activity in §6.4.2 and read through questions 5–8 in §6.5, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report template, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Complete activity in §6.4.2, finish final (Post-Lab) questions in §6.5, write lab summary, and submit completed lab report.

## 6.2 The Brightness of Stars

Astronomers describe the brightness of stars in two different ways: apparent brightness, based on how bright an object looks to us here on Earth, and intrinsic brightness (or luminosity), based on how bright an object actually is, independent of how far away it lies from a viewer. We have devised a scale of relative brightness called the “magnitude scale.” All objects of the same apparent magnitude appear equally bright when viewed from Earth. The 25 brightest stars in the sky are said to be first magnitude or brighter. The unaided eye can find about two thousand stars from first down to sixth magnitude in a dark sky, and large telescopes can reveal trillions more. In this lab exercise, you’ll learn how astronomers measure the magnitudes of stars.

There are two important features of the magnitude scale: 1) brighter objects have smaller magnitudes, and 2) the scale is not linear (it is logarithmic). This means that a first-magnitude star is brighter (not fainter) than a second-magnitude star, and it is more than

two times brighter than a second-magnitude star. If two stars differ by one magnitude, they differ in brightness by a factor of about 2.5; if they differ by two magnitudes, one is brighter than the other by a factor of  $2.5 \times 2.5 = 6.3$ ; if they differ by five magnitudes, one is brighter than the other by a factor of  $2.5^5 = 100$ ; if they differ by ten magnitudes, they differ by a factor of  $2.5^{10}$ , or  $100 \times 100$ , or 10,000. *An increment of one magnitude translates to a factor of 2.5 change in brightness.*

#### Example 6.1

A first-magnitude star is about 2.5 times brighter than a second-magnitude star, and 100 times brighter than a sixth-magnitude star. A sixth-magnitude star is 2.5 times brighter than a seventh-magnitude star, 100 times brighter than an eleventh-magnitude star, and 10,000 times brighter than a sixteenth-magnitude star.

If you are comfortable using exponents, you may express the change in relative brightness, of two stars ( $B_1$  and  $B_2$ ) as a function of the difference in their magnitudes ( $m_1$  and  $m_2$ ) as follows.

$$\frac{B_1}{B_2} = 10^{-\frac{m_1 - m_2}{2.5}}.$$

If we compare a sixth-magnitude star and a fainter seventh-magnitude star,  $m_1 - m_2 = 6 - 7 = -1$ , and

$$\frac{B_1}{B_2} = 10^{-\frac{-1}{2.5}} = 10^{+\frac{1}{2.5}} = 10^{0.4} = 2.51 \approx 2.5.$$

and we can confirm that the first star is 2.5 times brighter than the second. How much brighter is that seventh-magnitude star than a twelfth-magnitude star?

$$\frac{B_1}{B_2} = 10^{-\frac{7-12}{2.5}} = 10^{+\frac{5}{2.5}} = 10^2 = 100.$$

Astronomers use a similar magnitude scale to gauge intrinsic brightness, but it depends on a thought experiment. They imagine an object lies a certain distance away from Earth; a standard distance of ten parsecs (32.6 light years) is typically used. They then ask “how bright would this object appear from a distance of ten parsecs?” The resulting measure of brightness is called its absolute magnitude. While the Sun appears to be tremendously bright and thus has the smallest apparent magnitude of any star ( $-26.7$ ), if it were placed ten parsecs away, it would appear no brighter than a fifth-magnitude star. Given its close proximity, the Sun is extraordinarily special to us. Its apparent brightness exceeds that of the next brightest star, Sirius, by a factor of ten billion.

#### Example 6.2

The apparent magnitudes of the Sun and Sirius are  $-26.7$  and  $-1.6$  respectively. Note two things: 1) the brightest objects have the smallest magnitudes and magnitude values can be negative (smaller than zero), and 2) if two objects differ by 25 magnitudes – like the Sun and Sirius, the one with the smaller apparent magnitude appears ten billion times brighter than the one with the larger magnitude.

A star’s apparent brightness depends on both its intrinsic brightness and its distance from Earth. Viewed from 30 times further away, or from Pluto, the Sun would appear  $30^2$ , or 900

times fainter (though still very bright). What if we could view it from far outside the solar system?

#### Example 6.3

Viewed from a distance of 10 parsecs (32.6 light years), the Sun would look forty trillion times fainter. With a magnitude of 4.8, it would barely be visible to the unaided eye. This value represents the Sun's absolute magnitude. Comparing absolute magnitudes (imagining objects are viewed from the same distance) enables us to compare their intrinsic brightnesses, and to confirm that bright stars emit more energy per second than faint ones.

#### Example 6.4

Sirius is a bright, massive Main Sequence star, and emits twenty times more light than the Sun. If both stars were viewed from a standard distance of ten parsecs, Sirius would appear much brighter than the Sun (3.4 magnitudes brighter, given 1.4 magnitudes for Sirius versus 4.8 magnitudes for the Sun).

The magnitude scale is initially used to specify how bright a star appears, to quantify its apparent brightness. If the distance to the star is known, apparent magnitude and distance can be combined to compute an intrinsic brightness, or absolute magnitude. We can then easily convert absolute magnitude into energy output per second, or luminosity.

We can connect luminosity to other important physical quantities, by treating stars as idealized spherical radiators. The luminosity of a star depends primarily on its size and its surface temperature, in a fairly straightforward fashion. We can use the Stefan-Boltzmann Law to compute a star's luminosity  $L$ , where  $L = (4\pi\sigma)R^2T^4$  and  $4\pi\sigma$  is a constant,  $R$  is the star's radius, and  $T$  is its surface temperature. Given two of these three variables, we can use this relationship to estimate the third one.

## **6.3 The Main Sequence and the H-R Diagram**

A casual glance at the night sky reveals that stars span a wide range of apparent magnitudes (some are bright, and some are faint). Closer inspection suggests that they also have different colors. It turns out that the color of a star is related to its surface temperature. Blue stars (stars which radiate most of their light at blue wavelengths) are hot, yellow stars (like the Sun) are cooler, and red stars are the coolest of all. Suppose we plot apparent magnitude (vertically) and color (horizontally) for a set of stars. Does a pattern emerge? The answer is no – our graph is just a random scatter of points (see Figure 6.1a). If we instead plot absolute magnitude (or luminosity) versus color (creating an H-R Diagram), a pattern emerges.

While the diagram's overall appearance depends on the actual sample of stars we use, for many stellar populations lots of points lie within a narrow strip (as in Figure 6.1b), which we link with the Main Sequence phase of stellar evolution.

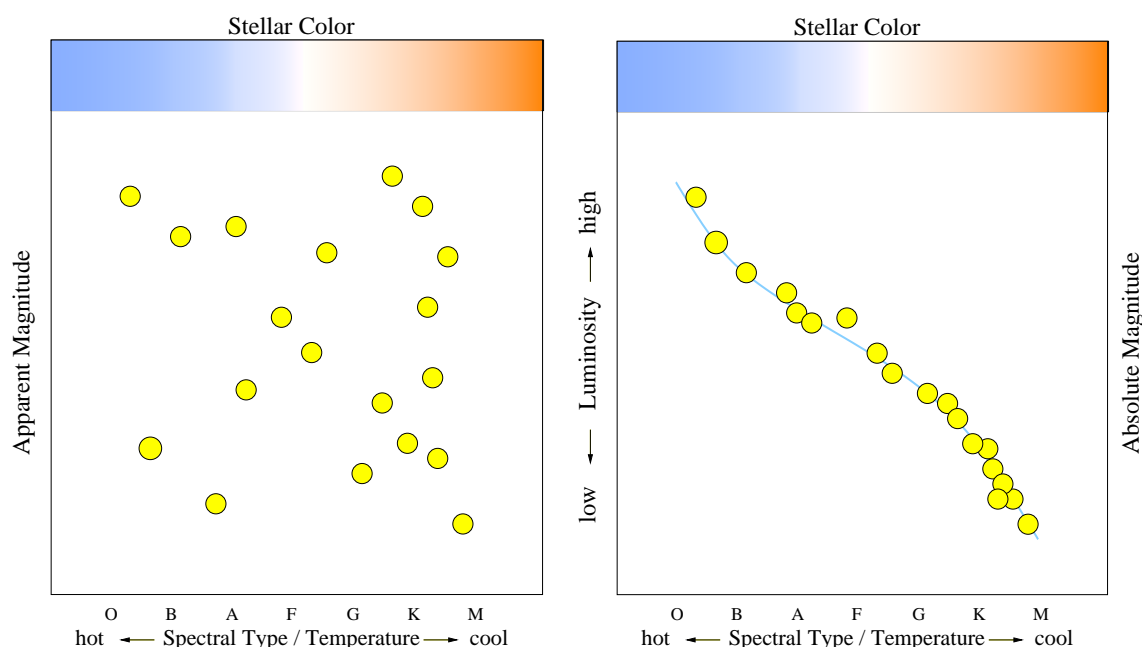


Figure 6.1: Plotting (a) apparent magnitude against color, we observe a random scattering of points. When (b) we convert to absolute magnitude, or luminosity, a clear pattern emerges. This is the fundamental form of an H-R Diagram.

### 6.3.1 The H-R Diagram and Stellar Properties Activity

In this activity we'll concentrate on two special samples of stars – the nearest stars, and those that appear the brightest from Earth – and consider H-R diagrams for each. These diagrams can be constructed in somewhat different ways. The ones we'll initially examine have luminosity (in units of the Sun's luminosity) plotted vertically versus surface temperature (plotted horizontally). The red X cursor initially marks the point plotted for the Sun, with a luminosity of  $10^0$ , or 1, and a surface temperature of 5800 kelvin (characteristic of a yellow star). Note that the Sun is a member of both the nearest star and the brightest star sample populations. Table 6.2 contains the basic stellar data for the Sun, collected in one place for easy reference.

Table 6.2: Basic stellar data for the Sun

Luminosity (solar units)	Radius (solar units)	Surface Temperature (K)	Spectral Type	Color (B–V)	Absolute Magnitude
1.00	1.00 <sup>a</sup>	5800	G2	+0.66	4.8

<sup>a</sup>The radius of the Sun is nearly 1/2 million miles.

Access the first web application (H-R Diagram) for this lab exercise from the GEAS project lab exercise web page (see the URL on page 126 in §6.1.2). Complete the following, filling in blanks as requested. (Each of the 36 filled blanks, and 2 circled answers, is worth 1/2 point.)

1. Beneath the H-R Diagram that appears on the right side of the screen, under “Plot Labels,” uncheck the box to the left of “Main Sequence.” The red line that will disappear

represents the Main Sequence. Stars plotted on (or very near to) it are in a fairly stable evolutionary phase. Re-check the box to get the Main Sequence back. Note that the Sun appears on this line – the initial red  $X$  represents its position. The Sun also lies on a second, green, line. Check the box to the left of “isoradius lines.” The parallel lines that disappear represent lines for groups of stars with the same radius. Re-check this box to get these lines back. The Sun appears on the line labeled  $1.0 R_{\odot}$ . All stars of this size, regardless of their luminosity or surface temperature, will lie along this line.

In which corner of the diagram (upper right, upper left, lower right, or lower left) would stars with radii 1000 times larger than that of the Sun be plotted? \_\_\_\_\_

In which corner would stars with radii 1000 times smaller than that of the Sun appear? \_\_\_\_\_

2. The H-R Diagram is fundamentally a plot of two variables (luminosity,  $L$ , versus surface temperature,  $T$ ), but it also contains a third important variable: radius,  $R$ . (Recall that we reviewed the relationship between these three variables in §6.2.) To investigate this relationship further, click on the red  $X$  for the Sun (think of it as a cursor). The “Cursor Properties” box at the lower left shows the corresponding values for  $T$ ,  $L$ , and  $R$ , with the latter two expressed in terms of values for the Sun (in solar units). Move the cursor (noting how the size of the first star in the “Size Comparison” box changes), to answer the following six questions (write your answers in the table below).

What is the relative luminosity (in solar units) of a star with the same surface temperature as the Sun but (a) only one-tenth the radius? (b) twice the radius?

What is the relative luminosity of a star which is the same size as the Sun but has (c) twice the surface temperature (11600 K, or 12000 K)? (d) only one-half the surface temperature (2900 K)?

Finally, what is the relative luminosity of a star with (e) one hundred times the Sun’s radius, but only one-half the surface temperature (2900 K)? (f) only one-hundredth its size, but twice the surface temperature (11600 K, or 12000 K)?

You may answer these six questions by moving the cursor to the positions of stars with the requested temperatures and radii on the H-R diagram. You may also choose to use the Stefan-Boltzmann relation expressing stellar luminosity as a function of stellar radius and temperature. If the luminosity, radius, and temperature of a star are expressed in solar units (relative to those of the Sun), then

$$L(L_{\odot}) = R^2(R_{\odot}) \times T^4(T_{\odot}).$$

For example, the luminosity of a star which is ten times larger than the Sun and twice as hot is

$$L(L_{\odot}) = 10^2 \times 2^4 = 100 \times 16 = 1600$$

and the star is thus 1600 times more luminous than the Sun.

Table 6.3: Basic stellar data for selected stars

Star	Luminosity (solar units)	Radius (solar units)	Surface Temperature (K)
Sun	1.00	1.00	5800
Star a		0.10	5800
Star b		2.00	5800
Star c		1.00	11600 (or 12000)
Star d		1.00	2900
Star e		100	2900
Star f		0.01	11600 (or 12000)

When you are finished, return the cursor to the position of the Sun. (Either reload the web-app in your browser, or select the “reset” option within the web-app.)

We hope that you’re beginning to realize that the H-R Diagram can take many forms. The vertically plotted variable can be luminosity, or it can be another quantity related to intrinsic brightness, such as absolute magnitude. The horizontally plotted variable is often surface temperature, but it can also be color, or spectral type. To illustrate these different forms of the diagram, do the following. Under “Plot Labels,” to the right of “x axis scale: temperature,” click the down arrow and select “spectral type” as the new x-axis label. Note that the Sun lies within the spectral type G (run a vertical line down from the  $X$  for the Sun and check where it intersects the x-axis).

3. Beneath the H-R Diagram under “Plotted Stars,” check the circle for “the nearest stars.” The properties of the nearest hundred or so stars will be shown. Examine the plot, and answer the following questions.

(a) Considering the measurement uncertainty inherent in the data, do most of the points lie on (or fairly near to) the Main Sequence line (within the Main Sequence strip)?

\_\_\_\_\_

If you can’t decide (or to double-check), under “Plot Labels,” click the box to the left of “show luminosity classes.” The green band that appears marked “Dwarfs (V)” is the Main Sequence strip.

(b) Match the four regions marked A, B, C, and D in Figure 6.2 with the areas where white dwarfs, blue supergiants, red giants, and red dwarfs are found.

Region A = \_\_\_\_\_, Region B = \_\_\_\_\_

Region C = \_\_\_\_\_, Region D = \_\_\_\_\_

What object’s position is marked by the  $X$ ? \_\_\_\_\_



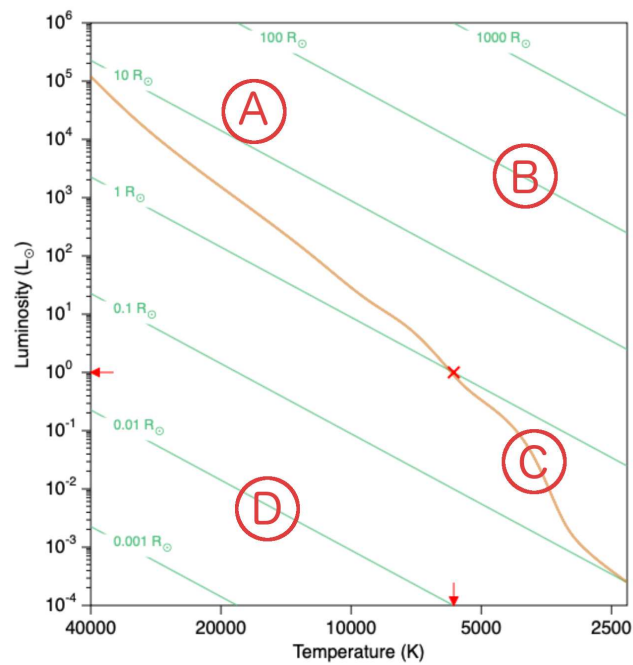


Figure 6.2: Consider which types of stars (white dwarfs, blue supergiants, red giants, and red dwarfs) are found in each of the four regions identified above.

What two lines intersect at this spot?

(c) The “Dwarfs (V)” label for the Main Sequence strip is unfortunate because the most luminous stars at the upper end of this strip have radii roughly \_\_\_\_\_ times that of the Sun – they are hardly “dwarfish.” But we still haven’t compared the Sun to typical stars, so let’s rectify that. Assume that the hundred or so stars nearest to us form a representative sample that we might find in a typical volume of our Milky Way galaxy. Let’s think about how the Sun compares to these stars.

(d) Of the hundred nearest stars, only \_\_\_\_\_ are obviously intrinsically brighter than the Sun.

(e) Move the cursor to the middle of the nearest star sample, where many of them are concentrated. This region is labeled \_\_\_\_\_ on the diagram. The luminosities for these typical stars are \_\_\_\_\_ that of the Sun’s, their radii are \_\_\_\_\_ that of the Sun’s, and their surface temperatures are around \_\_\_\_\_ K.

(f) The nearest star sample shown is technically incomplete; at least five nearby white dwarfs were omitted. The most famous is the companion to the bright, nearby star Sirius A; Sirius B has a luminosity of  $0.027 L_{\odot}$  and a surface temperature of 25,000 K. Moving the cursor to where this star would be plotted, its radius is \_\_\_\_\_ that of the Sun's (justifying the descriptive term dwarf). When finished, return the cursor to the position of the Sun.

Now consider another population of stars. Under "Plotted Stars," check the circle for "the brightest stars." The new points plotted on the diagram correspond to the 150 or so brightest stars which can be seen from Earth. Examine the plot and answer the following questions.

(g) Of these 150 brightest stars, only \_\_\_\_\_ is/are intrinsically fainter than the Sun. While all the nearest stars were predominantly Main Sequence (V) stars, the brightest stars sample also includes many objects of luminosity classes \_\_\_\_\_ and \_\_\_\_\_.

(h) Move the cursor to the star representing the most luminous star in this sample. Its luminosity is \_\_\_\_\_  $L_{\odot}$ , its radius is \_\_\_\_\_  $R_{\odot}$ , its surface temperature is \_\_\_\_\_ K, and its B-V color is \_\_\_\_\_. The region where the cursor is now located is labeled \_\_\_\_\_ on the diagram.

(i) Move the cursor to the star representing the largest star in this sample. Its luminosity is \_\_\_\_\_  $L_{\odot}$ , its radius is \_\_\_\_\_  $R_{\odot}$ , and its surface temperature is \_\_\_\_\_ K; its B-V color is \_\_\_\_\_. The region where the cursor is now located is labeled \_\_\_\_\_ on the diagram. When finished, return the cursor to the position of the Sun.

(j) There are \_\_\_\_\_ stars that are members of both samples (both nearest and brightest stars). Under "Plotted Stars," check the circle for "overlap" to verify your answer.

(k) In summary, the nearest / brightest (circle one) star sample is more representative of a typical volume of stars, whereas the nearest / brightest (circle one) star sample is a specially selected sample, heavily weighted towards relatively rare, highly luminous objects.

*Congratulations, you have completed the first of this lab's three activities. You may want to answer Post-Lab questions 1 through 3 on page 147 at this time.*

## 6.4 The Evolution of Stars

The Sun is a yellowish spectral type G star, with a diameter of 864,000 miles, a surface temperature of 5800 K, and an intermediate luminosity. Its physical properties place it in the middle of the H-R Diagram. To its lower right, cooler, redder Main Sequence stars (with spectral types of K and M) are plotted, while hotter, bluer Main Sequence stars (spectral types O, B, A, and F) are found to the upper left. Will the Sun's position in this diagram change as it ages? Yes! As it runs out of hydrogen fuel in its core, it will become a cooler, more luminous, and much bigger red giant star. Similar giants appear to the upper right of the Sun's current position (which means that they are brighter and redder than the Sun).

As the aging Sun runs out of fuel, it will eventually shed its bloated outer atmosphere and end its days as a white dwarf. White dwarfs are hotter, bluer and much less luminous than the Sun, so they are plotted to the lower left of the Sun's current position.

It would be fascinating to conduct a time-traveling experiment and observe how the Sun's position in the H-R diagram changed over time (charting its future path), but, alas, this is not possible. Though we can't conduct such a controlled experiment with the Sun, we can still get a good idea of its future evolutionary path.

Given all the diversity among observed stellar properties, you may be surprised to learn that it is the mass and the chemical composition of a star at formation which determine most of its other properties, and (provided it is not a member of a binary system) its subsequent life history. In addition to mass and chemical composition, two other variables can make stars differ: stars can have different ages (they form at different times) and they typically lie different distances away from us. Fortunately, astronomers can simplify our studies of stars and remove three of these four variables by using dense star clusters as "laboratories." In doing this, we assume that all of the stars in a given cluster have the same chemical composition and age, and lie the same distance from Earth. Mass is thus the critical variable.

The more massive a star is, the faster it burns through the hydrogen reserve in its core and evolves off the Main Sequence into a red giant. Figure 6.3 shows how a cluster's H-R diagram changes as it ages. We can't observe a single cluster for long enough to see these changes, as we'd need to observe it for millions of years. Instead, we can examine many clusters of varying ages and compare their H-R diagrams.

The diagrams shown in Figure 6.3 represent clusters which are 50 million, 500 million, two billion and six billion years old. We want you to appreciate how astronomers estimate how many more years the Sun, a lower intermediate mass star, will spend on the Main Sequence. There is growing evidence that the Sun was actually born in a cluster, one three to ten light years in size and containing 1500 to 3500 stars. It's thought that one of the most massive of these stars died in a cataclysmic supernova explosion. This event is thought to have occurred within just a few million years of the Sun's birth. Indeed, the telltale signs it left behind are the main evidence suggesting that the Sun was born in a star cluster.

Since all the stars in a cluster lie at essentially the same distance from Earth, we can use their apparent magnitudes to gauge their relative luminosities. We'll construct an H-R diagram using such data for stars in the Pleiades bright star cluster, in the constellation of Taurus, the bull. These data come from two telescope images taken of the cluster, one with a V (for visual) filter, and one with a shorter wavelength B (for blue) filter. (We need images taken in two filters to determine the colors of the stars.) Our Pleiades cluster H-R diagram (shown in Figure 6.4) plots V filter magnitudes vertically and colors horizontally (instead of surface temperature, or spectral type). The colors were obtained by measuring apparent magnitudes from both B filter and V filter images and subtracting these two magnitudes to produce a B-V color. (Note that "apparent magnitude" typically refers to an apparent brightness, often measured through a V filter, expressed on a magnitude scale.)

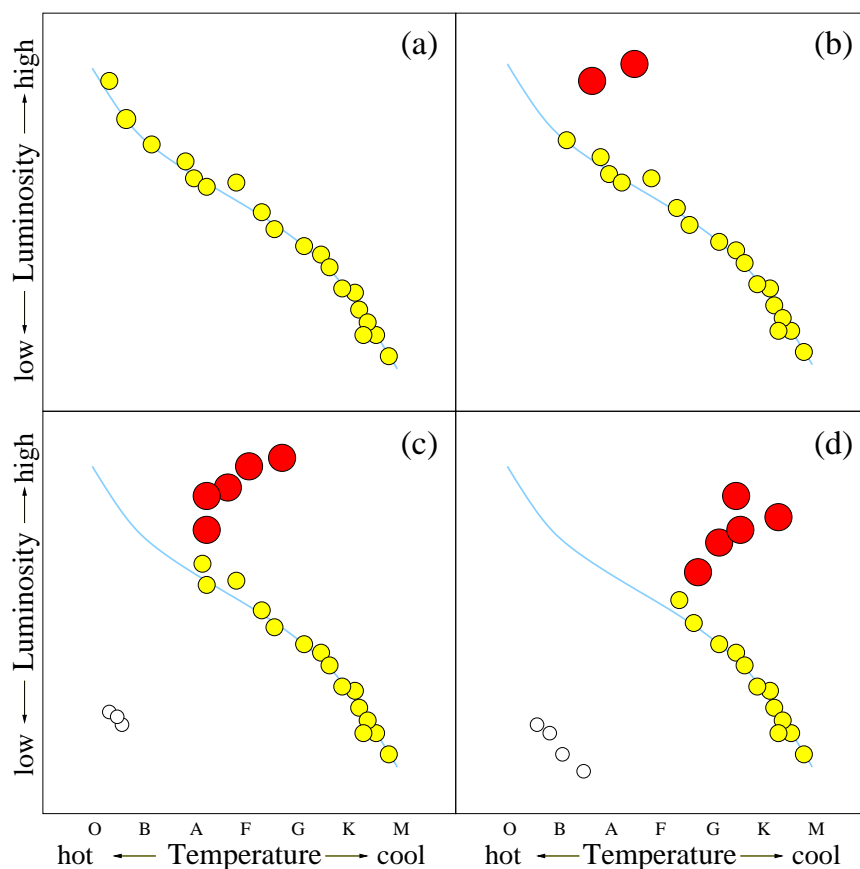


Figure 6.3: A star cluster’s H-R diagram evolves with time. (a) A young cluster, with plenty of hot, blue, high-mass stars. All objects shown are on the Main Sequence (the highest mass stars, exploding as supernovae, are not shown). (b) A fairly young cluster, missing only the highest mass stars which have gone supernova or evolved off Main Sequence to become giant stars. (c) A moderate-age cluster missing the highest, high, and a few intermediate mass stars which have evolved off the Main Sequence. Note the presence of both red giants and white dwarfs. (d) An older cluster, with many intermediate mass stars evolving off the Main Sequence and turning into giant stars. Note the increasing number of white dwarfs.

The Pleiades is a young cluster, and its H-R diagram will look similar to Figure 6.3(a). We can imagine that the Sun was once a young star in such a cluster. The Pleiades contains stars similar to the Sun in mass, luminosity, and color, as does the other star cluster whose H-R diagram we’ll study later (M67). How can we identify solar-like stars in these star clusters? We can simply search for Main Sequence objects of the appropriate color. While such stars have a characteristic intrinsic brightness, their apparent  $V$  magnitudes will depend on their distance from us, as Table 6.4 illustrates.

### 6.4.1 The H-R Diagram for the Pleiades, a Young Star Cluster

1. The Sun lies a mere 93 million miles from the Earth and seems incredibly bright, as illustrated by its very small apparent magnitude in Table 6.4. If we could shift the Sun farther and farther away from us, however, it would appear fainter and fainter in the sky. By

Table 6.4: The Apparent Magnitude of the Sun from Different Distances

Apparent magnitude	Distance	Significance
−26.7	93 million miles	Earth–Sun separation
4.8	10 parsecs	standard distance <sup>a</sup>
10.5	135 parsecs	distance to Pleiades cluster
14.2	770 parsecs	distance to M67 cluster

<sup>a</sup>Absolute magnitudes are calculated assuming this distance.

the time it approached the distance of the Pleiades cluster (at 135 parsecs, or 2,500 trillion miles), it would have an apparent magnitude of 10.5 and a small telescope would be needed to find it at all.

Check your understanding of the data listed in Table 6.4 by completing the following three statements. If the Sun were to be viewed from larger and larger distances, it would appear brighter / fainter / unchanged (circle one). Its apparent magnitude would increase / decrease / remain unchanged (circle one), while its absolute magnitude would increase / decrease / remain unchanged (circle one). (1 1/2 points total)

As you work through this activity, keep in mind that the faintest objects have the biggest magnitude values.

2. The Pleiades are sometimes called “The Seven Sisters” since this cluster contains seven stars which are bright enough to be found by eye on a clear, dark night. Use the data in Table 6.5 to complete the following questions and explore the properties of the cluster stars.

(a) Identify the seven brightest Pleiades stars from their apparent V magnitudes:

#\_\_\_\_\_, #\_\_\_\_\_, #\_\_\_\_\_, #\_\_\_\_\_, #\_\_\_\_\_, #\_\_\_\_\_, #\_\_\_\_\_. (1 point total)

Remember that the unaided eye can only see stars down to sixth magnitude, and stars fainter than that will have even larger magnitudes. The seven sisters should thus each have apparent magnitudes quite a bit smaller than 6.

(b) The brightest cluster star is #\_\_\_\_\_, with an apparent V magnitude of \_\_\_\_\_. (1 point total)

If this star also has an apparent B magnitude of 2.78, and a B–V color index of −0.09, explain how the color index (the B–V color) was computed. (1 point)

Some of the Pleiades stars listed are too faint to be seen without a telescope. Refer back to Example 6.1 (on page 128), and use apparent V magnitudes to make the following two comparisons.

(c) Star #1 is roughly \_\_\_\_\_ times brighter than star #4. (1/2 point)

(d) Star #\_\_\_\_\_ is almost 100 times brighter than star #15. (1/2 point)

Table 6.5: Apparent V Magnitudes of Bright Pleiades Stars

Star	V
1	2.87
2	3.62
3	3.70
4	3.87
5	4.18
6	4.30
7	5.09
8	5.45
9	5.46
10	5.65
11	5.76
12	6.17
13	6.43
14	7.24
15	8.86
16	10.48

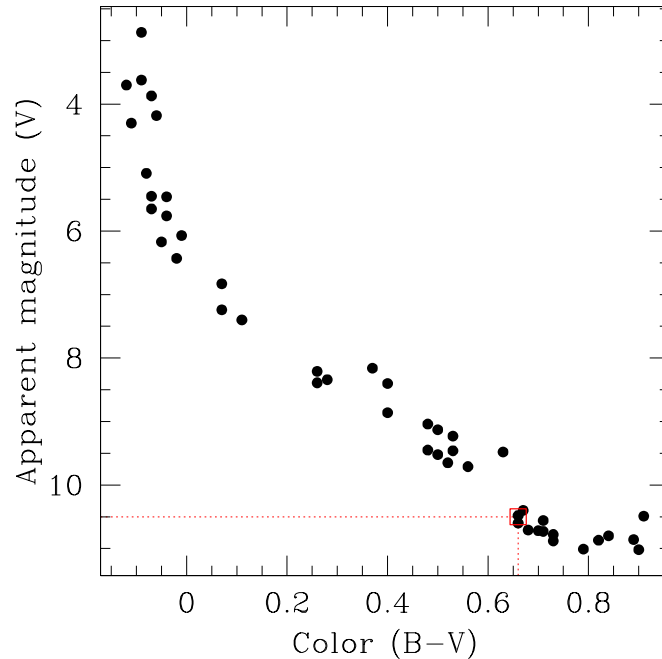


Figure 6.4: An H-R diagram for the Pleiades star cluster, with solar-type star #16 boxed in red. Note that the y-axis runs from large magnitudes to small, so that the brightest stars appear at the top of the figure.

Figure 6.4 shows an H-R diagram for the Pleiades cluster, constructed in part from the data in Table 6.5, and from observations of 31 additional cluster stars. Could you find the star most like the Sun in this diagram, or the star most like Sirius? Consider the examples below.

#### Example 6.5

Table 6.4 tells us that the Sun would have apparent (V) magnitude of roughly 10.5 if it were a member of the Pleiades cluster, and Table 6.2 gives its B–V color index as 0.66. Inspection of Table 6.5 reveals that star #16 (with an apparent V magnitude of 10.48) closely resembles the Sun. It too has a B–V color of 0.66, and it is boxed in red on Figure 6.4.

3. If the stars in the Pleiades cluster lay only ten parsecs from Earth, they would appear brighter / fainter / unchanged (circle one), though their absolute magnitudes would be brighter / fainter / unchanged (circle one). Solar-type star #16 would have an apparent

V magnitude of \_\_\_\_\_ and would just be visible to the unaided eye as a faint star. Star #16 is actually much farther away than ten parsecs (it lies around \_\_\_\_\_ parsecs away) so it appears much fainter. Its actual apparent V magnitude is \_\_\_\_\_. (2 1/2 points total)

Remember that any star which is fainter than sixth magnitude is not visible without a telescope.

#### Example 6.6

From Example 6.4, we know that Sirius is 3.4 magnitudes intrinsically brighter than the Sun. At the distance of the Pleiades, we would look for it on the Main Sequence at an apparent V magnitude of  $10.5 - 3.4 = 7.1$ . Star #14, with an apparent V magnitude of 7.24, is the closest match.

4. Imagine that you could go back to a time when the Sun was relatively young (say fifty million years old), and construct an H-R diagram for the star cluster in which it formed. This diagram would look like the one shown for the Pleiades in Figure 6.4, with star #16 representing the Sun, and the cluster's brightest star, Alcyone (star #1), highest up on the Main Sequence at the upper left.

What will happen to star #1 in the Pleiades (or to similar stars in the Sun's original cluster) as time marches on? Being 1000 times more luminous than the Sun, Alcyone is also much more massive, by a factor of seven. The more massive a star is, the faster it progresses through the phases of stellar evolution. Alcyone will spend less time in its stable (Main Sequence) phase, will run out of core nuclear fuel faster, and will move off the Main Sequence much sooner, compared to the Sun. While we think that the Sun will ultimately spend around ten billion years on the Main Sequence, a star as massive as Alcyone will spend less than one hundred million years there. Given its current properties, we think its age (and thus the age of the Pleiades cluster) is less than one hundred million years. This is less than the blink of a cosmic eye, compared to the age of the Universe (14 billion years).

(a) Review Figure 6.3. As a star cluster ages, its Main Sequence turn-off point steadily becomes brighter / fainter (circle one) and bluer / redder (circle one). The masses of the stars around the turn-off point decrease / increase (circle one). (1 point total)

(b) The difference between absolute and apparent magnitude is the same for all members of the Pleiades cluster, because it is purely a function of the cluster distance from Earth. Use the absolute magnitude of the Sun and the apparent magnitude of solar-type star #16 to estimate this difference, and then determine the absolute magnitude of star #1, Alcyone (the brightest star in the Pleiades). Show your work, as well as stating your final answer. (2 points)

The brightest, bluest star still remaining on the Main Sequence is sometimes used to define a cluster's turn-off point. The term "turn-off" comes from recognizing that this star will be the next one to leave (or turn off) the Main Sequence, and become a red giant. For the Pleiades, this turn-off point is currently at absolute magnitude \_\_\_\_\_ and B–V color \_\_\_\_\_. (1 point total)

(c) Review Example 6.6 (on page 139) and locate star #14 (resembling Sirius) in Figure 6.4. There are only \_\_\_\_\_ stars on the diagram that are brighter and bluer than it. (1 point)

Today, these massive, luminous blue stars give the Pleiades cluster its characteristic, youthful appearance. The cluster will look rather different in a few billion years, however, when these stars evolve off the Main Sequence. The next cluster that we will study, M67, lies in this age range. How different do you think its H-R diagram will be?

*Congratulations, you have completed the second of this lab's three activities. You may want to answer Post-Lab question 4 on page 148 at this time.*

### 6.4.2 An H-R Diagram for M67, an Older Star Cluster

You were given magnitudes for a number of stars in the Pleiades cluster in Table 6.5. We did not discuss how these magnitudes were obtained, but we shall do so now. You will measure apparent magnitudes (and record colors) for a set of stars in the star cluster M67 in the constellation of Cancer, the crab. As you work, remember that the faintest stars have the largest magnitude values.

1. Access the second web application for this lab exercise (M67 Cluster) from the GEAS project lab exercise web page (see the URL on page 126 in §6.1.2). This activity focuses on a color image of the star cluster M67. You will carefully measure the brightnesses of 12 individual stars, and use them as a basis for constructing an H-R diagram for the entire cluster. You will choose for yourself which of the hundreds of stars to measure, within certain guidelines.

When choosing stars to measure, try to avoid the brightest stars on the image (those which bleed sharp spikes of light into their nearest neighbors), stars which are very close to others and thus hard to isolate and sample without contamination, and the multitude of tiny, faint



red field stars in the background.

When you click on a star on the image of the sky, four changes will occur on the screen. A green circular aperture (a ring) will be drawn around the star on the image, to the left of the image a large green dot will appear at the position matching the brightness and color of the star on the H-R diagram and the plot labeled “distribution of counts” will be updated to show the radial distribution of light around the center of the aperture, and on the right an entry will be made and highlighted in the list of Marked Stars.

Start by clicking on the button labeled “Help,” and read through some basic information on how to use the web application efficiently. The “Zoom” option, for example, will double the size of the image on-screen, which can be quite useful when fine-tuning the position of an aperture around a star. Some options will be “grayed out” (hidden) when you are in the process of fitting an aperture, so remember to type “s” or click on the “Save aperture” button when you have finishing sizing your final aperture, so that the “Print,” “Write to disk,” and “Delete all apertures” options will become visible again. As you save an aperture, the aperture drawn on the image and the associated large dot on the H-R diagram will both turn from green to blue, and the star will no longer be highlighted in the list of Marked Stars.

2. Begin by identifying a solar-type star, one with properties similar to those of the Sun (as it would appear if it was placed within M67). A recent study found sixty of these within the cluster, so this should be a fairly straightforward exercise. Re-examining the data in Tables 6.2 and 6.4, we note that the Sun has a B–V color of 0.66, and at a distance of 770 parsecs (the distance to M67) it would have an apparent V magnitude of 14.2. We shall thus define a “solar-type” M67 star as one with a V magnitude between 13.7 and 14.7 and a B–V color between 0.56 and 0.76. Stars with magnitudes less than 13.7 are too bright, while those greater than 14.7 are too faint. Those with B–V colors less than 0.56 are too blue, and those greater than 0.76 are too red. Select an intermediate to faint yellowish looking star near the center of the image to measure.

(a) Use your mouse to click on the center of the star’s image. A green circle (an aperture) will appear around the star, and related data will appear in two charts: star # (for ID purposes),  $X$  and  $Y$  coordinates (indicating the position of the star on the image, where the origin is in the upper left corner), radius  $R$  of the aperture (an angular size expressed in arcseconds), corresponding apparent V magnitude, color index B–V, and the number of counts registered by the detector for the pixels enclosed by the aperture and their radial distribution around the center of the aperture. The color index is literally the difference between the number of counts recorded at blue wavelengths and the number recorded at visual (green) wavelengths, in units of magnitudes. Because fainter fluxes (less light) are equivalent to larger magnitudes, very blue objects have small B–V colors and very red ones have large B–V colors.

(b) Refine your aperture to better match the star location and contain all of its light by using the “Update Aperture” panel to the right of the image. From there you can shift your circle

left or right, or up or down, by clicking on the four rectangular directional arrow symbols, or you can increase or decrease its radius (to contain all of the light from the star, but none from its neighbors) via the two circular up or down arrow symbols. You can duplicate these actions using the arrow keys, and the plus and the minus keys, on your keyboard.

Note how the counts and the corresponding apparent magnitudes and colors change as you vary the aperture properties. The distribution of counts around the center of the aperture should be sharply peaked when your aperture is properly centered. (Some of the brightest stars will have flat, or slightly dimpled, cores because they are so bright. Don't worry about these artifacts.) When you have centered the aperture as best you can, and adjusted its size to include all significant "starlight" counts, but not too many "dark sky background" counts and light from neighboring objects, examine its V magnitude and B-V color in the chart of Marked Stars. Do they match the definition of a solar-type star? If not (and in general if you wish to abandon your attempt to measure any star) click the "delete aperture" button in the "Update Aperture" panel (or type the letter "d"), and choose another candidate to continue your search for a solar-type star.

#### Example 6.7

Suppose you select star #132, and measure a V magnitude of 15.42 and B-V color of 0.76. This star lies within the color range of a solar-type star (as  $0.56 \leq 0.76 \leq 0.76$ ), but it is too faint ( $14.7 < 15.42$ ). You then look for a brighter star with a similar color and settle on star #152. It has a V magnitude of 14.21 and a B-V color of 0.54. This star lies within the required magnitude range ( $13.7 \leq 14.21 \leq 14.7$ ), but is too blue ( $0.54 < 0.56$ ), so you look for a redder star of similar brightness and settle on star #123. Your measurement yields a V magnitude of 14.02 and a B-V color of 0.62. This star is a solar-type star ( $13.7 \leq 14.02 < 14.7$ , and  $0.56 \leq 0.62 \leq 0.76$ ).

(c) When you find an acceptable star, click the "Save" button in the Update Aperture panel. The green aperture will turn blue, and the data for the star will be permanently fixed in the data chart.

There is a second chart, a graph showing the radial distribution of counts around the center of the selected aperture. This plot illustrates the distribution of light within the aperture (the green circle) as defined by two short green bars, while the plot extends a bit farther out to show how much light lies just beyond the aperture. This will help you select the best size for the aperture, a decision that determines how much light to count when measuring the star's brightness.

As you vary the position and size of the aperture around each star and simultaneously watch the displayed H-R diagram, you'll note that the point where the star is plotted in this diagram typically varies in the vertical direction, since the measured brightness varies depending on the size and position of the aperture.

3. After you've found and marked an acceptable solar-type star, you'll repeat this basic process to find 11 more stars that fit into four categories. We want you to find at least two

stars fainter and bluer than the Sun, at least two fainter and redder than the Sun, at least two brighter and bluer than the Sun, and at least two brighter and redder than the Sun. Try to guess whether each star that you select is brighter or fainter, and redder or bluer, than the Sun before reading its magnitude and color from the chart. We want to confirm that you can correctly compare the brightnesses and colors of stars, so be sure to place each of the 11 additional stars that you mark into the appropriate category.

#### Example 6.8

Let us re-examine the first star which we rejected in our search for a solar-type star. We can classify star #132 as both fainter and redder than the Sun, since for apparent V magnitudes,  $14.2 < 15.42$ , and for B–V colors,  $0.66 < 0.76$ . This star thus qualifies for the redder, fainter category.

As you select your set of 12 stars, sort them into the five defined categories below. You will receive one point for the solar-type star and two points for each of the other four categories, if all stars are correctly categorized.

(a) The solar-type star is #\_\_\_\_\_. (This star should have a V magnitude between 13.7 and 14.7, and a B–V color between 0.56 and 0.76.)

(b) The 2 to 4 stars fainter and bluer than solar are #\_\_\_\_\_. (These stars are fainter than a V magnitude of 14.2, so  $14.2 < V$ , and they have colors bluer than 0.66, so  $B-V < 0.66$ .)

(c) The 2 to 4 stars fainter and redder than solar are #\_\_\_\_\_.

(d) The 2 to 4 stars brighter and bluer than solar are #\_\_\_\_\_.

(e) The 2 to 4 stars brighter and redder than solar are #\_\_\_\_\_.

You will print out your completed data chart and add it as a figure in your lab report. In the corresponding figure caption, identify the solar-type star by its ID, and also state which of the other 11 stars fall into each of the other four categories.

4. After all 12 stars have been measured and marked, points for an additional 400 stars within the M67 cluster (down to an apparent magnitude of 16) will suddenly appear on the H-R diagram plot. These stars were marked in the same way you marked your set, and their magnitudes were measured similarly. Keep the diagram on the screen for a few minutes; it will take awhile to properly interpret it, assess the quality of its data, and extract important information from it. If you have trouble reading it from within the application, go ahead and save a copy as a file on your local disk (or print out a paper copy). This version should be easier to view, as it will be somewhat larger. Go ahead and save a copy of your list of Marked Stars as well, as you will need it for your lab report.

5. Your first task in interpreting the H-R diagram for M67 is to find the Main Sequence, which is not quite as obvious as it was for the much closer Pleiades cluster (recall Figure 6.3). There

are several important reasons for this. Some stars that we think lie in M67 may actually be closer foreground stars or more distant background stars; some stars which appear to be isolated may actually be unresolved binary stars; there may be small variations in chemical composition (the ratio of various metals) within this stellar population; and measurement uncertainties (every measured quantity has some uncertainty to it) and procedural errors (errors in technique) may exist. To simplify this task, note the four stellar evolutionary tracks (drawn in four colors), which merge into a single track in the diagram's lower center right. Let them define the center of the Main Sequence for low-mass stars.

6. When measuring the magnitudes of stars in M67, any errors in the sizes or positions of the stellar apertures will lead to poor measurements. Consider this point when answering the following four questions. (1 point each).

(a) What happens if an aperture is too small? Where will the associated star appear on the H-R diagram, relative to its correct position?

(b) What happens if an aperture is too large? Does it matter whether the extra space is dark sky, or contains a neighboring object?

(c) What happens if the aperture is offset from the center of the star?

(d) What happens if you place an aperture directly between two stars?

If you are unsure about any of your answers, go ahead and experiment by shifting one of your apertures on and off of its target, and examine the result in the radial plot of stellar fluxes. (Be sure to reset your aperture to its correct position and size when you are done experimenting!)

7. Having made our measurements and considered common sources of error, we can now use our derived H-R diagram to estimate the age of the M67 cluster. Examine the four evolutionary tracks (drawn in four colors) showing the expected brightnesses and colors of stars as they turn off the Main Sequence and evolve (moving upwards and to the right) into red giant stars. These tracks are based on sophisticated models of stellar evolution, which simulate the nuclear processes by which stars burn their fuel. Each track has a stellar age associated with it.

The low-mass stars at the bottom part of the Main Sequence (where the four tracks effectively merge into a single line) are of no help here. Instead, look above and to the right of this region – where stars “turn off” the Main Sequence as they become red giants – and decide which one of four tracks (if any) best fits the *edge* of the region which contains the giant stars. Base your age estimate for M67 on the corresponding age for that track, or interpolate between two adjoining tracks if you think that the best fit to the data lies between them.

Discuss your results (your age estimate) for M67 below, explaining how you came to your conclusion. Note the particular features on the H-R diagram which were most important to your decision-making process. (6 points)

8. Compare your results for M67 with those for the Pleiades, with respect to the following five factors. (1 point each)

(a) Cluster turn-off point:

(b) Presence of red giants:

(c) Presence of red dwarfs:

(d) Presence of massive blue Main Sequence stars:

(e) Age of cluster:

9. Besides answering the remaining Post-Lab questions (see below), you need to save two things for presentation in your lab report: (a) the M67 data chart for your 12 stars, and (b) the M67 cluster H-R diagram. You may also include any radial charts of flux around a particular aperture, if you wish to do so to help in answering the four questions in part 7 above.

*Congratulations, you have completed the third of this lab's three activities. You may want to complete your answers to Post-Lab questions 5 through 7 on page 149 at this time.*

## 6.5 Final (Post-Lab) Questions

*Questions 1 through 3 relate to the materials discussed in §6.3.1. Review this section if you are unsure of your answers.*

1. How does the Sun compare to the other members of the nearest star sample? If one assumes this sample is representative of typical stars found throughout the Universe, to what extent is the Sun a typical star? (3 points)

2. Consider the brightest stars in the sky, and why they appear so bright. Three students debate this issue. Student A: “These stars must be very close to us. That would make them appear brighter to us in the sky.” Student B: “These stars are intrinsically very luminous, so they emit a tremendous amount of energy.” Student C: “I think it’s because these stars are very close and very luminous.” Use what you’ve learned in this lab to support the views of one of the three students and answer the question “Why do the stars which appear the brightest in the night sky seem so bright?” (3 points)

3. Are these apparently bright stars very common (do stars like them make up a large percentage of all stars)? Explain your reasoning. (3 points)

*Question 4 relates to the materials discussed in §6.4.1. Review this section if you are unsure of your answer.*

4. Consider how the H-R diagram of the Pleiades would look far in the future.

(a) Suppose all of the Main Sequence stars above solar-type star #16 had run out of fuel and left the Main Sequence. What other regions in the diagram (besides the Main Sequence) would now be populated? (1 point)

(b) How old would this cluster be? Explain. (1 point)



*Questions 5 through 7 relate to the materials discussed in §6.4.2. Review this section if you are unsure of your answers.*

5. The best explanation for why the H-R diagram for M67 does not include any white dwarf stars is (a) this cluster is not old enough for any of its stars to have evolved to this stage, or (b) the data this H-R diagram is based on only includes stars brighter than an apparent V magnitude of 16, and we expect any white dwarfs in M67 to be fainter than this. Explain your choice of answer. (2 points)

6. When measuring apparent magnitudes for stars in M67, if two equal-mass stars were tightly clustered in a binary system and could not be separated (they appeared as one star), where would their combined properties place them on the H-R diagram (versus where they would be placed if they were separable)? (2 points)

7. The constellation of Cancer contains the star cluster M44, which, like the Pleiades, is visible to the unaided eye on a clear night.

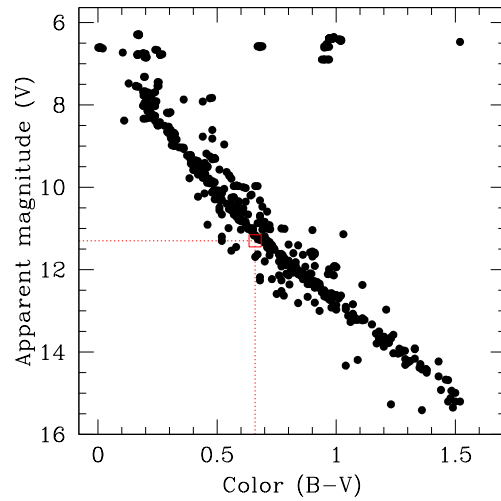


Figure 6.5: An H-R diagram for the M44 star cluster, with a solar-type star boxed in red. Note that the y-axis runs from large magnitudes to small, so that the brightest stars appear at the top of the figure.

(a) Using its H-R diagram in Figure 6.5, compare this cluster's age with that of the Pleiades and M67 clusters. Explain how you arrived at your conclusion. (2 points)

(b) Is M44 closer to or farther away from us than the Pleiades, and closer to or farther away from us than M67? Explain your answer. (2 points)

## 6.6 Summary

After reviewing this lab's goals (see §6.1.1), summarize the most important concepts discussed in this lab and discuss what you have learned (include both techniques for data analysis and scientific conclusions). (25 points)

Be sure to consider and answer the following questions.

- How are H-R diagrams constructed? Why are they useful, and how are they used? Start with the necessary telescope observations and required data, explain how they are obtained, and then proceed to the use and significance of the derived diagrams.
- Astronomers say that the Sun has another five billion or so years remaining of stable, well-behaved existence on the Main Sequence. If someone were to express skepticism, saying, "How can we possibly know that?" what would you say in response?

Make use of the H-R diagram constructed for M67 as an illustrative example, and reference the three figures that you created at the end of §6.4.2.

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be roughly 300 to 500 words long.

## 6.7 Extra Credit

The brightest star in the sky, Sirius, has a faint companion (Sirius B) whose existence was predicted from orbital irregularities by Friedrich Bessel in 1844. It is ten magnitudes fainter than Sirius, and was at first assumed to be a cool, red dwarf. In 1914, observations showed that it was both hot and roughly the size of Earth. How was this discovery made? Describe the observations which were made, and the mathematical argument behind the size estimate. In 2005, the mass and size of Sirius B were measured to unprecedented precision. How was this done? (5 points)

Be sure to cite your references, whether they are texts or URLs.

## Lab 7

# Hubble's Law and the Cosmic Distance Scale

### 7.1 Introduction

There are tens of satellites (moons) orbiting the planets of the solar system, a handful of planets in orbit around the Sun, and over one hundred billion stars like the Sun which make up our Milky Way galaxy. As we advance from satellites to planets to stars to galaxies we note that the masses and sizes of the bodies grow tremendously, as do the distances between them. How can we measure large distances in the Universe?

We can bounce radar signals off of solar system objects to find out how far away they lie from Earth, and use parallax measurements to find the distances to nearby stars (taking advantage of their apparent movements across the sky as the Earth changes its position around the Sun). Parallax measurements have been extended to stars out to 500 light-years by the Hipparcos satellite (and may be pushed out to 30,000 light-years, if the planned Gaia mission succeeds). Parallax is an excellent technique for mapping the distribution of stars within our Milky Way galaxy – but how can we chart the rest of the Universe?

As the 20<sup>th</sup> century began many astronomers thought that everything observed through telescopes, including the numerous “spiral nebulae” like the famous faint swirl in the constellation Andromeda, was contained within our Milky Way galaxy. Others argued that spiral nebulae were separate galaxies, lying far beyond the Milky Way. The issue came to a head in the famous Shapley-Curtis debate of 1920, illustrating the need for deeper, higher resolution observations to answer the question. Edwin Hubble soon showed that the observable Universe extends far beyond the Milky Way. He used new techniques to measure the distances to distant objects, employing Cepheid variable stars as “standard candles”.

An entire family of distance determination methods is based on the concept of a standard candle. Simply put, if we can identify a class of objects with near-identical properties then those which appear the smallest and faintest must lie furthest away from us. If you have ever gauged the distance to an approaching car at night from the brightness of its headlights, you've used this technique! We assume that the brightness of these objects obeys an inverse square law, being inversely proportional to the square of the distance ( $F \propto 1/d^2$ ), and that their apparent size varies inversely with distance ( $\theta \propto 1/d$ ). If your standard candle is a 100-watt light bulb, for example, then if it shifts to a position twice as far away from you it will appear one-fourth as bright (like a 25-watt bulb would at its initial position).

Cepheid variable stars are important examples of standard candles, used to determine accurate distances to stars within the Milky Way and within nearby galaxies like the Andromeda galaxy. The amount of light output by these massive stars can vary by a factor of two or three, over a regular period of time extending from a few days to a few months. In 1908, Henrietta Leavitt discovered that this period of variability was determined by the intrinsic luminosity of the star – the more energy the star pumped out, the longer it took to vary from bright to faint and back to bright again, like clockwork. The brightest of these stars are 10,000 times more luminous than the Sun, so they can be seen from quite a distance.

With modern telescopes, we can observe Cepheid variable stars out to 30 megaparsecs (Mpc), or 100 million light-years. This allows us to determine distances to galaxies hosting such stars which lie within 30 Mpc of the Milky Way, covering a number of nearby isolated galaxies, groups of galaxies, and even the closest large galaxy clusters like Virgo, containing hundreds of galaxies. Thirty megaparsecs might seem like a large distance (equal to 300 times the size of the Milky Way), but in the grand scheme of things it barely covers our own cosmic backyard.

We can employ this technique at larger distances by finding brighter standard candles, in the form of supernovae. Type Ia supernovae are stars which (for a brief period of time) can be as bright as an entire galaxy. They occur as part of the end-state process for low-mass white dwarf stars, when mass accreted from a stellar companion (a second star) fuels a short, immense explosion. These objects consistently achieve a known peak luminosity, and thus when we observe them we know how far away they lie. (As they are all the same intrinsic brightness, the further they lie from Earth the fainter they appear.) The most distant supernovae studied lie an amazing 1,700 Mpc away from us.

Supernovae explosions are wonderful probes of the distant Universe, but they are rare and can be difficult to find. We thus employ additional techniques to measure the distances to many galaxies, taking advantage of the known properties of the galaxies themselves. We will explore some of these techniques within this laboratory exercise. Within large clusters of galaxies (home to hundreds or even thousands of galaxies), one almost always finds a bright elliptical galaxy in the center of the cluster. These brightest cluster galaxies (BCGs) have remarkably consistent intrinsic properties. Because they lurk in the cores of clusters and consume unwary interlopers (yes, it's a dog-eat-dog world, and large galaxies are literally cannibals), they tend to grow to a certain size and give off a certain amount of energy.

These galaxies exhibit uniform properties, meaning that they appear to be the same size and emit the same amount of energy when viewed from the same distance. (They are standard candles in a sense, but they vary too much to truly deserve the name. You might think of them as sub-standard candles built in a galactic factory, one with no routine inspections or quality controls.)

We will also estimate distances to galaxies using a cosmological method based on a discovery made by Edwin Hubble in 1930. While determining the distances to nearby galaxies, he realized that the derived distances correlated with another observable property – the velocity at which the galaxies appeared to recede. Hubble plotted recessional velocity  $v$  as a function of distance  $d$ , as shown in Figure 7.1, and discovered that the most distant galaxies are also moving the fastest. We call this relationship Hubble’s Law, and these diagrams Hubble diagrams, to honor his insight.

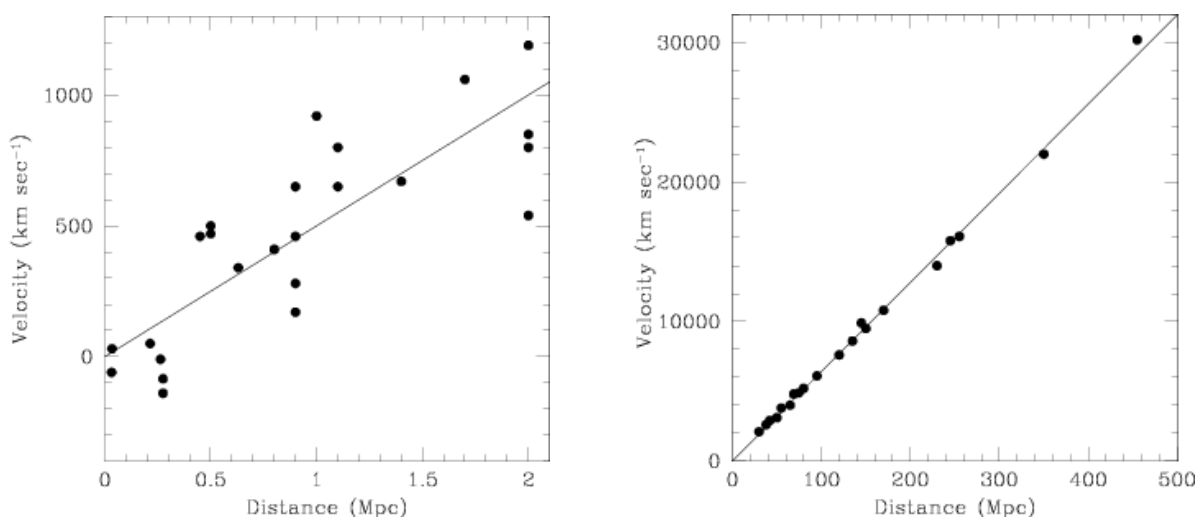


Figure 7.1: Hubble diagram showing Edwin Hubble’s original data set (left), and modern data (right) extending out to larger distances. The  $x$ -axis shows the distances to nearby galaxies determined by a variety of methods, and the  $y$ -axis shows their recessional velocities. The slope of a line fit to the data has units of  $y$  over  $x$ , or  $\text{km sec}^{-1}$  per Mpc, and is called the Hubble constant ( $H_0$ ).

We interpret these recessional velocities as evidence that the entire Universe is expanding outward (the distances between all galaxies increases with time). We do not assume that we are situated at the center of the Universe and everything is moving away from us, having learned from our previous “the Earth is the center of the world” phase (so 16<sup>th</sup> century).

We can fit the slope (the change in  $y$  over the change in  $x$ ) of the data in Figure 7.1, and find the relationship between distance  $d$  and velocity  $v$ . We call this slope value  $H_0$  (the Hubble constant) in honor of Edwin Hubble, where

$$H_0 = \frac{v}{d}, \quad \text{and so } v = H_0 d. \quad (7.1)$$

The current accepted value of  $H_0$  is  $72 \text{ km sec}^{-1}$  per Mpc. We can also solve this equation for  $d$ , so if we know the velocity  $v$  of a nearby galaxy, then it lies at a distance  $d$  such that

$$d = \frac{v}{H_0}. \quad (7.2)$$

We will examine several Hubble diagrams in this laboratory exercise, determining the distances to galaxies by measuring changes to their spectra due to recessional velocities.

### 7.1.1 Goals

The primary goals of this lab are to appreciate that the Universe is expanding, and to understand and evaluate various techniques for determining distances to galaxies.

### 7.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL.

<http://astronomy.nmsu.edu/geas/labs/labs.html>

You will also need a computer with an internet connection, and a calculator (or calculator app).

### 7.1.3 Primary Tasks

You will study Hubble diagrams based on Cepheid variable star data and Type Ia supernovae observations. You will then measure the apparent sizes and fluxes for a sample of brightest cluster galaxies (BCGs), and compare the derived distances with those that you find from redshifted spectra.

### 7.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that §7.9 contains 5 extra credit points.

Table 7.1: Breakdown of Points

Activity	Hubble diagrams	Distances	Comparisons	Questions	Summary
Section	§7.3.1, §7.3.2,	§7.4	§7.5	§7.7	§7.8
Page	161, 164	167	173	178	180
Points	14	27	22	13	24



### 7.1.5 Timeline

Week 1: Read sections §7.1–§7.4, complete activities in §7.3 and §7.4, and begin final (Post-Lab) questions in §7.7.

Week 2: Read §7.5–§7.6, complete activities in §7.5, finish final (Post-Lab) questions in §7.7, write lab summary, and submit completed lab report.

## 7.2 Determining Galaxy Redshifts and Velocities from Spectra

A Hubble diagram shows the distances to galaxies versus their recessional velocities (how quickly they and the Milky Way are separating from each other). We can determine these velocities by examining galaxy spectra and determining redshifts. Let’s walk through this process, explaining our terms as we go.

If you’ve ever listened to the siren of a speeding police car, ambulance, or fire truck, you’ve heard the high-pitched sound as it approaches you drop to a lower frequency as it passes by and recedes. This “Doppler Effect” is due to successive sound waves from the approaching source piling up in time (as each new wave travels a shorter distance to reach you), so that your ear absorbs more of them with every second. Once the siren starts to move away from you, the sound waves start to space out again (as they have to cover more and more ground to reach you) and the siren seems to drop in pitch.

Light waves undergo a similar effect. While it’s most natural to talk about sound waves changing in pitch (or frequency), the analogous effect for light waves is typically described as a change in wavelength. We detect visual light emitted from approaching sources to be shifted in wavelength toward the blue end of the spectrum, and we find light from sources moving away from us to be redshifted to longer wavelengths. The amplitude of the shift is defined by a change in wavelength,  $\Delta\lambda$ . The ratio of  $\Delta\lambda$  to the original wavelength of the light (called  $\lambda_{rest}$ , as it is emitted by a source at rest with respect to an observer) is defined as redshift ( $z$ ). We can write this as an equation:

$$z = \frac{\Delta\lambda}{\lambda_{rest}}. \quad (7.3)$$

The redshift  $z$  is the change in wavelength of a spectral feature, relative to its wavelength at rest. The larger the redshift, the faster the object is moving away from the observer.

### Example 7.1

Hydrogen is the most common element in the Universe, and so the stars and gas clouds within galaxies frequently absorb or emit light at the wavelengths at which hydrogen atoms absorb and emit radiation. The hydrogen beta ( $H\beta$ ) line is one such feature, found at a wavelength corresponding to blue-green light. In an object at rest with respect to an observer, this line

is observed at its rest wavelength of  $4861\text{\AA}$  (we say  $\lambda_{rest} = 4861\text{\AA}$ ). Suppose that we observe the  $H\beta$  line in the spectrum of a galaxy and it appears instead at a wavelength of  $5347\text{\AA}$  ( $\lambda_{obs} = 5347\text{\AA}$ ). The observed wavelength is longer, as the light from the galaxy is being redshifted to longer wavelengths. To determine the galaxy redshift, we first need to find the change in wavelength,  $\Delta\lambda$ . We recognize that

$$\Delta\lambda = \lambda_{obs} - \lambda_{rest} = 5347\text{\AA} - 4861\text{\AA} = 486\text{\AA}. \quad (7.4)$$

How large a shift is  $486\text{\AA}$ ? Remember that the range of the human eye extends from violet down to red wavelengths, covering roughly  $3,500\text{\AA}$ . A shift of  $486\text{\AA}$  would turn a blue beam of light to green, or yellow light to orange – a noticeable effect.

Dividing by  $\Delta\lambda$  by  $\lambda_{rest}$ , we see that the redshift for this object is

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{486\text{\AA}}{4861\text{\AA}} = 0.10. \quad (7.5)$$

### Example 7.2

At speeds much less than the speed of light (redshifts  $z \leq 0.10$ ), the redshift of a galaxy is equivalent to its velocity  $v$  in units of the speed of light,  $c$ . We say that  $z = v/c$ , so the velocity of the galaxy relative to an observer is

$$v = z \times c = z \times 300,000 \text{ km sec}^{-1} \quad (7.6)$$

as the speed of light,  $c$ , is  $300,000 \text{ km sec}^{-1}$ .

How quickly is the galaxy described in Example 7.1 moving away from us? With a redshift of 0.10,

$$v = z \times c = 0.10 \times 300,000 \text{ km sec}^{-1} = 30,000 \text{ km sec}^{-1}. \quad (7.7)$$

Due to the overall expansion of the Universe, this galaxy and the Milky Way are separating at one-tenth the speed of light!

Recall that the “spectrum” of a galaxy is simply a plot of the amount of light that it emits as a function of wavelength. A continuous (or continuum) spectrum is one which varies smoothly and slowly as a function of wavelength, appearing in the visual as a rainbow containing all of the saturated colors. Imagine that there was a sudden gap in a continuum spectrum, where the light was removed at a particular wavelength. We would say that it contained an “absorption” feature because light had been absorbed out at that wavelength, leaving an empty dark space. When we observe stars, we often observe a gap at the wavelength of the  $H\beta$  line, where hydrogen in the stellar atmosphere has absorbed photons emitted by the stellar core.

### Example 7.3

Consider the spectrum of a galaxy at a redshift  $z \approx 0.10$  ( $z = 0.095$ ), shown in Figure 7.2. We have marked seven significant absorption and emission lines in this spectrum, and identified the elements responsible for them (hydrogen, oxygen, magnesium, sodium, and sulfur). The

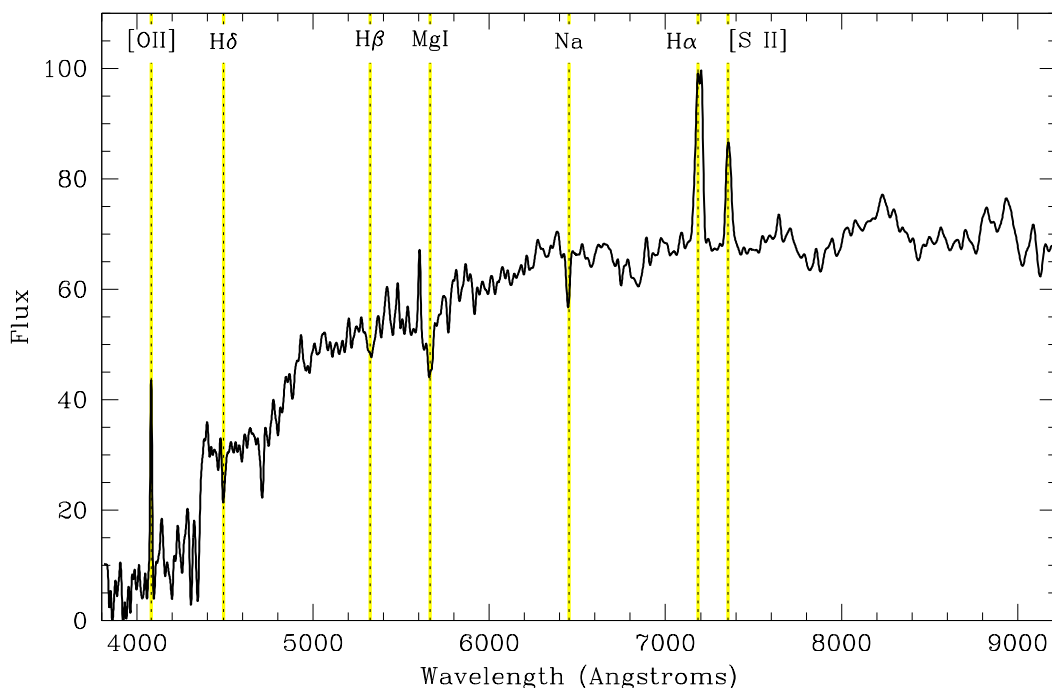


Figure 7.2: The optical spectrum of a nearby galaxy, observed by the Sloan Digital Sky Survey (SDSS). This plot shows the amount of light emitted by the galaxy as a function of wavelength. The spectrum has a smooth underlying shape which rises gently toward longer (redder) wavelengths, with emission and absorption features superimposed on top of it at particular wavelengths. We have identified several of these key features by name: the oxygen [OII] line, the hydrogen Balmer lines  $H\gamma$ ,  $H\beta$ , and  $H\alpha$ , the magnesium Mg I line, the sodium Na line, and the sulfur [S II] line. As we know the rest wavelengths of these lines, by finding their observed wavelengths we can determine the redshift of the galaxy.

presence of these lines tells us that these elements are present in this galaxy. Note how, at each of the marked wavelengths, there is either a peak or a drop in the spectral flux, indicating an emission or absorption feature. Find the  $H\beta$  line, and determine its observed wavelength along the  $x$ -axis – if you estimated a value around  $5325\text{\AA}$ , you did well! We determined in Example 7.1 that a galaxy with a redshift of 0.10 should show the  $H\beta$  line at a wavelength of  $5347\text{\AA}$ , so we can tell that this galaxy has a redshift slightly lower than 0.10.

In this laboratory exercise, you will compare real galaxy spectra like this to a rest-frame spectrum showing the wavelengths and intensities of various features as they would appear in a galaxy at rest with respect to our Milky Way galaxy. You will align the spectra by shifting the rest-frame spectrum back and forth in wavelength to match the observed galaxy spectrum, in order to determine its redshift, as described in Figure 7.3.

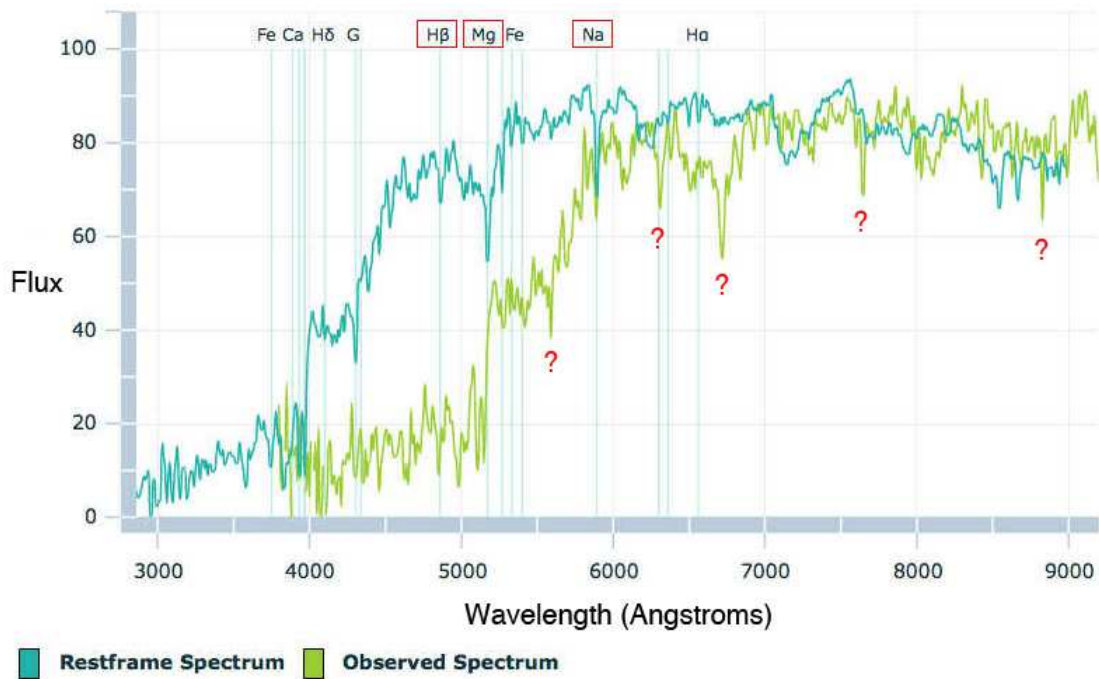


Figure 7.3: The optical spectrum of a nearby galaxy observed by the SDSS, and a spectrum of a similar galaxy as it would appear at rest with respect to us. We can align the spectra by moving the rest-frame spectrum rightwards, shifting each feature to longer wavelengths. Can you find the  $4861\text{\AA}$   $H\beta$  line, the  $5173\text{\AA}$  magnesium (Mg) line, and the  $5893\text{\AA}$  sodium (Na) line in the observed spectrum? What is the approximate redshift of this galaxy?

## 7.3 Constructing Hubble diagrams

We will now examine Hubble diagrams based on distances to galaxies drawn from Cepheid variable stars and from Type Ia supernovae, and recessional velocities measured from galaxy spectra. As you examine the relationship between distance and velocity, think about the quality of the fit to the data points.

If all of the data points were to fall exactly along the best-fit line, what would that mean? It would suggest that the observational data were of extremely high quality, and also that the model represented by the line (see equation 7.1) encapsulated all of the relevant physics governing the relationship between galaxy distances and velocities. If the data points appeared to be scattered randomly, what would that mean? It would suggest that the observational data were corrupted (damaged) or of vastly substandard quality, and/or that the hypothesis behind our physical model was not correct.

As with many things in life, the reality of the situation lies between the extremes. We observe a certain amount of scatter within the distance–velocity relationship, but the underlying connection is clearly present. A correlation coefficient ( $R$ , varying between 100% if  $y$  can be completely predicted from  $x$  and 0% for no connection) can help us to quantify the level of agreement between the plotted variables.

What are the primary causes of the scatter (noise) in these Hubble diagrams? First, the difficulty in determining accurate distances (and velocities) for these galaxies can produce data-driven offsets from an underlying theoretical relation. Second, there are additional factors that can affect the relationship between distance and velocity, factors not included in the Hubble Law. Some neighboring galaxies exhibit *negative* rather than positive velocities, because at short distances the attractive force of gravity can overwhelm the universal force of expansion. Galaxies within clusters can also have anomalous velocities because they dive into the cluster cores at high speeds relative to their nearest neighbors, drawn by the massive bulk of the cluster itself. Third, at large distances (beyond 400 Mpc) the relationship between velocity and distance becomes more complicated, due to cosmological factors related to the curvature of the Universe (space is not completely flat on large size scales).

### 7.3.1 A Hubble Diagram for Nearby Galaxies with Cepheids

Let's work through an example concerning Cepheid variable star data, to better understand how to derive distances to these types of stars.

#### Example 7.4

In Examples 7.1 through 7.3 we discussed how to determine redshifts for galaxies based on spectral features. We can combine such velocities with distances derived for Cepheid variable stars within the same galaxies to create Hubble diagrams. Figure 7.4 shows six successive images of a single Cepheid star within the galaxy M100, observed repeatedly over a two-month period with the Hubble Space Telescope. Note how the star dims and then brightens over time. It appeared rather bright on April 23, fainter on May 4, and was barely visible on May 9. By May 16 it had begun to brighten again, in a pattern that repeats every 53 days. Twenty such stars were observed within this galaxy, and a distance was derived by averaging the results of analyzing each star's light curve (see Figure 7.5).

Because Cepheid variable stars which are intrinsically brighter (which emit more light) have longer periods of variability, we can determine how far such a star lies from Earth by comparing its observed flux with that predicted from various distances. We simply shift the star back and forth across the Universe until its observed flux matches the light level predicted for a star with its period of variability at that distance. The longer the period, the more light the star emits, and the further it must lie from Earth.

Figure 7.6 combines distances from Cepheid stars observed within 23 galaxies, plotting them against recessional velocities derived from the galaxy spectra. The most distant galaxies in the sample have spectral lines which are redshifted the farthest to longer (redder) wavelengths. Because Cepheid stars in all of the galaxies have the same properties, we can use them to determine distances in a uniform fashion.

1. Complete the following statements after studying Figure 7.6. (3 points)

(a) The galaxy M100 is one of the most ( nearby / distant ) galaxies for which Cepheid-derived distances are shown.

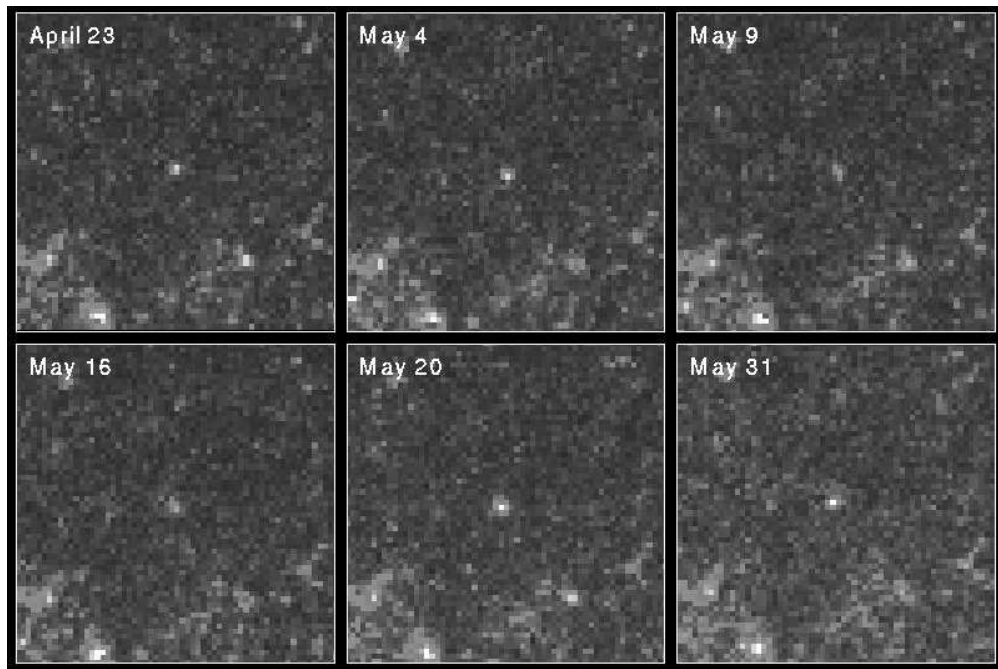


Figure 7.4: Six images of a variable star in the galaxy M100, taken by the Hubble Space Telescope over a two-month period in 1994. Observe how the variable star, centered in each frame, grows fainter and then brighter over time.

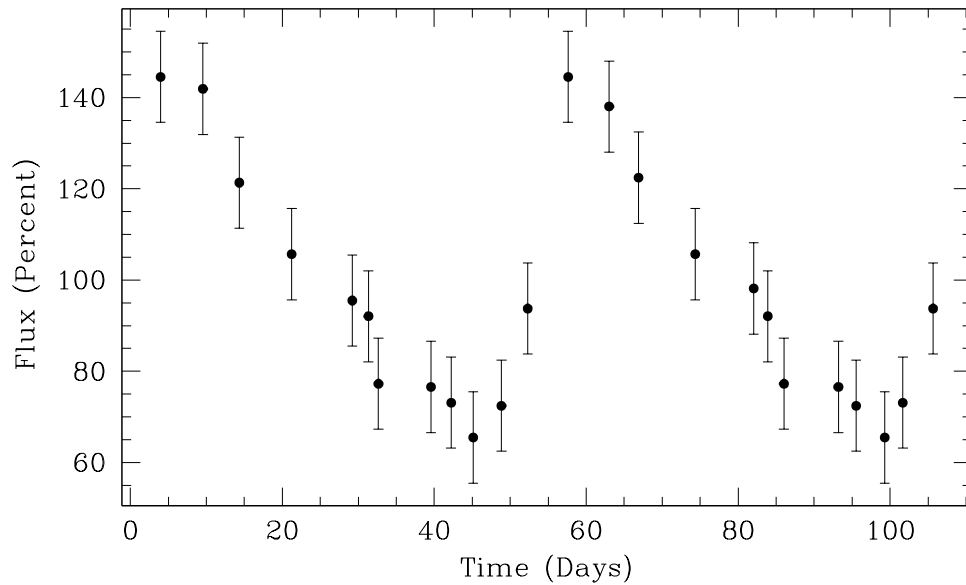


Figure 7.5: The light curve for a variable star in the galaxy M100, showing the change in flux over time. A flux value of 100% corresponds to the average brightness. The star doubles in brightness during each periodic variation and then decays again, taking 53 days to complete the pattern. Intrinsically brighter stars have longer periods of variability.

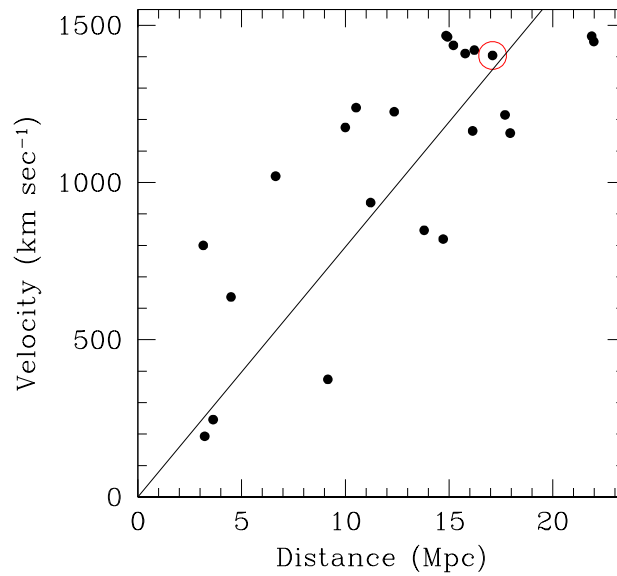


Figure 7.6: A Hubble diagram showing the relationship between galaxy distance and recessional velocity, where distances were derived from the period-luminosity relationship for Cepheid variable stars identified within each galaxy. The M100 point is circled in red, and based in part on the data shown in Figures 7.4 and 7.5. The slope of the relationship is  $79 \pm 4 \text{ km sec}^{-1}$  per Mpc, and the correlation coefficient  $R$  is 78%, indicating that there is a strong positive relationship between velocity and distance.

(b) A galaxy lying 10 Mpc away from us should have a recessional velocity of roughly \_\_\_\_\_ . (Please attach units to your value.)

(c) The modern value for  $H_0$  derived here is much ( higher / lower ), by a factor of roughly \_\_\_\_\_ , than that found in Hubble's original data (shown in Figure 7.1).



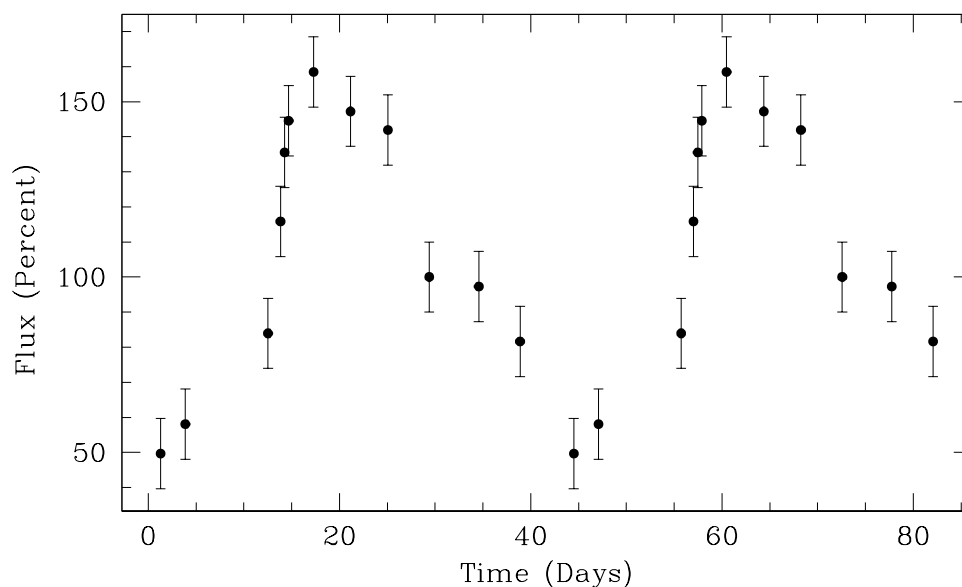


Figure 7.7: The light curve for a second variable star in the galaxy M100. What is the period of variability for this star? Is it intrinsically brighter or fainter than the star shown in Figure 7.5?

2. Figure 7.7 shows the light curve for a second Cepheid variable star in the M100 galaxy. (2 points)

- The period of variability for this star is \_\_\_\_\_ (to the nearest day or two).
- This star is intrinsically ( brighter / fainter ) than the one shown in Figure 7.5.

### 7.3.2 A Hubble Diagram for More Distant Galaxies with Type Ia Supernovae

Variable stars are too faint to be observed beyond distances of 30 Mpc, but supernovae explosions can be detected out to 1,700 Mpc (over a considerable fraction of the known Universe). Figure 7.8 is a Hubble diagram constructed from galaxy distances based on individual supernovae events. When we compare it to Figure 7.6, we notice immediately that it extends out more than ten times further in distance and has a much higher correlation coefficient (the relationship between  $d$  and  $v$  is less noisy). Given the increased range (supernovae can be detected from much farther away, because they are so much brighter) and higher correlation coefficient, why don't we just use the supernovae technique to find distances to all galaxies?

The key point to consider is that the brighter supernovae are much easier to detect than Cepheid variable stars, *when they are present*. Astronomers have found more than fifty Cepheid variable stars in M100 alone, and they are even easier to find in galaxies which lie closer to the Milky Way. Once a Cepheid star has been identified, we can derive its distance by observing it as few as ten times over a several-month period to determine its period of



variability and apparent brightness. If we later discovered an error in the observations, we could go back at any time to repeat them.

A supernovae explosion occurs once in every hundred years per galaxy, however, and lasts for only a handful of days. They are rare events, and so we have observed very few of them nearby. Imagine the space surrounding the Milky Way galaxy as a series of nested concentric spherical shells, each one larger than the next. Because each successive shell has a larger radius and lies farther from the Milky Way, it contains a larger volume and holds more galaxies. The odds of our detecting a supernovae in our lifetimes actually increase if we look further away from the Milky Way, even though the supernovae become fainter, just because there are more galaxies available at larger distances to host them.

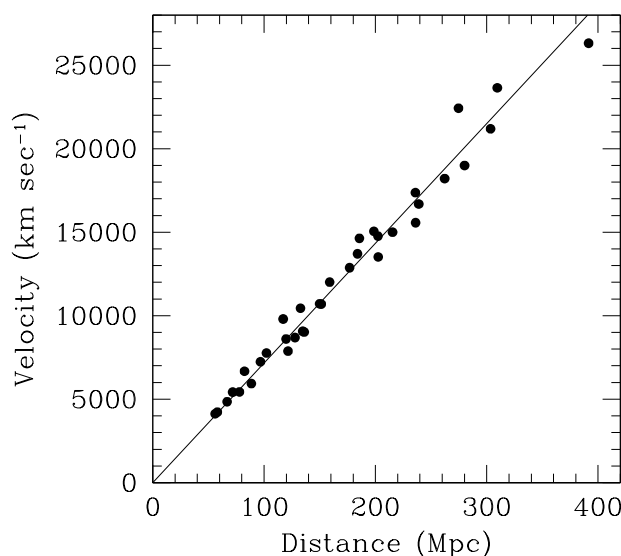


Figure 7.8: A Hubble diagram showing the relationship between galaxy distance and recession velocity, where distances were derived from the observed peak luminosities for supernovae observed within each galaxy. The slope of the relationship is  $71.7 \pm 0.8$  km sec<sup>-1</sup> per Mpc and the correlation coefficient  $R$  is 99%, indicating that there is a very strong positive relationship between velocity and distance.

1. What is the redshift  $z$  for the most distant galaxy shown in Figure 7.8? (1 point)
2. Figures 7.6 and 7.8 illustrate the process of deriving values for the Hubble constant ( $H_0$ ) from samples of galaxies containing Cepheid variable stars and supernovae. In each case, is the derived value for  $H_0$  consistent with the accepted value of 72 km sec<sup>-1</sup> per Mpc? (Are the differences more or less than  $2\sigma$ ?) Please show your work. (3 points)

3. How many galaxies appear in Figure 7.6 and also appear in Figure 7.8? Why might even a small number of galaxies in which we had detected both Cepheid variable stars and a supernovae be of particular interest? (Hint: How do we know that these distance estimation techniques are accurate?) (1 point)

4. How many supernovae could have been seen in the nearest 10 galaxies since the telescope was invented in 1600? How many could have been observed with modern telescopes and recording devices (over the last 30 years)? (3 points)

5. There is an unwritten rule of courtesy among astronomers that if a supernovae occurs nearby, anyone with time on a major telescope in the next few days observes it and circulates the images worldwide. Why do you think we do this? (1 point)

## 7.4 Brightest Cluster Galaxies as Distance Indicators

We refer to the brightest galaxy in a cluster of hundreds (or even thousands) of galaxies as a BCG. For example, the giant elliptical galaxy known as M87 is the BCG in the nearest large cluster of galaxies, a dense grouping of over 1,300 galaxies known as the Virgo Cluster. You will examine a series of images of BCGs taken by the Sloan Digital Sky Survey (SDSS). The BCG will appear at the center of each image, surrounded by other fuzzy amber-colored galaxies within the same cluster.

We will determine the distances to BCGs three different ways, determining a “D-size” distance based on the angular size of each galaxy, a “D-flux” distance based on the measured brightness of each galaxy, and a “D-z” distance based on the galaxy redshift. The following three examples will help us to better understand how each distance is determined.

### Example 7.5

We can use the apparent (observed) sizes of BCGs to determine how far away they are, by assuming that these galaxies are all the same actual size. Galaxies which appear larger must then lie closer to us, and those which appear smaller must lie at greater distances. We say that the relationship between the apparent size  $\theta$  and distance D-size is

$$\theta \propto \frac{1}{\text{D-size}}. \quad (7.8)$$

For nearby galaxies, a galaxy which is twice as far away looks half as big, while one which is half as far away appears twice as large. Let’s assume that all BCGs are 20 kiloparsecs (kpc) in size, 130% the size of the Milky Way. A BCG at a distance of 50 Mpc would then have an angular radius of 82'' (82 arcseconds, where there are 3,600 arcseconds in a degree), while one at 100 Mpc would have a size of 41''.

$$\frac{\text{D-size}}{50} = \frac{82}{\theta} \quad (7.9)$$

where D-size is measured in units of megaparsecs and  $\theta$  in units of arcseconds, so that

$$\text{D-size (Mpc)} = \frac{4,100}{\theta('')}. \quad (7.10)$$

As we examine galaxies at larger distances (above 400 Mpc), the relationship between angular size and distance becomes slightly more complex, because we have to take into effect cosmological factors (space is curved, and the Universe was smaller in the past). These are small corrections, however. Of more importance is the fact that all BCGs are not the exact same size. If a particular galaxy is smaller than 20 kpc in size, for example, we will overestimate the distance to it with this technique.

### Example 7.6

We can also use the apparent (observed) brightnesses of BCGs to determine how far away they are, by assuming that these galaxies all emit the same amount of light. Galaxies which appear brighter must then lie closer to us, and those which appear fainter must lie at greater

distances. We say that the relationship between the apparent brightness  $f$  and distance D-flux is

$$f \propto \frac{1}{\text{D-flux}^2}, \text{ or } \sqrt{f} = \frac{1}{\text{D-flux}}. \quad (7.11)$$

For nearby galaxies, a galaxy which is twice as far away looks four times fainter, while one which is half as far away appears one-fourth as bright. Let's assume that all BCGs are three times brighter than the Milky Way. A BCG at a distance of 50 Mpc would then be observed to add 2,120,000 counts to our calibrated SDSS images, while one at 100 Mpc would add only 530,000 counts. (Each "count" in the calibrated images corresponds to a number of photons detected at the telescope.)

$$\frac{\text{D-flux}}{50} = \sqrt{\frac{2,120,000}{f}} \quad (7.12)$$

where D-flux is measured in units of megaparsecs and  $f$  in units of counts, so that

$$\text{D-flux (Mpc)} = \sqrt{\frac{5.30 \times 10^9}{f(\text{counts})}}. \quad (7.13)$$

As we examine galaxies at larger distances, the relationship between flux and distance also becomes slightly more complex due to cosmological effects, but these are still small corrections. The fact that all BCGs are not equally bright is much more important. If a particular galaxy gives off more light than expected, for example, we will underestimate the distance to it with this technique.

#### Example 7.7

We can use the observed redshifts of BCGs to determine how far away they are, by assuming that the Universe is expanding. Galaxies with smaller redshifts must then lie closer to us, and those with larger redshifts must lie at greater distances. We say that the relationship between the redshift  $z$  and distance D-z is

$$z \propto \text{D-z}. \quad (7.14)$$

For nearby galaxies, a galaxy which is twice as far away has twice as large a redshift, while one which is half as far away has half as large a redshift. The critical difference between this technique and those for D-size and D-flux, however, is that we don't need to make any assumptions about the physical properties of the galaxy in order to determine its distance from its redshift. From Hubble's Law, we know that

$$v = H_0 \times \text{D-z}, \text{ and as } z = \frac{v}{c}, z = \frac{H_0}{c} \times \text{D-z}. \quad (7.15)$$

A BCG at a distance of 50 Mpc would then have a redshift of 0.012, while one at 100 Mpc would have a redshift of 0.024.

$$\frac{\text{D-z}}{50} = \frac{z}{0.012} \quad (7.16)$$

where  $D_z$  is measured in units of megaparsecs and  $z$  is dimensionless, so that

$$D_z \text{ (Mpc)} = \frac{c}{H_0} z = 4,200 z. \quad (7.17)$$

As we examine galaxies at larger distances ( $z > 0.10$ , beyond 400 Mpc), the relationship between redshift and distance becomes slightly more complex, because we again have to factor in cosmological effects.

### 7.4.1 Finding Distances to Brightest Cluster Galaxies

We are going to estimate the distances to 34 BCGs, using images and spectra from the Sloan Digital Sky Survey (SDSS). You will estimate  $D_{\text{size}}$ ,  $D_{\text{flux}}$ , and  $D_z$  for ten galaxies, and then combine your results with measurements that we have already made for another 24 galaxies.

#### Measuring $D_{\text{size}}$ and $D_{\text{flux}}$ from Galaxy Images

Reload the GEAS project lab exercise web page (see the URL on page 156 in §7.1.2), and click on the link for this exercise labeled “Web application #1 (galaxy distances I).”

We will now measure the angular sizes of our BCGs, by fitting an ellipse of appropriate size and orientation to each one of them. As we do so, we’ll also measure the amount of light contained within this elliptical aperture to see how bright they are. We’ll assume that each one of the galaxies has a radius of 20 kpc and would produce 2,120,000 counts in our images at a distance of 50 Mpc, and calculate distances accordingly. (This calculation will differ slightly from the simplified models presented in Examples 7.6 through 7.7 to take in to account cosmological effects, but the basic idea will be the same.)

#### Imaging Tool Tips

The image tool interface contains five primary panels, as well as a set of key options across the top of the screen. Start by clicking on the button labeled “Help” to learn about the basic properties of the tool. The three top panels all have to do with the image of the galaxy. The left panel contains the controls to adjust the position, size, and orientation of the green ellipse on top of the galaxy image shown in the middle panel. Each image is 100'' arcseconds wide, so if a particular central BCG pictured appears smaller than the others that means that it would appear smaller on the sky as well (the images are all the same size). The right panel shows the distribution of light within the ellipse. It is split in half along its shortest dimension, called the minor axis, so if you have centered it properly on the galaxy the radial plot should appear symmetric about the point  $x = 0$ .

The bottom two panels contain a table of galaxy properties, including the position of the ellipse on the image, its position angle ( $0^\circ$  for a tall ellipse and  $90^\circ$  for a fat one), its axial ratio (0 for a skinny ellipse, 1 for a circular one), the length of the semi-major axis (the longest radius you can fit into the ellipse),  $D_{\text{size}}$  based on the semi-major axis, the amount

of light contained within the ellipse, and D-flux based on the light. On the right we have the spectrum of each galaxy. As you work with each image, examine the spectrum as well to see if there are any correlations – can you make any predictions about the spectra based on the images, or vice versa?

1. Once in a great while, a network error will interfere with the loading of one of the galaxy images in this tool. A warning message will appear when you first load the tool if this occurs. *If this happens to you, force your browser to reload the application (most will do so if you hold down the control key or the shift key while reloading the page).* If you do not do this, then at least one of your radial plots will not contain any data and you will not be able to complete this exercise.

2. Examine the presence of the green ellipse on top of the image of Galaxy #1. We can vary its eccentricity by changing the axial ratio in the Control Panel, and rotate it in place by changing the position angle. The four arrows shift the ellipse left or right and up or down, and the two buttons to their right make the ellipse grow bigger or smaller. Try changing all of the setting in the Control Panel until you are confident that you understand how they work.

3. Select “hide ellipse” from the list of options at the top of the screen and then “zoom in” to inspect the galaxy image more closely. Then select “show ellipse” to draw the elliptical aperture again.

4. Decide whether the galaxy is more oval or more round, and set the axial ratio for the ellipse accordingly (0 for very elongated, 1 for round). Identify the longest axis of the galaxy, and rotate the ellipse to match by varying its position angle.

5. Use the four position arrows to move the aperture around so that its center coincides with the center of the galaxy (the bright core).

6. Use the two remaining arrows on the Control Panel to increase or decrease the size of the ellipse. Your goal is to have the aperture contain all of the light from the galaxy, including its faint (nearly-invisible) outer regions, and a minimum of light from foreground Milky Way stars, other galaxies, and the background sky.

7. Use the radial distribution of counts plot to finalize the aperture. Make the aperture large enough that the counts are distributed symmetrically on either side of the central position (where  $x = 0$ ). Set the size so that the boundaries (shown as two green vertical lines on the counts plot) are placed where the number of counts starts to rise noticeably upward relative to the background level. You may sometimes need to decrease the aperture size to keep a foreground star or another galaxy outside of the ellipse; sometime you can avoid doing this by instead changing its position angle.

8. Steps 6 and 7 above involve making difficult decisions, and compromising your desire to include all of the galaxy flux with your need to include as little as possible of the light from other stars and galaxies. The key is to be consistent in your approach and to treat all ten

galaxies uniformly. Try to place the ellipse to mark the same brightness level on all images, as best you can. For some galaxies there will be no “perfect” solution; we have given you some difficult cases to consider to illustrate the type of decisions that are often made when dealing with real astronomical data. Read through the following questions before you fit the galaxy images, and keep them in mind as you work. When finished, you’ll have two different distance determinations (D-size and D-flux) for all ten galaxies, based on their apparent sizes and apparent brightness.

1. Save a copy of the final galaxy data table and include it in your lab report. (7 points)
2. Save a copy of the image for Galaxy #10 and include it in your lab report. (1 point)
3. As you decrease the size of the elliptical aperture on a galaxy image, D-size ( increases / decreases / remains constant ) and D-flux ( increases / decreases / remains constant ). (1 point)
4. If the size of a galaxy is over-estimated (the aperture is larger than the galaxy), D-size will be ( too large / too small ). (1 point)
5. If the size of a galaxy is under-estimated (the aperture is smaller than the galaxy), D-flux will be ( too large / too small ). (1 point)
6. Most of the light from the galaxies is uniformly distributed in smooth ellipses around the cores. Why is there a second peak in the radial counts plot for Galaxy #6? (1 point)
7. Which of the ten galaxies posed the largest challenges to fit? Which was the easiest to analyze? Explain your choices. (2 points)

### Measuring D-z from Galaxy Spectra

Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled

“Web application #2 (galaxy distances II).”

We will now measure the redshifts of our BCGs, by taking a rest-frame spectrum of a typical BCG (the spectrum as it would appear for a galaxy at rest with respect to the Milky Way) and redshifting it ourselves to match the spectrum of each of our ten BCGs. (The calculation of D-z will differ slightly from the simplified model presented in Example 7.8, to take in to account cosmological effects, but the basic idea will be the same.)

### Spectral Tool Tips

The spectral tool interface contains four primary panels, as well as a set of key options across the top of the screen. Start by clicking on the button labeled “Help” to learn about the basic properties of the tool. The largest panel (on the left) shows the spectrum of each galaxy (in pale green) and the rest-frame spectrum of a typical BCG (in pale blue). The controls at the bottom of the panel allow you to shift the rest-frame spectrum left and right (by defining a redshift for it) until it matches up with the observed spectrum.

Use the slider bar to set an approximate value, and then “touch up” the redshift with the more delicate button controls on either side. Note that several strong emission and absorption features have been marked by element in the rest-frame spectrum (Fe for iron, Ca for calcium,  $H\alpha$  through  $H\delta$  for hydrogen, Mg for magnesium, and Na for sodium). If these features are strong, then these elements are present in the galaxy.

You can position the rest-frame spectrum two ways: either pay attention to the broad, underlying shape of the continuum (the overall shape of the spectrum), or select the deepest absorption lines in the two spectra and match them up. To fine-tune your redshift value, use the outermost redshift button controls to make small shifts until the correlation coefficient ( $R$ ) is maximized. When this value, printed next to the redshift on the bottom of the panel, is as close to 100% as possible, you will have found the best redshift for the galaxy. The second panel (from left to right) is a visual representation of the correlation coefficient – try to raise the value of the “correlation thermometer” all the way (to 100%) in each case.

A data table on the right records the galaxy redshifts and derived distances D-z, as well as the final correlation coefficients. In the lower right corner you will find an image of each galaxy. Keep an eye on the images as you fit the spectra, and try to identify trends that you see between the two types of data.

Read through the following questions before you fit the galaxy spectra, and keep them in mind as you work. When finished, you’ll have a third distance determination (D-z) for all ten galaxies, based on their redshifts.

1. Save a copy of the final galaxy data table and include it in your lab report. (7 points)
2. Save a copy of the spectrum for Galaxy #9 and include it in your lab report. Are you more confident in your estimates of D-z or of D-size and D-flux for this galaxy? Why? (2 points)



3. As you increase the redshift for a galaxy spectrum, D-z ( increases / decreases / remains constant ). (1 point)
4. More distant galaxies appear ( smaller / larger ) on the images, and their spectra shift to the ( left / right ) to ( shorter / longer ) wavelengths. (1 point)
5. Which of the ten galaxies posed the largest challenges in determining redshifts? Which was the easiest to fit? Explain your choices. (2 points)

## 7.5 How Reliable Are Your Distance Determinations?

Which of the three distance estimates for BCGs, D-size based on angular size, D-flux based on apparent brightness, and D-z based on redshift, is the most accurate? Review Examples 7.5–7.7 on pages 167–168, and complete the questions below to explore this topic.

### 7.5.1 Evaluating Distances Based on Apparent Sizes of BCGs

As Example 7.5 explains, to determine the distances to BCGs from their angular sizes (their apparent sizes on the sky) we must assume that they are all the same actual size. (If viewed from the same distance, they would all appear equally large.) Is this a good assumption? Figure 7.9 shows the distribution of galaxy sizes throughout the sample.

1. Two-thirds of the BCGs will lie within  $1\sigma$  of the mean size, between \_\_\_\_\_ kpc and \_\_\_\_\_ kpc. For the remaining third, our D-size distance estimates would be more than 30% too low or more than 66% too high (see Example 7.5). (1 point)

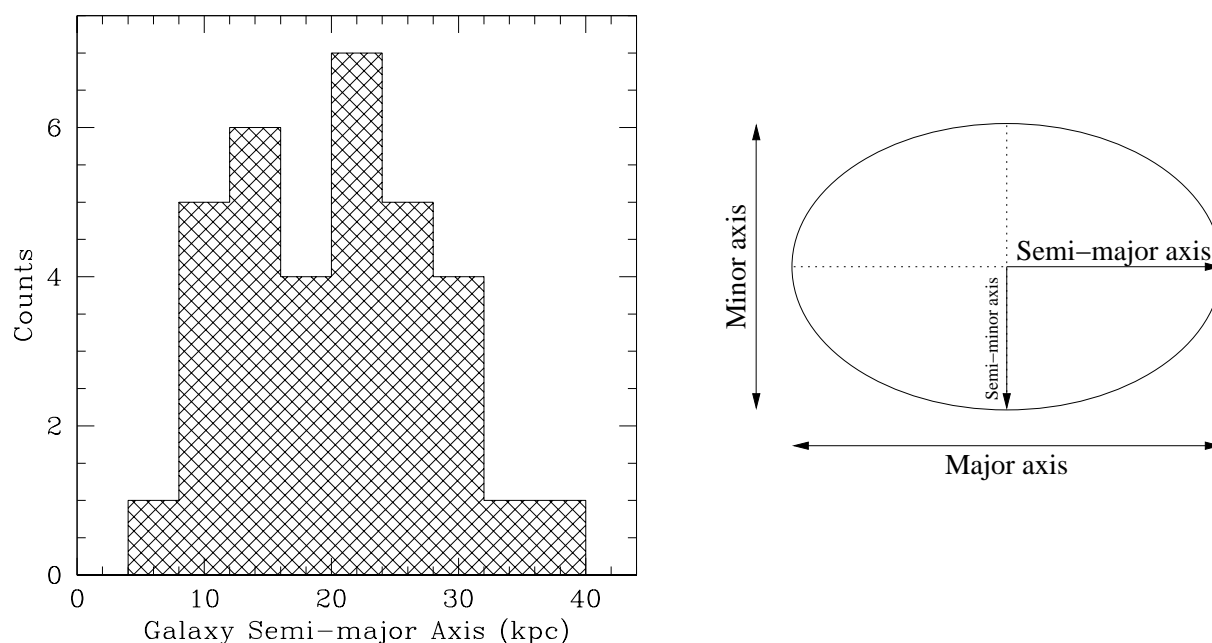


Figure 7.9: This histogram (left) shows the distribution of physical sizes within our sample of 34 BCGs. Semi-major axes are plotted in units of kiloparsecs (kpc), with an average value of  $20 \pm 8$  kpc. The figure on the right illustrates the definition of the semi-major axis for an ellipse (the largest radial length you can fit inside it).

### 7.5.2 Evaluating Distances Based on Apparent Brightnesses of BCGs

As Example 7.6 explains, to determine the distances to BCGs from their apparent brightnesses (how bright they appear from Earth) we must assume that they all emit the same amount of light. (If viewed from the same distance, they would all appear equally bright.) Is this a good assumption? Figure 7.10 shows the distribution of galaxy luminosities throughout the sample.

1. Two-thirds of the BCGs will lie within  $1\sigma$  of the mean luminosity, between \_\_\_\_\_ and \_\_\_\_\_ times as bright as the Milky Way. For the remaining third, D-flux would be more than 16% too low or more than 33% too high (see Example 7.6). (1 point)
2. ( D-size / D-flux ) is more accurate, on average. (1 point)

### 7.5.3 Comparing Distance Estimators

We have estimated the distance to each BCG in three ways, from their sizes, their fluxes, and their redshifts. How do these techniques compare? To compare all three methods, we will plot each type of distance estimate against a “gold standard” based on measurements

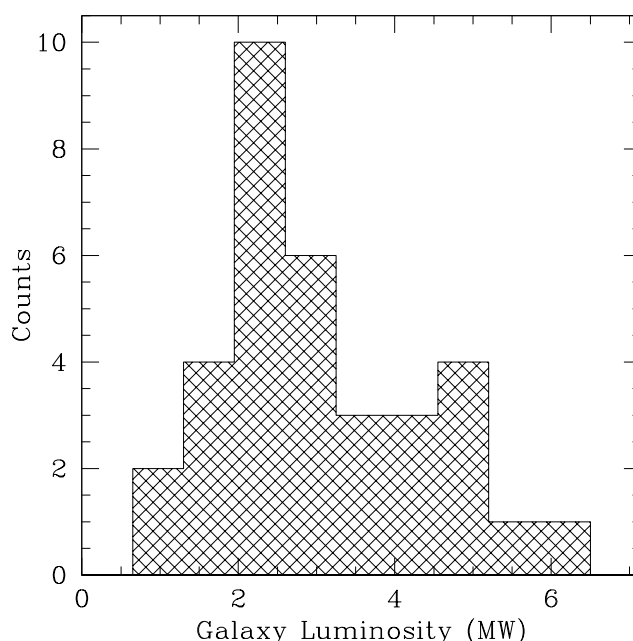


Figure 7.10: This histogram shows the distribution of luminosities within our sample of 34 BCGs. A BCG with a luminosity of unity would emit as much light as the Milky Way galaxy. The average value is  $3.0 \pm 1.3$ , relative to the Milky Way.

of the distances to each galaxy cluster. Each of these measurements are based on multiple galaxies within each cluster, so they can leverage multiple measuring techniques and benefit from combining data taken for all of the galaxies within each cluster.

Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled “Web application #3 (distances plotting tool).” Enter your distance measurements for each BCG in the box labeled “Distances to plot: D-size, D-flux, and D-z,” placing three measurements on a line for each of your ten BCGs.

Once you have done so you will create a plot comparing D-size, D-flux, and D-z in turn against the distance to each cluster. Each plot window will contain points for 34 BCGs, the ten that you fit and 24 more that we analyzed using the same techniques. Your data will be shown as large magenta points, while the other 24 BCGs will appear as smaller red points.

Let’s examine these plots, and see how well our distance estimators worked.

1. Be sure to save a JPG- or PNG-format copy of the plot to your computer, to include in your lab report. (7 points)
2. Do the ten points from your data set appear to follow the same trend as the 24 additional points? If not, how do they differ, and why do you think this might have occurred? (1 point)

3. If all of the distance estimators were “perfect”, what values would you expect for a slope and a  $y$ -intercept for each window? (1 point)
  
4. The two distance estimators ( D-size / D-flux / D-z ) had the worst rms deviations (highest  $\sigma$  values) when plotted against the cluster distances. (1 point)
  
5. Distance estimator ( D-size / D-flux / D-z ) had the best rms deviation when plotted against the cluster distances. Do the points in this plot also appear to follow the best-fit line most closely, just looking by eye? ( yes / no ) (1 point)
  
6. Why do you think that this technique produced the best results? (2 points)

### 7.5.4 The Range of BCG Properties

Figures 7.9 and 7.10 illustrate the fact the BCGs exist over a significant range of sizes and luminosities. By assuming that they were all the same size and emitted the same amount of light, we were able to make only rough approximations of their distances. Let’s now plot size against luminosity in Figure 7.11, and learn a bit more about these galaxies.

1. Is the most luminous BCG also the largest one? ( yes / no ) (1 point)
  
2. Is the least luminous BCG also the smallest one? ( yes / no ) (1 point)
  
3. What percentage of the 34 galaxies are either brighter and smaller than average, or fainter and larger than average? (2 points)
  
4. In general, BCGs which are intrinsically brighter than average are also ( larger / smaller ) than average, while those which are intrinsically fainter than average are also ( larger / smaller ) than average. Explain this result, assuming that the luminosity of a galaxy is just the total amount of light emitted by all of its stars. (2 points)

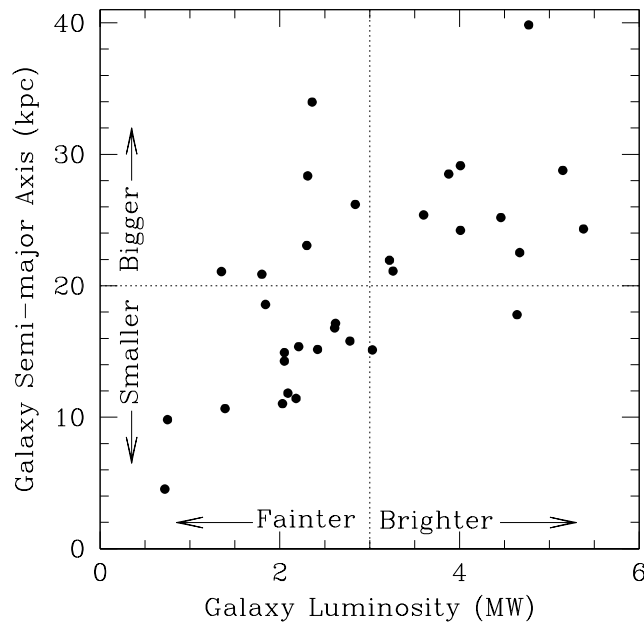


Figure 7.11: The distribution of physical sizes and luminosities within our sample of 34 BCGs, with the same units as those used in Figures 7.9 and 7.10. The horizontal dotted line indicates the semi-major axis of the average BCG (20 kpc), so galaxies above this line are larger than average and galaxies below this line are smaller than average. The vertical dotted line indicates the luminosity of the average BCG (three times that of the Milky Way), so galaxies to the right of this line are brighter than average and galaxies to the left of this line are fainter than average.

## 7.6 Telescopes Are Time Machines

We’ve been working with observational data that support one of the greatest discoveries of all time, the fact that the Universe is expanding outward. Galaxies are moving further and further away from us with every passing second, and also moving further and further away from each other. Not only are most galaxies rushing away from us – as shown by all those redshifted spectra – those farther away are moving proportionally faster.

### Example 7.8

Though you may not have realized it, as you worked with images and spectra of distant galaxies you were traveling through time. It takes light a whole second to travel just 300,000 kilometers. When we examine the light emitted by distant galaxies which lie hundreds or thousands of megaparsecs away from us, we see them not as they are today, but as they were billions of years in the past.

The light emitted by a galaxy with a redshift of 0.10 was emitted more than a billion years ago (1.1 billion years), and the photons from one at a redshift of 0.30 left their home more than three (3.3) billion years in the past. Imagine an astronomy student living in a galaxy with a redshift of 0.40 examining images of our own Milky Way galaxy – from their point of

view, our solar system would still be in the throes of formation, and life on Earth would be an unknown possibility of the far-flung future!

## 7.7 Final (Post-Lab) Questions

1. In the 1930–1950 era the Andromeda Galaxy was thought to lie a mere 750,000 light years away, but this distance was increased to two million light years in later years. What happened? The Andromeda Galaxy did not leap and skip across the cosmos! The calibration for the Cepheid period-luminosity relationship improved, however, changing all distances which depended on it. Similar corrections occurred in the 1970–2000 era, producing large changes in cosmologically-based distances determined from measured redshifts, as the value of the important constant known as \_\_\_\_\_ also became more accurate. (1 point)
2. How do the BCGs differ qualitatively and quantitatively from our Milky Way galaxy? (Consider galaxy sizes, luminosities, colors, types, and environments.) (2 points)
3. What is the redshift of the most distant BCG that you examined? Roughly how long ago was the light captured in the image that you analyzed emitted by the galaxy? (2 points)
4. If it takes light a full second to travel 300,000 kilometers, how long does it take to travel one parsec, or even 400 Mpc, in units of years? (A parsec is equal to  $3 \times 10^{13}$  kilometers, and there are  $\pi \times 10^7$  seconds in a year.) (3 points)

5. We can use the Hubble constant  $H_0$  to estimate the age of the Universe. We observe that the Universe is expanding; clusters of galaxies are all moving apart from one another. In the past the galaxies were closer together, and if we go back far enough in time we get to a point where this expansion started. We call that event – the start of the expansion of the Universe – the “Big Bang.” As the Universe expands, more distant galaxies are observed to be moving away faster since there is more expanding space between us and them. There is thus a correlation between the distance and the time. Let’s see how that works!

If a friend says that they drove at a velocity  $v$  of 60 miles per hour and covered a distance  $d$  of 120 miles, you instantly know their travel time  $t$  (two hours). Velocity equals distance per unit time, so

$$v = \frac{d}{t}, \quad \text{and } t = \frac{d}{v}. \quad (7.18)$$

In this case,

$$t = \frac{d}{v} = \frac{120 \text{ miles}}{60 \text{ miles per hour}} = \frac{120}{60} \text{ hours} = 2 \text{ hours}. \quad (7.19)$$

We can perform the exact same calculation for galaxies, using the fact that  $H_0 = v/d$ .

$$t = \frac{d}{v} = \frac{1}{H_0}. \quad (7.20)$$

Until now we have expressed the units of  $H_0$  as  $\text{km sec}^{-1}$  per Mpc. We need to cancel the units of length (kilometers and megaparsecs), leaving seconds (a unit of time).

$$t = \frac{d}{v} = \frac{1}{H_0} = \frac{1}{72 \text{ km sec}^{-1} \text{ per Mpc}} = \frac{1 \text{ Mpc-sec}}{72 \text{ km}}. \quad (7.21)$$

(a) One parsec is equal to  $3 \times 10^{13}$  kilometers, so how many kilometers are there in a megaparsec (in  $10^6$  parsecs)? (1 point)

Let’s call this value  $L$ . We can now replace the Mpc in our expression for  $t$  with “ $L$  kilometers”.

$$t = \frac{1 \text{ Mpc-sec}}{72 \text{ km}} = \frac{1}{72} \frac{L \text{ km-sec}}{\text{km}} = \frac{L \cancel{\text{km-sec}}}{72 \cancel{\text{km}}} = \frac{L}{72} \text{ seconds}. \quad (7.22)$$

(b) There are  $\pi \times 10^7$  seconds in a year, so what is the age of the Universe, in units of billions of years? (3 points)

$$t = \frac{L}{72} \text{ seconds} \times \frac{\text{year}}{\pi \times 10^7 \text{ seconds}} = \frac{L}{72\pi \times 10^7} \text{ years}. \quad (7.23)$$

(c) If the Hubble constant  $H_0$  were twice as large, the age of the Universe would be ( twice as large / half as large / unchanged ). (1 point)

## 7.8 Summary

After reviewing this lab's goals (see §7.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. (24 points)

Be sure to cover the following points.

- Describe various methods astronomers employ to obtain distances to celestial objects.
- Define “redshift”, and discuss how it is determined from a galaxy spectrum.
- Explain what Hubble diagrams, Hubble's Law, and the Hubble constant are, how they relate to determining distances to other galaxies, and why they are important.
- Discuss how the distance to the brightest galaxy in a cluster (BCG) can be determined several ways (by assuming that all BCGs are the same size, or that they emit the same amount of energy, or by measuring their redshifts), and how reliable the derived distances are in each case.

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

## 7.9 Extra Credit

The Hubble constant  $H_0$  determines both the size of the Universe and its age. For example, the extent of the observable Universe can be described in terms of the distance to the galaxy with the greatest measured redshift. This redshift tells us how far away the galaxy lies, and how much time has passed since the light that we see from it was emitted (the “look-back” time). Both values depend on  $H_0$ .

Research this topic briefly, and provide the latest numbers associated with the most distant galaxy ever observed and the age of our Universe. Describe how these values would change if the value of  $H_0$  was revised to be greater or smaller. (5 points)



# Appendix I

## Keywords

**Absolute dating** – Absolute dating refers to the process of assigning absolute ages to physical samples (such as rocks) or to geologic events. These ages are usually based on radioactive decay rates.

**Altitude** – The altitude of an object in the sky is the number of degrees which it lies above the horizon. At local noon the Sun could have an altitude of 90 degrees (lying directly overhead), and as it sets the altitude falls to a value of zero.

**Amazonian Period** – The last of three major periods in Martian history, lasting from 1.8 billion years to the present. This era is named for Amazonis Planitia, a low plain in the northern hemisphere. Amazonian surfaces exhibit a varied morphology, but contain relatively few impact craters. The bulk of the historical record of cratering on these surfaces has been erased by lava flows, glacial activity, and even occasional liquid water flows. Much of the northern hemisphere is thought to have been resurfaced during the Amazonian Period.

**Aperture** – A circle, or ellipse, with an open center. Astronomers often place an aperture around a single star or galaxy, and then add up all the light contained within this region in order to determine how bright the object is.

**Arcminute** – An arcminute is a unit of angular size, equal to 1/60 of a degree (recall that there are 90 degrees in a right angle). There are 60 arcseconds in an arcminute (see below).

**Arcsecond** – An arcsecond is a unit of angular size, equal to 1/60 of an arcminute or 1/3600 of a degree (recall that there are 90 degrees in a right angle). Astronomers often measure the angular separation between neighboring objects on the sky in units of arcseconds.

**Astronomical unit** – The average distance between the Earth and the Sun, equal to  $1.5 \times 10^8$  kilometers.

**Asymmetry Index (AI)** - The percentage variation in the difference between the flux emitted at one position within a galaxy and at a corresponding position located  $180^\circ$  around the galaxy center-point. This index tends to be quite low for elliptical galaxies because they have smooth, symmetric light profiles, higher for spiral galaxies due to the spiral arm structure in the disks, and higher still for interacting galaxies, which can appear quite distorted.

BCG – The brightest cluster galaxy (BCG) is the brightest galaxy within a cluster of galaxies. It typically resides in the core of the cluster. BCGs are known for having moderately uniform properties (such as linear size and luminosity).

Big Dipper – The Big Dipper is made up of the seven brightest stars in the northern hemisphere constellation Ursae Majoris (the Great Bear). It is both large and bright, and so forms a useful landmark in the northern sky. The two stars which form the outer lip of the “dipper” (bowl) shape lie along a line which points toward the North Star.

Billion – Ten raised to the ninth power, or 1,000,000,000.

Blue supergiants – Rare hot, blue stars with very high mass and luminosity that are ten to fifty times the Sun’s size. In the H-R diagram, they occupy a region above the Main Sequence and on the left.

Caldera – A caldera is a circular crater, the relic of a volcanic explosion or the collapse of a volcanic cone. Caldera is also the Spanish word for cauldron, in reference to the basin-shape of the depression.

Cassiopeia – Cassiopeia is a northern hemisphere constellation which looks like a letter “W”, making it easy to identify. It is located opposite to the Big Dipper (on the other side of the North Star).

Celestial equator – The celestial equator is the equatorial band of the celestial sphere (see below). It lies in the same plane as the Earth’s equator.

Celestial sphere – The celestial sphere is an imaginary construct designed to aid in visualizing the positions and movements of objects through the sky. It comprises a spherical surface with an arbitrarily large radius, which is centered at the center of the Earth. It has a celestial equator which lies in the same plane as the Earth’s equator, and celestial north and south poles which extend along the Earth’s rotational axis above and below the Earth’s North and South Poles. Objects in the sky can be projected onto the celestial sphere, and their motions understood in the context of the Earth’s motions around the Sun.

Central peak – A central mountain peak is produced when molten excavated material from an impact blast flows back towards the center of the excavation and creates an uplift. This typically occurs at the center of large craters, those which are 40 kilometers or more in diameter.

Central vent volcano – A volcano with a central vent is constructed as debris and lava are ejected from an upthrust, cylindrical vent, forming a symmetrical structure around it.

Color index – A number used to gauge a star’s color, or relative intensity, at two wavelengths. Often based on the difference between how bright a star appears in two different filters, e.g. B–V for the blue and visual filters.

Concentration Index (CI) – The ratio of the amount of flux emitted by the inner 30% of a galaxy and by the total galaxy. Because elliptical galaxies are centrally concentrated they have higher CI values than spiral galaxies, which generally have more slowly decaying, exponential light profiles across their disks.

Correlation coefficient – The correlation coefficient  $R$  is a measure of the strength of the relationship between two variables  $x$  and  $y$ . It ranges from -1 to 1, where +1 indicates the strongest possible positive correlation (as  $x$  increases, so does  $y$ ), zero indicates no predictive relationship between quantities, and -1 indicates the strongest possible negative correlation (as  $x$  increases,  $y$  decreases). Correlation coefficients are well-suited for determining zero-point offsets in periodic relationships (such as syncing sine waves to remove phase offsets).

Cosmology – Cosmology is the study of the structure and the evolution of the entire Universe.

Crater – A crater is a generally circular surface depression, caused by an impact or an explosion, a volcano, or a geyser.

Crystalline rock – Crystalline, or igneous, rock forms as molten rock cools and crystallizes. It is generally harder and denser than sedimentary rock. Granite is an example of crystalline rock.

Culmination – See transit.

Data set – A data set is a collection of measurements made within an experiment.

Declination – The declination of an astronomical body is its height (in degrees) above the plane defined by the Earth's Equator. It runs from  $90^\circ$  (due north) to  $-90^\circ$  (due south).

Degree – A unit used to measure angles. There are 90 degrees in a right angle, and 360 degrees in a full circle.

$\Delta\lambda$  – The shift in wavelength for an absorption or emission feature in a spectrum between its observed ( $\lambda_{obs}$ ) and its rest-frame ( $\lambda_{rest}$ ) wavelengths.

Density – The density of a three-dimensional object is equal to its mass divided by its volume (or the mass per unit volume), and is typically measured in units of grams per cubic centimeter, or kilograms per cubic meter. The density of water is one  $\text{g cm}^{-3}$ , while rock is roughly three times more dense, and steel has a density of eight  $\text{g cm}^{-3}$ . One could also refer to the surface density of features found on a planetary surface, for example, with units of counts (the number of features) per unit area.

Distance Scale – The distance scale, or the cosmic distance ladder, is the combination of various techniques used to determine distances to cosmological objects such as stars and galaxies.

Ecliptic plane – The plane in which the Earth orbits about the Sun (inclined by  $23^\circ$  to the plane containing the Earth's equator).

Ejecta – Ejecta refers to a blanket of material surrounding a crater that was excavated during an impact event. The ejecta will become thinner at increasing distances from a crater.

Equator – The Equator is the area on the surface of the Earth within the plane which is perpendicular to the rotation axis (running through the North and South Poles).

Error bar – An error bar is a symbol attached to a point on a plot, which shows the associated error (how much the point have might shifted in position due the way in which it was measured). It often resembles a small bar (or line) placed on one side or another of the point value.

Flux - The flux from a celestial object is the amount of emitted light that is observed, by eye or through a telescope, at a certain distance.

Galaxy – A galaxy is a gravitationally bound set of stars, gas, and dust, spanning up to hundreds of kiloparsecs in size and containing thousands to billions of stars. Our galaxy is called the Milky Way.

Galaxy Cluster – A galaxy cluster contains hundreds or thousands of galaxies, all bound together by their combined gravitational attraction.

Gibbous – A moon or planet in the gibbous phase appears more than half, but less than fully, illuminated.

Hertzsprung-Russell Diagram (H-R Diagram) – A plot of intrinsic brightness (luminosity or absolute magnitude) versus color index (or the analogous surface temperature or spectral class) for stars, used to study stellar evolution for stars of various types and for clusters of stars.

Hesperian Period – The second of three major periods in Martian history, lasting from 3.5 till 1.8 billion years ago. This era is named for Hesperia Planum, an elevated plain in the southern hemisphere. The largest volcano on Mars, Olympus Mons, was active during this time. Surfaces dating back to the Hesperian lack the pattern of large, densely packed craters characteristic of the older Noachian era, as many were erased by intense volcanic activity. Large bodies of water, and catastrophic releases of water, were also thought to be common, and carved channels in regions like the Chryse Palitia basin. The southern hemisphere contains many regions thought to date back to the Hesperian Period.

Histogram – A histogram is a plot which shows the number of measurements of a particular quantity which fall within bins defined to extend over the range of measured values. The bin size should be selected so that the bins with the largest number of measurements within them hold a statistically meaningful number of measurements, and should also not be smaller than the precision (the resolution) of the measurements.

Horizon – The horizon is the boundary observed between the Earth and the sky. It extends in all directions (north, south, east, and west) around an observer.

Hubble constant – The Hubble constant,  $H_0$ , is the slope of the relationship between recessional velocity  $v$  and distance  $d$  observed for nearby galaxies (those within 400 megaparsecs of the Milky Way galaxy). The current accepted value for  $H_0$  is  $72 \text{ km sec}^{-1}$  per megaparsec.

Hubble diagram – A Hubble diagram is a plot of recessional velocity  $v$  versus distance  $d$  for nearby galaxies.

Hubble's Law – Hubble's Law is the relationship  $v = H_0 d$ , observed between recessional velocity  $v$  and distance  $d$  for nearby galaxies.

Impact crater – An impact crater is one produced by the collision of an object with a planetary body's surface.

Intrinsic – Inherent, or natural. Astronomers distinguish between the intrinsic brightness of a star, or how much energy it radiates, and the apparent brightness, or how bright it appears when observed from Earth. Intrinsic properties of stars are absolute, while apparent properties change depending on how far away the stars are from us.

Inverse square law for light – The observed intensity of light emitted by an object varies inversely with the square of the distance from an observer ( $f \propto 1/d^2$ ). A star or galaxy placed twice as far away from us would thus appear one-fourth as bright.

JPG format – Images are often stored on computer disks in JPG-format files, a format which allows the files to be stored and transferred from computer to computer without loss of information. A JPG-format file should have a file name which ends with the extension “.jpg,” so that the image analysis and display packages can recognize its contents.

Kelvin (K) – A temperature unit, similar to degree Fahrenheit (commonly used in the United States) or Celsius. Astronomers use the Kelvin scale to describe how hot or cool stars are. Stars cooler than 4000 kelvin appear reddish, and those hotter than 7500 kelvin appear bluish. The Sun lies in between these extremes, with a temperature of 5800 kelvin.

Kinetic energy – Kinetic energy is commonly referred to as energy of motion, and is equal to one-half of an object's mass times the square of its velocity.

$\lambda_{obs}$  - The wavelength of an absorption or emission feature in a spectrum at which it is observed to occur within a celestial object moving at some velocity with respect to the observer.

$\lambda_{rest}$  - The wavelength of an absorption or emission feature in a spectrum at which it is observed to occur within a celestial object at rest with respect to the observer.

Latitude – The latitude of a location on Earth is the number of degrees which it lies above the plane of the Equator. It takes on values between  $90^\circ$  (North Pole) to  $-90^\circ$  (South Pole).

Lava – Lava flows are streams of liquid rock, or magma, which reach the surface of a terrestrial body through volcanic eruption.

Law of Cross-Cutting Relations – The law of cross-cutting relations states that any geological feature that cuts across another geological feature must be younger (must have formed later) than the feature it disturbed.

Light curve – A light curve is a plot of the observed brightness of a star (plotted on the vertical  $y$ -axis) as a function of time (plotted on the horizontal  $x$ -axis).

Light year – A unit of distance (not time), equal to the distance which light travels in a year. One light year is equal to 0.307 parsecs.

Linear size – The linear size of an object is its length, in units of length such as centimeters (small) or miles (large).

Little Dipper – The Little Dipper is made up of some of the brightest stars in the northern hemisphere constellation Ursae Minoris (the Little Bear). The North Star, Polaris, located almost due north, is the brightest star in the Little Dipper. It can be found at the end of the handle.

Logarithm – The word logarithm comes from the Greek words for *proportion* and *number*, and means “a number that indicates a ratio.” In the expression  $x = 10^e$ , the exponent  $e$  is the base 10 logarithm of the number  $x$ . When we plot numbers on a logarithmic scale, we can compare data over wide ranges on a single plot. As ones moves along a logarithmic axis by set amounts, one multiplies by a certain factor (rather than adding a certain amount, as is done along a linear axis).

Luminosity – A measure of intrinsic brightness defined by how much energy a star (or other object) radiates into space per second.

Main Sequence – A narrow region running across the H-R diagram, where hydrogen-burning stars are found. As stars grow old and run out of fuel, they evolve away from the Main Sequence.

Magnitude, absolute – The brightness of an object on the logarithmic magnitude scale, as observed from a distance of ten parsecs. This provides a measure of intrinsic brightness.

Magnitude, apparent – The brightness of an object based on the logarithmic magnitude scale, as observed from Earth. Two equivalent stars (with the same absolute magnitude) will have different apparent magnitudes if one lies closer to Earth than the other does.

Magnitude scale – A logarithmic scale for gauging the brightness of astronomical objects. It is based on historical measurements done by eye in which first magnitude stars were the brightest and sixth the faintest, so brighter objects have smaller magnitude values.

Major axis – The major axis of an ellipse is its longest side, the longest line segment which can be placed within it (passing from one side through the center to the other side). It is perpendicular to the minor axis.

**Maria** – The lunar maria are the dark, smooth regions thought to represent ancient lava flows. They are generally younger than the heavily cratered highlands. They were immortalized in Bernstein and Sondheim’s 1956 *West Side Story*.

**Mean value** – The mean value  $\mu$  of a set of  $N$  repeated measurements  $m_i$  is defined to be the unweighted average, or

$$\mu = \frac{1}{N} (m_1 + m_2 + m_3 + \dots + m_N) = \frac{1}{N} \sum_{i=1}^N m_i.$$

**Measurement error** – Measurement error refers to the precision with which a set of measurements were made (to how many decimal places the measured values were recorded).

**Mercator projection** – A Mercator projection is a cylindrical map of a spherical surface, such as the surface of a planet. By convention, west and east run from left to right, with north at the top and south at the bottom. Because lines of constant latitude are spread out across the entire plot, the regions near to the poles are greatly extended in width relative to those at the equator. (This is why Greenland, Iceland, and Antarctica, for example, appear so huge on Mercator projections of the Earth’s surface.)

**Meridian** – A meridian is an arc which projects from the North Pole to the South Pole and passes directly overhead for an observer. All observers located along a given meridian share a common longitude (the distance they lie east of the Royal Greenwich Observatory in England), but have unique longitudes corresponding to how far north or south of the Earth’s equator they lie.

**Meteor** – The term meteor is used to refer to a particle of debris (space dust) which has entered the atmosphere of a planet or satellite. It also refers to the visible path left by such an object.

**Meteoroid** – A meteoroid is a particle of rocky or metallic debris found in space. If a meteoroid enters the Earth’s atmosphere it becomes a meteor, and upon landing any surviving remnant is called a meteorite.

**Million** – Ten raised to the sixth power, or 1,000,000.

**Milky Way** – The Milky Way is the name of our own galaxy, a barred intermediate-type spiral.

**Minor axis** – The minor axis of an ellipse is its shortest side, the shortest line segment which can be placed within it (passing from one side through the center to the other side). It is perpendicular to the major axis.

**Model** – A model fit is a mathematical expression which attempts to reproduce the relationship between two or more variables.

**Morphology** – Shape, or form.



Mu – The Greek letter “μ” ( $\mu$ ), often associated with the average value of a set of measurements.

Natural variation – Natural variation refers to the intrinsic width of a distribution of a measured property.

Network valley – A network valley is one of a set of branching valleys found on Mars, with a resemblance to terrestrial river drainage basins. They are usually less than five kilometers wide, though they may extend for thousands of kilometers in length.

Noachian Period – The first of three major periods in Martian history, dating from formation epoch 4.5 billion years ago to 3.5 billion years ago. This era is named for Noachis Terra, a large southern hemisphere highland. Surfaces dating back to the Noachian are covered with many craters, and exhibit the largest impact craters, but widespread evidence for water erosion suggests that the planetary surface was warm and wet during this time. They are found predominantly in the southern hemisphere.

North Celestial Pole – The North Celestial Pole (NCP) is the projection of the Earth’s North Pole upon the celestial sphere. One can think of it as the extension of the Earth’s rotational axis arbitrarily high above the North Pole. The North Star, Polaris, lies very close to the NCP on the sky.

North Star – The North Star, or Polaris (the pole star), is a star which currently happens to lie almost due north of our planet, above the North Pole and along the Earth’s rotational axis. Because of its location it is always above the horizon for observers in the northern hemisphere (and never above the horizon for those in the south). The northern night sky appears to revolve around this star, moving counter-clockwise in a full circle once every 24 hours. Because the earth’s rotation axis wobbles, over tens of thousands of years it points slightly away from, and then back toward, the North Star. In ten thousand years, the title of North Star will be given to another, neighboring star in the vicinity.

Outflow channel – An outflow channel is a particular type of surface feature found on Mars. Outflow channels are wide and long, and contain streamlined remnants of ancient features which have been sculpted by the passage of fluids (such as lava or water flows). They can extend over hundreds of kilometers in length, and can be up to a few hundred kilometers wide as well. Kasei Vallis is a prominent example of such.

Parallax – A technique for estimating the distances to objects, by measuring their apparent angular shifts on the sky relative to distant objects when they are observed from two separated locations.

Parsec – A unit of distance defined as the distance at which an object exhibits a parallax shift of one arcsecond. As the Earth rotates around the Sun and shifts by a length of one astronomical unit, a star which lies one parsec away from Earth will appear to shift by one arcsecond across the sky. One parsec is equal to 3.26 light years or 206,265 astronomical units.



**Perturbation** – A perturbation is a disturbance (in the force, or elsewhere). When the orbit of an astronomical body varies slightly (wobbling, or shifting back and forth), we often describe the variation as a perturbation.

**Phase** – For a periodic function, such as sine or cosine, the word phase is often used to define the shift of the function away from the default zero point by a fraction of a full period. For the Moon, we generally describe its appearance as it shifts from shadow into full illumination and back over the course of a lunar month in terms of the new, quarter, gibbous, and full phases.

**Plate tectonics** – Plate tectonics defines a theory of planetary surface dynamics in which a planet's outer skin (the lithosphere) is broken into plates. These plates are driven by internal heat, and shift and interact in various ways (including collisions).

**PNG format** – Images are often stored on computer disks in PNG-format files, a format which allows the files to be stored and transferred from computer to computer without loss of information. A PNG-format file should have a file name which ends with the extension “.png,” so that the image analysis and display packages can recognize its contents.

**Polaris** – see North Star.

**Precision** – The precision of a measurement is defined as the smallest change in its value which can be observed with a given experimental technique. A ruler with markings every millimeter (mm), for example, could carry a precision of  $\pm 0.5$  mm.

**Projectile** – A projectile is an object that is launched or dropped into space, or into an atmosphere. It is sometimes called an impactor, once it has struck a surface.

**Proto-planetoid** – This word is a combination of “proto”, meaning first or earliest, and “planetoid,” meaning minor planet. The proto-planetoids in the early solar system were the largest of the building blocks which combined (through collisions) to form the major planets which we know, and love, today.

**Radial** – Extending outward from a common center. When a child draws the Sun as a little dot with arrows going out in all directions to show the sunlight escaping, they are drawing radial lines. To plot the radial distribution of light within an aperture centered on an image of a star, imagine placing a series of thin rings around the star. Average the brightness of the light contained within the left half and the right half of each ring, and plot these values from left to right according to how far each ring lies from the center of the star.

**Radian** – A unit used to measure angles. There are  $\pi/2$  degrees in a right angle, and  $2\pi$  radians in a full circle.

**Radioactive** – An radioactive isotope of a particular element is unstable, and will decay into other elements and isotopes over time. We define the “half-life” of a radioactive sample as the amount of time in which half of its atoms will decay into another state. By comparing

the relative amounts of various isotopes of key radioactive elements, we can often determine absolute ages for samples of various materials.

Rays – Rays are bright linear streaks extending radially outward from certain craters, most notably young ones like Tycho and Copernicus on the Moon. They indicate the presence of thin deposits of lighter material.

Red dwarfs – Cool, red, low luminosity stars with less mass and smaller sizes than the Sun. In the H-R diagram, they are Main Sequence objects, located to the lower right of the Sun's position. Because red dwarfs are such low-mass stars, they spend much more time on the Main Sequence than solar-mass or more massive counterparts.

Red giants – Cool, red, high-luminosity stars that are hundreds of times the Sun's size. In the H-R diagram, they occupy a region well off the Main Sequence to the upper right. The progenitors of red giants are Main Sequence stars, which burn through their hydrogen reserves and then move into the giant phase.

Redshift – The redshift  $z$  of a galaxy is defined as  $\Delta\lambda/\lambda_{rest}$ , the ratio of the shift in wavelength  $\Delta\lambda$  observed for a spectra feature of rest-frame wavelength  $\lambda_{rest}$ .

Relative dating – Relative dating refers to the process of placing an event along a time line relative to other events (before them or after them), without defining a specific time for any event.

Rille – The word rille (also rill) is used to denote long trenches, or brooks or streams, and was often used in describing lunar features seen through the first telescopes.

River delta – A river delta is land formed from sediment (silt) that builds up at the mouth of a river, where it flows into an ocean or other large body of water.

RMS (root mean square) deviation – The rms deviation is the square-root of the average square of the offsets in  $y$  between a set of  $N$  data points and a fit function. For a linear fit, where  $y = mx + b$ ,

$$\text{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^N [y_i - (mx_i + b)]^2}.$$

Scablands – Scablands are erosive features, composed of flat, elevated land characterized by poor soil and little or no vegetation, marked by dry channels which formed through the action of glaciers. The Channeled Scablands of the state of Washington were created by the Missoula Floods during the Pleistocene era, and are the most well-known of such features.

Scatter plot – See  $xy$  plot.

Sedimentary rock – Sedimentary rock, such as limestone or sandstone, is rock that was originally laid down as horizontal sediment (deposited by water, air, or ice). Contrast it with igneous rock, which is formed by the cooling of molten rock.

Semi-major axis – The semi-major axis of an ellipse is half the length of the major axis.

Semi-minor axis – The semi-minor axis of an ellipse is half the length of the minor axis.

Sextant – A mechanical device used to calculate the angle on the sky between two objects, or the altitude of an object (its distance above the horizon).

Shield volcano – A shield volcano is one that has built up from fluid lava flows. The name comes from the distinctive large, extended size and low height (low profile), giving rise to a shape that resembles the shield of a warrior.

Sigma – The Greek letter “s” ( $\sigma$ ), often associated with a measurement of a standard deviation.

Slope – The slope  $m$  of a line is the change in  $y$  divided by the change in  $x$ , or for two points along the line with coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ ,

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Small angle approximation – For small angles (less than 10 degrees, or  $\pi/18$  radians), the tangent of the angle is roughly equal to the angle itself, measured in radians.

South Celestial Pole – The South Celestial Pole (SCP) is the projection of the Earth’s South Pole upon the celestial sphere. One can think of it as the extension of the Earth’s rotational axis arbitrarily high above the South Pole.

Spectral class – A classification based on the appearance of a stellar spectrum, analogous to the temperature sequence, with blue O class stars being hottest, yellow G stars like the Sun being intermediate, and red M stars being cooler.

Standard candle – A class of objects assumed to be of uniform brightness. Any variation in observed brightness for a set of standard candles can be attributed to the distance to each object.

Standard deviation – The standard deviation  $\sigma$ , also called the spread, of a set of  $N$  repeated measurements  $m_i$  with an mean (average) value  $\mu$  is defined as

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (m_i - \mu)^2}.$$

Star – A hot, glowing, spherical mass of gas, dominated by hydrogen. Stars are typically found in stable configurations in which the inward-directed force of gravity is balanced by the outward radiation pressure due to nuclear fusion reactions in the cores.

Star cluster – A group of hundreds or thousands of stars bound together by gravity, which formed at a single epoch from a giant cloud of interstellar gas and dust.

Stefan-Boltzmann Law – A mathematical relationship describing the behavior of spherical, idealized radiators (a.k.a. stars), connecting luminosity  $L$ , temperature  $T$ , and radius  $R$ :  $L = (4\pi\sigma)T^4R^2$ , where  $\sigma$  is the Stefan-Boltzmann constant.

Stellar – Relating to a star or to stars.

Stellar evolution – The process by which a star changes in size, luminosity, temperature, and appearance, as it ages and consumes its fuel. The speed of these changes is driven primarily by stellar mass. The most massive stars may shine for only a few million years, while the least massive could last hundreds of billions of years.

Systematic error – A systematic error is one which biases all of a set of measurements in the same fashion (as opposed to making some smaller and some larger).

Terraces – Terraces are stair-like levels in the sloped walls of craters.

Terrestrial – The term “terrestrial” means Earth-like or pertaining to the Earth’s surface. Planets with rocky surfaces are sometimes called Terrestrial planets, in contrast to the Jovian gas giants.

Topography – A topographical representation of a region involves a detailed physical description, including the relative positions and elevations of features.

Transit – An astronomical object transits when it passes across the observer’s meridian, an arc of constant longitude (the east/west coordinate) along the surface of the Earth connecting the North and South Poles and the observer’s location. An object which is transiting lies either to the north, to the south, or directly overhead of an observer. An object can be transiting for one observer, but appear far to the east or west in the sky for an observer at another location.

Trench – A trench is a long ditch, or a long steep-sided valley.

Tributary – A tributary is a stream that flows into a larger body of water.

Turn-off point – The point on the H-R diagram for a particular star cluster where its stars are evolving off of the Main Sequence and becoming red giants. The location, usually specified by the corresponding color index, depends on the cluster’s age.

Ursae Majoris – A large, bright constellation in the northern hemisphere, named “the Great Bear.” The well-known Big Dipper is a part of Ursae Majoris.

Ursae Minoris – A constellation in the northern hemisphere, named “the Little Bear.” The well-known Little Dipper is a part of Ursae Minoris, as is Polaris, the North Star.

Variable star – A star which varies periodically in luminosity over time.

Velocity – The velocity of an object is its speed in a particular direction. It has units of

distance traveled per unit time, such as miles per hour, or centimeters per second.

Velocity of recession – The velocity of an object which appears to be moving away from us. For galaxies, recessional velocities can be measured from spectral redshifts.

Watershed – A watershed is an elevated ridge of land which divides two regions which drain into separate rivers, or a single region which drains into a river or other body of water.

White dwarfs – Hot, low-luminosity stars that are much smaller than the Sun (they are Earth sized!) These old, dying stars are gradually cooling, and growing fainter with time. They are the end-states for intermediate- and low-mass Main Sequence stars which have passed through the giant phase.

$xy$  plot – A plot which shows the relationship between two variables by plotting one along an  $x$ -axis and the other along a  $y$ -axis is commonly called an  $xy$ , or scatter, plot.

$y$ -intercept – The  $y$ -intercept  $b$  of a line is the  $y$  coordinate of the point on the line for which  $x = 0$ , or for two points along the line with slope  $m$  and coordinates  $(x_1, y_1)$  and  $(x_2, y_2)$ ,

$$b = y_1 - mx_1 = y_1 - \left( \frac{y_2 - y_1}{x_2 - x_1} \right) x_1.$$

Zenith – The zenith is the direction pointing directly overhead for an observer.

Zenith distance – The zenith distance is measured in degrees, and is equal to the angle on the sky between an observer's zenith and an object. The sum of the zenith distance and the altitude is always equal to  $90^\circ$ , for an object.



# Appendix II

## Supplies

You will need the following supplies to perform the physical experiments contained within this laboratory exercise manual. Additional details can be found in the lists of materials at the front of each individual exercise.

- a 30-foot long tape measure, or a shorter tape measure (or a yardstick) and at least 15 feet of string
- a 3"-8" ruler
- additional measurement tools, such as a stopwatch or a kitchen or jeweler's scale
- a half-cup of pinto beans OR a bag of marshmallows OR several handfuls of gravel OR 30 bushes or trees OR a herd of amiable dairy cows OR access to a parking lot full of cars OR a few shelves of books OR 30 friendly people OR two bags of tortilla chips OR 30 coins of a single denomination
- two quarts of clean, dry, loose sand
- two cups of salt, sugar, or colored sand
- a cooking pan, paint tray, or box
- a cup or large spoon, small brush, and sieve
- several dense round projectiles
- one piece of 30-inch by 6-inch and two pieces of 10.5-inch by 6.5-inch thick cardboard
- the cardboard tube from the center of a roll of paper towels
- a needle, a pin, and one 18-inch and two 9-inch pieces of brightly colored thread
- a pair of scissors, and a roll of tape
- a drinking straw and a full soft drink can, or a chopstick and a soup can
- a straight drinking straw
- a page or two of newspaper
- 3 paper clips
- 3 coins
- a pencil or pen
- a calculator (or calculator app)
- a flashlight (or flashlight app)