## Lab 5

## Parallax Measurements and Determining Distances

### 5.1 Overview

Exercise five centers on a hands-on activity where students perform their own parallax measurements, measuring angular shifts in nearby object positions based on changes in observer vantage point and then connecting their experiment to larger-scale parallax measurements conducted on semi-yearly timescales to measure analogous shifts for nearby Milky Way stars.

As in exercise two, students build their own measuring device and then use it to conduct an astronomical experiment. In this case the device (shown in Figure 5.1) is similar to a surveyors transit. Students begin with a rigid piece of cardboard 30 inches wide and six inches across. They mark two observing positions with upright posts formed by paper clips, separated by a distance of two feet. This separation length is analogous to the two astronomical units that separate the Earths positions at six-month intervals. The right-hand post is also the origin of a small paper protractor that is taped down to the cardboard so that the two posts both lie along the $0^{\circ}-180^{\circ}$ axis.

Students then align a nearby foreground object (within thirty feet of the transit device) so that it lines up with a distant object (at least 200 feet away) when viewed from the left-hand post. When they move two feet to the right and work from the right-hand post, the two objects will no longer line up. They measure the angular gap between the two objects (a gap which was zero at the left-hand post) and then use these data to determine the distance to the nearby object.

We use a slightly more complicated transit device when working face-to-face with students. This version has a protractor fixed at both vantage positions. This enables students to


Figure 5.1: The parallax transiting device, used to determine the distances to nearby objects by measuring their angular positions from two vantage points. The device has a base formed from a piece of cardboard 30 inches long. A paper protractor is taped to the right side, and an "X" is marked exactly 24 inches to the left of its origin. This two-foot distance represents the two astronomical units between the Earth's position around the Sun during January and July. Straightened paper clips are attached perpendicular to the surface at the protractor origin and the "X" mark, to mark sight-lines. A piece of thread is doubled and secured at the protractor origin, to create two threads that can be rotated around the protractor to mark various angles between $0^{\circ}$ and $180^{\circ}$.
confirm directly that the nearby and distant objects lie at the same protractor angle when viewed from the left-hand post, and to see for themselves that the angular position the distant object is the same when viewed from either vantage point. We remove the second protractor from the device when students are working alone, to minimize confusion about having a measuring tool on which no measurements need to be made to complete the experiment. (The second protractor is a good aid to comprehension, but can complicate the experimental process for students who are not confident in visualizing the entire experiment ahead of time.) It also requires that the two protractor axes be aligned precisely with each other along the length of the cardboard.

As in previous exercises, measurements are made repeatedly to estimate measurement errors, and to attach error bars to derived distances to objects. Students compare their parallaxderived distances to directly-measured distances. If their measurements differ by more than five standard deviations they critique they critique their experimental design, set-up, and process to hypothesize about possible significant causes for the differences. As in previous exercises, success is not defined by whether or not they can determine the exact same distance to an object via the two techniques. We are more concerned with developing their skills to quantitatively compare measurements, to determine whether or not measurements agree, and to judge for themselves the strengths and the limitations of their experiments.

The results of the experiment are quite good in most cases. The differences between distance derivations are a few percent for distances out to ten feet, and stay below $20 \%$ at the largest distances ( 30 feet) where the measured angular shifts are only a few degrees. Roughly $3 / 4$ of our students report differences of less than two standard deviations, and the remaining $1 / 4$ are able to suggest reasonable causes for their larger disagreements.

We have found that there are typically numerous safe, well-lit locations available to students to conduct their parallax experiments. Multi-story buildings with large windows provide access to views of lamp posts, radio towers, and mountains, and nearby object samples can draw on existing distributions of sign posts and trees or be formed from water bottles sitting on stools, for example. Libraries of all sorts are popular experimental sites, and librarians are usually supportive of our students' educational efforts. During daylight hours on days with good weather, the entire experiment can be conducted outdoors. Our video tutorial follows a pair of students working through the transit device construction and the complete process of the doing the experiment for clarity, as well as emphasizing safe and unsafe locations to use.

The experiment proper goes more smoothly and faster with a partner, one who needs no training in astronomy to be of assistance in shifting objects and holding down ends of measuring tapes. We have found that older children can function well in this role, and students who are parents are often pleased to be able to involve their children in their studies this way (as can also be done in exercises one, two, and three).

A potential sticking point in early versions of this exercise involved the use of trigonometry. We worked with multiple pilot groups of students who demonstrated no familiarity with or comprehension of sines or tangents. We discovered that these same students were rather more comfortable discussing ratios of heights to widths for triangles. They could understand the effect on an angle of changing the height or width of the triangle that defined it, so we recast our geometric discussions and figures into this language. To get around the problem of students being unable to calculate tangents of angles reliably on their calculators (and them not knowing whether they were working in degrees or radians), we include a look-up table that lists height-to-width ratios (aka tangents) for angles between $0^{\circ}$ and $20^{\circ}$.

The final component of the exercise has students apply the same parallax technique to nearby stars, converting between observed angular shifts in units of arc-seconds to distances of order parsecs. Students work with text-based questions and also measure stellar shifts from stellar diagrams.

### 5.2 Learning Objectives

After completing this laboratory exercise, the student should be able to do the following:

- Use a ruler to measure linear distances, and a protractor to measure angular separations.
- Describe the effect of varying the observational baseline or the distance to an object on its apparent change in angular position against a panorama of distant objects.
- Understand the concept of a parallax measurement, and explain how the parallax technique can be used to determine the distances to solar system and galactic objects.
- Visualize the connection between an astronomical unit, an arcsecond, and a parsec, and use the first two to define the third.
- Connect the differences between repeated measurements of a single quantity to a measure of error or accuracy.
- Plot two variables against each other, and decide whether a straight line is a good fit to the data over a certain range.
- Determine the slope and zero point of a linear relationship between two variables, and then translate from one variable to the other (apply the relationship $y=m x+b$ to find $y$ for a given value of $x$ ).


### 5.3 Teaching Toffees

Teaching toffees are practical tips based on the hard-won experience of fellow instructors.
For all GEAS lab exercises:
Please have students access the list of components for the lab exercise (lab chapter, video tutorial, web applications, report template) directly from the GEAS website so that they use the current version of all tools.

By having them save and share the lab report template, they will have their work backed up automatically and instructors will be able to observe their progress $24 / 7$. This allows instructors to check for appropriate progress towards intermediate goals over a two-week period, and for students and instructors to interact asynchronously through threaded comments placed in the report margins at exactly the point where a question occurs. Note that the Lab \#1 video tutorial describes how to work with Google resources, and there is a two-page handout on the lab website as well.

Students sometimes try to complete the lab exercise by skipping over the lab chapter entirely and working from just the report template. This is not an effective learning strategy, nor does it save time. The report template contains only abbreviated forms of the questions with spaces for answers, but none of the context or framework for the questions.

We strongly encourage you to strongly encourage all of your students to watch the video tutorial before beginning to work on the exercise. If a picture is worth a thousand words then a video might be worth a million words. Each tutorial is designed to introduce the scientific concepts that motivate the companion exercise. Because students may be working remotely on their own, it also focuses on the most potentially challenging aspects of each project and walks them through key steps by showing them what they look like in practice.

### 5.4 Keywords

Arcminute - An arcminute is a unit of angular size, equal to $1 / 60$ of a degree (recall that there are 90 degrees in a right angle). There are 60 arcseconds in an arcminute (see below).

Arcsecond - An arcsecond is a unit of angular size, equal to $1 / 60$ of an arcminute or $1 / 3600$ of a degree. Astronomers often measure the angular separation between neighboring objects on the sky in units of arcseconds.

Astronomical unit - The average distance between the Earth and the Sun, equal to $1.5 \times 10^{8}$ kilometers.

Degree - A unit used to measure angles. There are 90 degrees in a right angle, and 360 degrees in a full circle.

Light year - A unit of distance (not time), equal to the distance which light travels in a year. One light year is equal to 0.307 parsecs.

Parallax - A technique for estimating the distances to objects, by measuring their apparent angular shifts on the sky relative to distant objects when they are observed from two separated locations.

Parsec - A unit of distance defined as the distance at which an object exhibits a parallax shift of one arcsecond. As the Earth rotates around the Sun and shifts by a length of one astronomical unit, a star which lies one parsec away from Earth will appear to shift by one arcsecond across the sky. One parsec is equal to 3.26 light years or 206,265 astronomical units.

Radian - A unit used to measure angles. There are $\pi / 2$ degrees in a right angle, and $2 \pi$ radians in a full circle.

Small angle approximation - For small angles (less than 10 degrees, or $\pi / 18$ radians), the tangent of the angle is roughly equal to the angle itself, measured in radians.

### 5.5 Relevant Lecture Chapters

This laboratory exercises draws upon the material in Chapter 10: Geocentric and Heliocentric Models. There are loosely related concepts for visualization of objects and orientations in Chapter 7: The Celestial Sphere, and Chapter 8: Planetary Orbits.

### 5.6 References and Notes

1. This laboratory exercise draws from an in-class parallax exercise taken from the 2007 New Mexico State University Astronomy Department ASTR110G laboratory manual.
2. Figure 5.1 through Figure 5.9 and the attached protractor design are shown courtesy of Nicole Vogt.
