Lab 7

Hubble's Law and the Cosmic Distance Scale

7.1 Introduction

There are tens of satellites (moons) orbiting the planets of the solar system, a handful of planets in orbit around the Sun, and over one hundred billion stars like the Sun which make up our Milky Way galaxy. As we advance from satellites to planets to stars to galaxies we note that the masses and sizes of the bodies grow tremendously, as do the distances between them. How can we measure large distances in the Universe?

We can bounce radar signals off of solar system objects to find out how far away they lie from Earth, and use parallax measurements to find the distances to nearby stars (taking advantage of their apparent movements across the sky as the Earth changes its position around the Sun). Parallax measurements have been extended to stars out to 500 light-years by the Hipparcos satellite (and may be pushed out to 30,000 light-years, if the planned Gaia mission succeeds). Parallax is an excellent technique for mapping the distribution of stars within our Milky Way galaxy – but how can we chart the rest of the Universe?

As the 20^{th} century began many astronomers thought that everything observed through telescopes, including the numerous "spiral nebulae" like the famous faint swirl in the constellation Andromeda, was contained within our Milky Way galaxy. Others argued that spiral nebulae were separate galaxies, lying far beyond the Milky Way. The issue came to a head in the famous Shapley-Curtis debate of 1920, illustrating the need for deeper, higher resolution observations to answer the question. Edwin Hubble soon showed that the observable Universe extends far beyond the Milky Way. He used new techniques to measure the distances to distant objects, employing Cepheid variable stars as "standard candles".

An entire family of distance determination methods is based on the concept of a standard candle. Simply put, if we can can identify a class of objects with near-identical properties then those which appear the smallest and faintest must lie furthest away from us. If you have ever gauged the distance to an approaching car at night from the brightness of its headlights, you've used this technique! We assume that the brightness of these objects obeys an inverse square law, being inversely proportional to the square of the distance ($F \propto 1/d^2$), and that their apparent size varies inversely with distance ($\theta \propto 1/d$). If your standard candle is a 100-watt light bulb, for example, then if it shifts to a position twice as far away from you it will appear one-fourth as bright (like a 25-watt bulb would at its initial position).

Cepheid variable stars are important examples of standard candles, used to determine accurate distances to stars within the Milky Way and within nearby galaxies like the Andromeda galaxy. The amount of light output by these massive stars can vary by a factor of two or three, over a regular period of time extending from a few days to a few months. In 1908, Henrietta Leavitt discovered that this period of variability was determined by the intrinsic luminosity of the star – the more energy the star pumped out, the longer it took to vary from bright to faint and back to bright again, like clockwork. The brightest of these stars are 10,000 times more luminous than the Sun, so they can be seen from quite a distance.

With modern telescopes, we can observe Cepheid variable stars out to 30 megaparsecs (Mpc), or 100 million light-years. This allows us to determine distances to galaxies hosting such stars which lie within 30 Mpc of the Milky Way, covering a number of nearby isolated galaxies, groups of galaxies, and even the closest large galaxy clusters like Virgo, containing hundreds of galaxies. Thirty megaparsecs might seem like a large distance (equal to 300 times the size of the Milky Way), but in the grand scheme of things it barely covers our own cosmic backyard.

We can employ this technique at larger distances by finding brighter standard candles, in the form of supernovae. Type Ia supernovae are stars which (for a brief period of time) can be as bright as an entire galaxy. They occur as part of the end-state process for low-mass white dwarf stars, when mass accreted from a stellar companion (a second star) fuels a short, immense explosion. These objects consistently achieve a known peak luminosity, and thus when we observe them we know how far away they lie. (As they are all the same intrinsic brightness, the further they lie from Earth the fainter they appear.) The most distant supernovae studied lie an amazing 1,700 Mpc away from us.

Supernovae explosions are wonderful probes of the distant Universe, but they are rare and can be difficult to find. We thus employ additional techniques to measure the distances to many galaxies, taking advantage of the known properties of the galaxies themselves. We will explore some of these techniques within this laboratory exercise. Within large clusters of galaxies (home to hundreds or even thousands of galaxies), one almost always finds a bright elliptical galaxy in the center of the cluster. These brightest cluster galaxies (BCGs) have remarkably consistent intrinsic properties. Because they lurk in the cores of clusters and consume unwary interlopers (yes, it's a dog-eat-dog world, and large galaxies are literally cannibals), they tend to grow to a certain size and give off a certain amount of energy.

These galaxies exhibit uniform properties, meaning that they are they appear to be the same size and emit the same amount of energy when viewed from the same distance. (They are standard candles in a sense, but they vary too much to truly deserve the name. You might think of them as sub-standard candles built in a galactic factory, one with no routine inspections or quality controls.)

We will also estimate distances to galaxies using a cosmological method based on a discovery made by Edwin Hubble in 1930. While determining the distances to nearby galaxies, he realized that the derived distances correlated with another observable property – the velocity at which the galaxies appeared to recede. Hubble plotted recessional velocity v as a function of distance d, as shown in Figure 7.1, and discovered that the most distant galaxies are also moving the fastest. We call this relationship Hubble's Law, and these diagrams Hubble diagrams, to honor his insight.

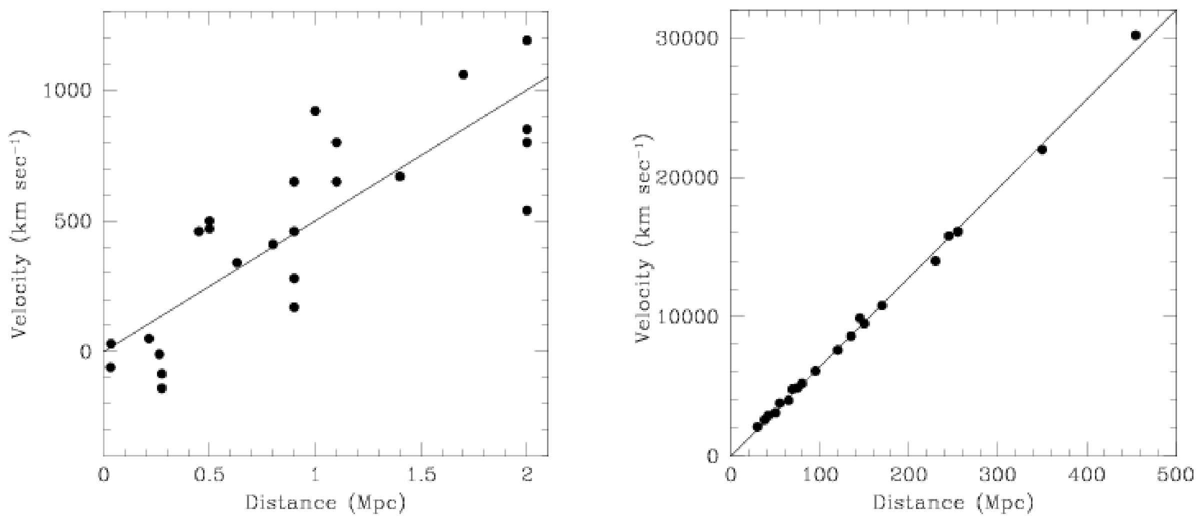

Figure 7.1: Hubble diagram showing Edwin Hubble's original data set (left), and modern data (right) extending out to larger distances. The x-axis shows the distances to nearby galaxies determined by a variety of methods, and the y-axis shows their recessional velocities. The slope of a line fit to the data has units of y over x, or km sec^{-1} per Mpc, and is called the Hubble constant (H_0).

We interpret these recessional velocities as evidence that the entire Universe is expanding outward (the distances between all galaxies increases with time). We do not assume that we are situated at the center of the Universe and everything is moving away from us, having learned from our previous "the Earth is the center of the world" phase (so 16^{th} century).

We can fit the slope (the change in y over the change in x) of the data in Figure 7.1, and find the relationship between distance d and velocity v. We call this slope value H_0 (the Hubble constant) in honor of Edwin Hubble, where

$$H_0 = \frac{v}{d}, \quad \text{and so } v = H_0\, d. \tag{7.1}$$

The current accepted value of H_0 is 72 km sec^{-1} per Mpc. We can also solve this equation for d, so if we know the velocity v of a nearby galaxy, then it lies at a distance d such that

$$d = \frac{v}{H_0}. \tag{7.2}$$

We will examine several Hubble diagrams in this laboratory exercise, determining the distances to galaxies by measuring changes to their spectra due to recessional velocities.

7.1.1 Goals

The primary goals of this lab are to appreciate that the Universe is expanding, and to understand and evaluate various techniques for determining distances to galaxies.

7.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL.

http://astronomy.nmsu.edu/geas/labs/labs.html

You will also need a computer with an internet connection, and a calculator (or calculator app).

7.1.3 Primary Tasks

You will study Hubble diagrams based on Cepheid variable star data and Type Ia supernovae observations. You will then measure the apparent sizes and fluxes for a sample of brightest cluster galaxies (BCGs), and compare the derived distances with those that you find from redshifted spectra.

7.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. Note that §7.9 contains 5 extra credit points.

Table 7.1: Breakdown of Points

Activity	Hubble diagrams	Distances	Comparisons	Questions	Summary
Section	§7.3.1, §7.3.2,	§7.4	§7.5	§7.7	§7.8
Page	9, 12	15	21	26	28
Points	14	27	22	13	24

7.1.5 Timeline

Week 1: Read sections §7.1–§7.4, complete activities in §7.3 and §7.4, and begin final (Post-Lab) questions in §7.7.

Week 2: Read §7.5–§7.6, complete activities in §7.5, finish final (Post-Lab) questions in §7.7, write lab summary, and submit completed lab report.

7.2 Determining Galaxy Redshifts and Velocities from Spectra

A Hubble diagram shows the distances to galaxies versus their recessional velocities (how quickly they and the Milky Way are separating from each other). We can determine these velocities by examining galaxy spectra and determining redshifts. Let's walk through this process, explaining our terms as we go.

If you've ever listened to the siren of a speeding police car, ambulance, or fire truck, you've heard the high-pitched sound as it approaches you drop to a lower frequency as it passes by and recedes. This "Doppler Effect" is due to successive sound waves from the approaching source piling up in time (as each new wave travels a shorter distance to reach you), so that your ear absorbs more of them with every second. Once the siren starts to move away from you, the sound waves start to space out again (as they have to cover more and more ground to reach you) and the siren seems to drop in pitch.

Light waves undergo a similar effect. While it's most natural to talk about sound waves changing in pitch (or frequency), the analogous effect for light waves is typically described as a change in wavelength. We detect visual light emitted from approaching sources to be shifted in wavelength toward the blue end of the spectrum, and we find light from sources moving away from us to be redshifted to longer wavelengths. The amplitude of the shift is defined by a change in wavelength, $\Delta\lambda$. The ratio of $\Delta\lambda$ to the original wavelength of the light (called λ_{rest}, as it is emitted by a source at rest with respect to an observer) is defined as redshift (z). We can write this as an equation:

$$z = \frac{\Delta\lambda}{\lambda_{rest}}. \tag{7.3}$$

The redshift z is the change in wavelength of a spectral feature, relative to its wavelength at rest. The larger the redshift, the faster the object is moving away from the observer.

Example 7.1

Hydrogen is the most common element in the Universe, and so the stars and gas clouds within galaxies frequently absorb or emit light at the wavelengths at which hydrogen atoms absorb and emit radiation. The hydrogen beta ($H\beta$) line is one such feature, found at a wavelength corresponding to blue-green light. In an object at rest with respect to an observer, this line is observed at its rest wavelength of $4861\AA$ (we say $\lambda_{rest} = 4861\AA$). Suppose that we observe the $H\beta$ line in the spectrum of a galaxy and it appears instead at a wavelength of $5347\AA$ ($\lambda_{obs} = 5347\AA$). The observed wavelength is longer, as the light from the galaxy is being redshifted to longer wavelengths. To determine the galaxy redshift, we first need to find the change in wavelength, $\Delta\lambda$. We recognize that

$$\Delta\lambda = \lambda_{obs} - \lambda_{rest} = 5347\AA - 4861\AA = 486\AA. \tag{7.4}$$

How large a shift is $486\AA$? Remember that the range of the human eye extends from violet down to red wavelengths, covering roughly $3{,}500\AA$. A shift of $486\AA$ would turn a blue beam of light to green, or yellow light to orange – a noticeable effect.

Dividing by $\Delta\lambda$ by λ_{rest}, we see that the redshift for this object is

$$z = \frac{\lambda_{obs} - \lambda_{rest}}{\lambda_{rest}} = \frac{\Delta\lambda}{\lambda_{rest}} = \frac{486\AA}{4861\AA} = 0.10. \tag{7.5}$$

<u>Example 7.2</u>

At speeds much less than the speed of light (redshifts $z \leq 0.10$), the redshift of a galaxy is equivalent to its velocity v in units of the speed of light, c. We say that $z = v/c$, so the velocity of the galaxy relative to an observer is

$$v = z \times c = z \times 300,000 \text{ km sec}^{-1} \tag{7.6}$$

as the speed of light, c, is 300,000 km sec^{-1}.

How quickly is the galaxy described in Example 7.1 moving away from us? With a redshift of 0.10,

$$v = z \times c = 0.10 \times 300,000 \text{ km sec}^{-1} = 30,000 \text{ km sec}^{-1}. \tag{7.7}$$

Due to the overall expansion of the Universe, this galaxy and the Milky Way are separating at one-tenth the speed of light!

Recall that the "spectrum" of a galaxy is simply a plot of the amount of light that it emits as a function of wavelength. A continuous (or continuum) spectrum is one which varies smoothly and slowly as a function of wavelength, appearing in the visual as a rainbow containing all of the saturated colors. Imagine that there was a sudden gap in a continuum spectrum, where the light was removed at a particular wavelength. We would say that it contained an "absorption" feature because light had been absorbed out at that wavelength, leaving an empty dark space. When we observe stars, we often observe a gap at the wavelength of the $H\beta$ line, where hydrogen in the stellar atmosphere has absorbed photons emitted by the stellar core.

<u>Example 7.3</u>

Consider the spectrum of a galaxy at a redshift $z \approx 0.10$ ($z = 0.095$), shown in Figure 7.2. We have marked seven significant absorption and emission lines in this spectrum, and identified the elements responsible for them (hydrogen, oxygen, magnesium, sodium, and sulfur). The

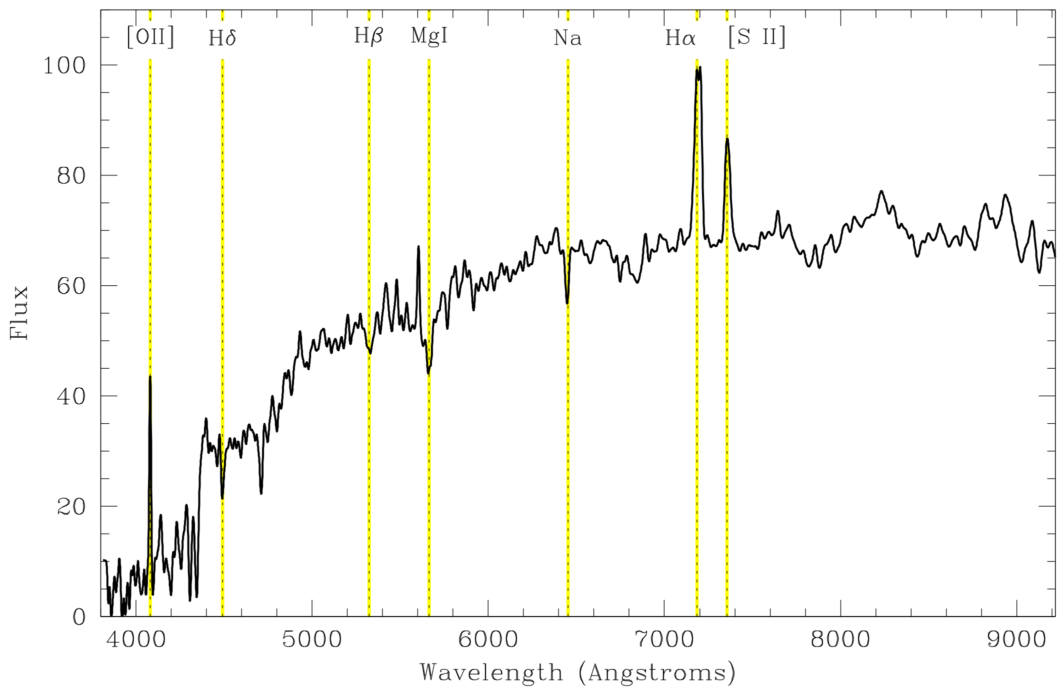

Figure 7.2: The optical spectrum of a nearby galaxy, observed by the Sloan Digital Sky Survey (SDSS). This plot shows the amount of light emitted by the galaxy as a function of wavelength. The spectrum has a smooth underlying shape which rises gently toward longer (redder) wavelengths, with emission and absorption features superimposed on top of it at particular wavelengths. We have identified several of these key features by name: the oxygen [OII] line, the hydrogen Balmer lines $H\gamma$, $H\beta$, and $H\alpha$, the magnesium Mg I line, the sodium Na line, and the sulfur [S II] line. As we know the rest wavelengths of these lines, by finding their observed wavelengths we can determine the redshift of the galaxy.

presence of these lines tells us that these elements are present in this galaxy. Note how, at each of the marked wavelengths, there is either a peak or a drop in the spectral flux, indicating an emission or absorption feature. Find the $H\beta$ line, and determine its observed wavelength along the x-axis – if you estimated a value around $5325\mathring{A}$, you did well! We determined in Example 7.1 that a galaxy with a redshift of 0.10 should show the $H\beta$ line at a wavelength of $5347\mathring{A}$, so we can tell that this galaxy has a redshift slightly lower than 0.10.

In this laboratory exercise, you will compare real galaxy spectra like this to a rest-frame spectrum showing the wavelengths and intensities of various features as they would appear in a galaxy at rest with respect to our Milky Way galaxy. You will align the spectra by shifting the rest-frame spectrum back and forth in wavelength to match the observed galaxy spectrum, in order to determine its redshift, as described in Figure 7.3.

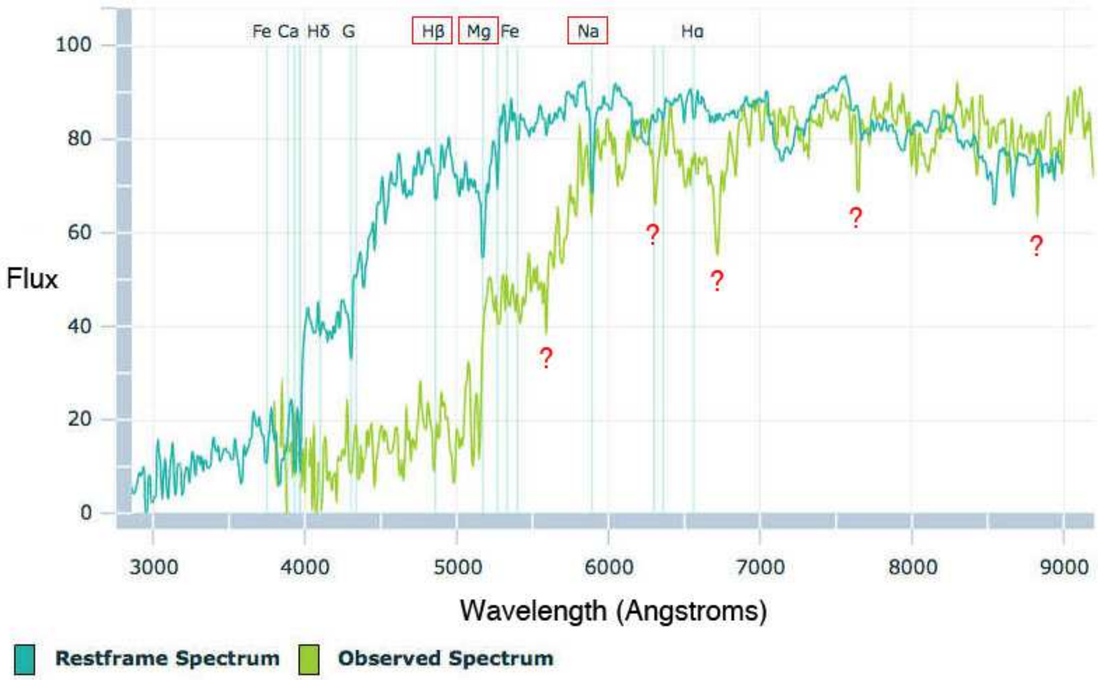

Figure 7.3: The optical spectrum of a nearby galaxy observed by the SDSS, and a spectrum of a similar galaxy as it would appear at rest with respect to us. We can align the spectra by moving the rest-frame spectrum rightwards, shifting each feature to longer wavelengths. Can you find the 4861$\mathring{A}$ $H\beta$ line, the 5173$\mathring{A}$ magnesium (Mg) line, and the 5893$\mathring{A}$ sodium (Na) line in the observed spectrum? What is the approximate redshift of this galaxy?

7.3 Constructing Hubble diagrams

We will now examine Hubble diagrams based on distances to galaxies drawn from Cepheid variable stars and from Type Ia supernovae, and recessional velocities measured from galaxy spectra. As you examine the relationship between distance and velocity, think about the quality of the fit to the data points.

If all of the data points were to fall exactly along the best-fit line, what would that mean? It would suggest that the observational data were of extremely high quality, and also that the model represented by the line (see equation 7.1) encapsulated all of the relevant physics governing the relationship between galaxy distances and velocities. If the data points appeared to be scattered randomly, what would that mean? It would suggest that the observational data were corrupted (damaged) or of vastly substandard quality, and/or that the hypothesis behind our physical model was not correct.

As with many things in life, the reality of the situation lies between the extremes. We observe a certain amount of scatter within the distance–velocity relationship, but the underlying connection is clearly present. A correlation coefficient (R, varying between 100% if y can be completely predicted from x and 0% for no connection) can help us to quantify the level of agreement between the plotted variables.

What are the primary causes of the scatter (noise) in these Hubble diagrams? First, the difficulty in determining accurate distances (and velocities) for these galaxies can produce data-driven offsets from an underlying theoretical relation. Second, there are additional factors that can affect the relationship between distance and velocity, factors not included in the Hubble Law. Some neighboring galaxies exhibit *negative* rather than positive velocities, because at short distances the attractive force of gravity can overwhelm the universal force of expansion. Galaxies within clusters can also have anomalous velocities because they dive into the cluster cores at high speeds relative to their nearest neighbors, drawn by the massive bulk of the cluster itself. Third, at large distances (beyond 400 Mpc) the relationship between velocity and distance becomes more complicated, due to cosmological factors related to the curvature of the Universe (space is not completely flat on large size scales).

7.3.1 A Hubble Diagram for Nearby Galaxies with Cepheids

Let's work through an example concerning Cepheid variable star data, to better understand how to derive distances to these types of stars.

Example 7.4

In Examples 7.1 through 7.3 we discussed how to determine redshifts for galaxies based on spectral features. We can combine such velocities with distances derived for Cepheid variable stars within the same galaxies to create Hubble diagrams. Figure 7.4 shows six successive images of a single Cepheid star within the galaxy M100, observed repeatedly over a two-month period with the Hubble Space Telescope. Note how the star dims and then brightens over time. It appeared rather bright on April 23, fainter on May 4, and was barely visible on May 9. By May 16 it had begun to brighten again, in a pattern that repeats every 53 days. Twenty such stars were observed within this galaxy, and a distance was derived by averaging the results of analyzing each star's light curve (see Figure 7.5).

Because Cepheid variable stars which are intrinsically brighter (which emit more light) have longer periods of variability, we can determine how far such a star lies from Earth by comparing its observed flux with that predicted from various distances. We simply shift the star back and forth across the Universe until its observed flux matches the light level predicted for a star with its period of variability at that distance. The longer the period, the more light the star emits, and the further it must lie from Earth.

Figure 7.6 combines distances from Cepheid stars observed within 23 galaxies, plotting them against recessional velocities derived from the galaxy spectra. The most distant galaxies in the sample have spectral lines which are redshifted the farthest to longer (redder) wavelengths. Because Cepheid stars in all of the galaxies have the same properties, we can use them to determine distances in a uniform fashion.

1. Complete the following statements after studying Figure 7.6. (3 points)

(a) The galaxy M100 is one of the most (nearby / distant) galaxies for which Cepheid-derived distances are shown.

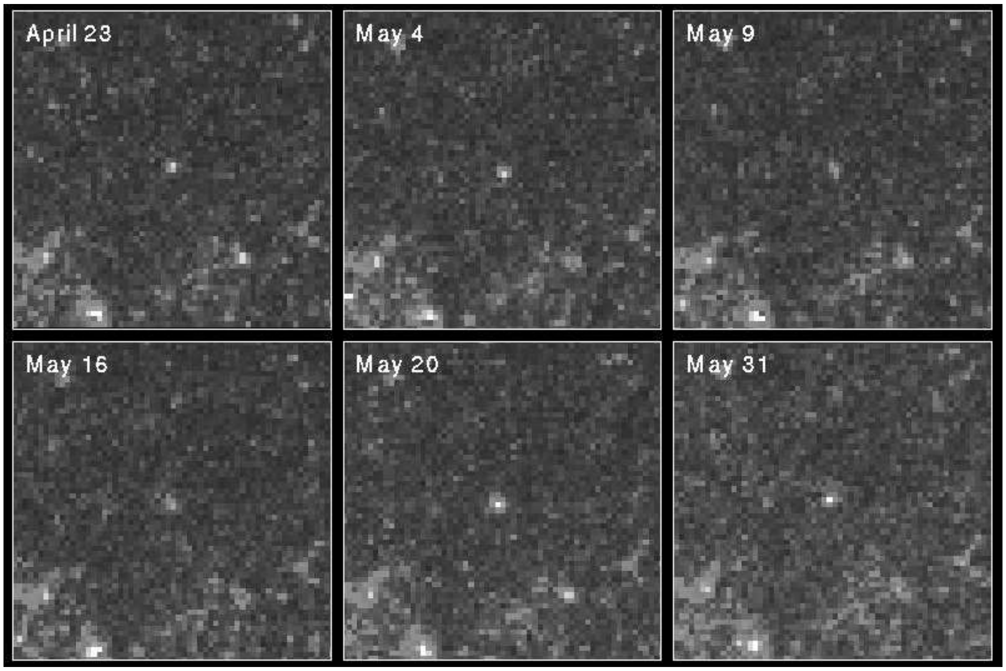

Figure 7.4: Six images of a variable star in the galaxy M100, taken by the Hubble Space Telescope over a two-month period in 1994. Observe how the variable star, centered in each frame, grows fainter and then brighter over time.

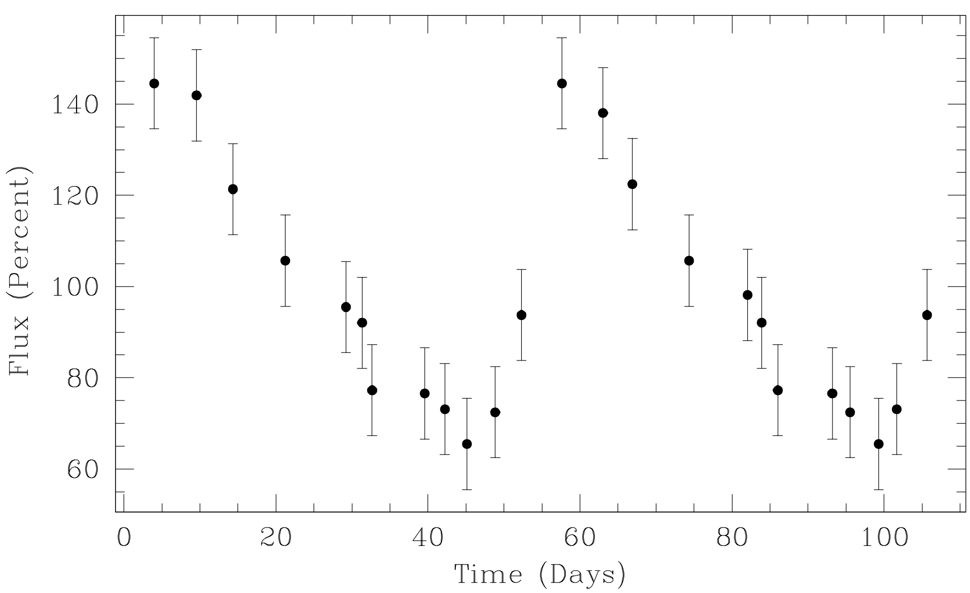

Figure 7.5: The light curve for a variable star in the galaxy M100, showing the change in flux over time. A flux value of 100% corresponds to the average brightness. The star doubles in brightness during each periodic variation and then decays again, taking 53 days to complete the pattern. Intrinsically brighter stars have longer periods of variability.

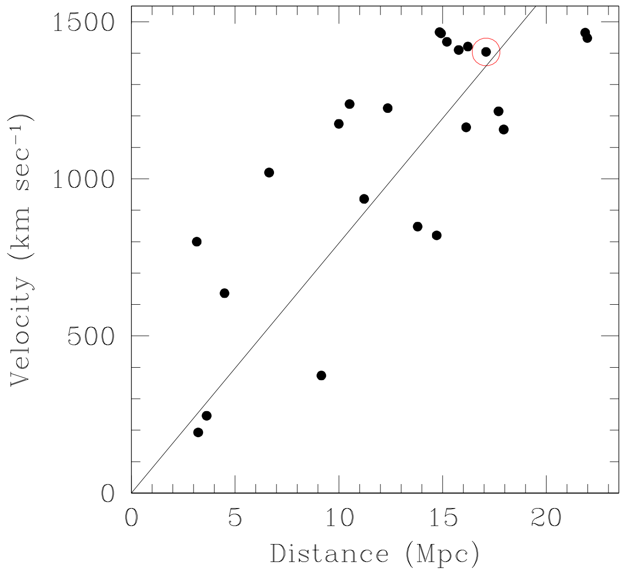

Figure 7.6: A Hubble diagram showing the relationship between galaxy distance and recessional velocity, where distances were derived from the period-luminosity relationship for Cepheid variable stars identified within each galaxy. The M100 point is circled in red, and based in part on the data shown in Figures 7.4 and 7.5. The slope of the relationship is 79 $\pm$ 4 km sec^{-1} per Mpc, and the correlation coefficient R is 78%, indicating that there is a strong positive relationship between velocity and distance.

(b) A galaxy lying 10 Mpc away from us should have a recessional velocity of roughly

__________________________. (Please attach units to your value.)

(c) The modern value for H_0 derived here is much (higher / lower), by a factor of roughly

_____________, than that found in Hubble's original data (shown in Figure 7.1).

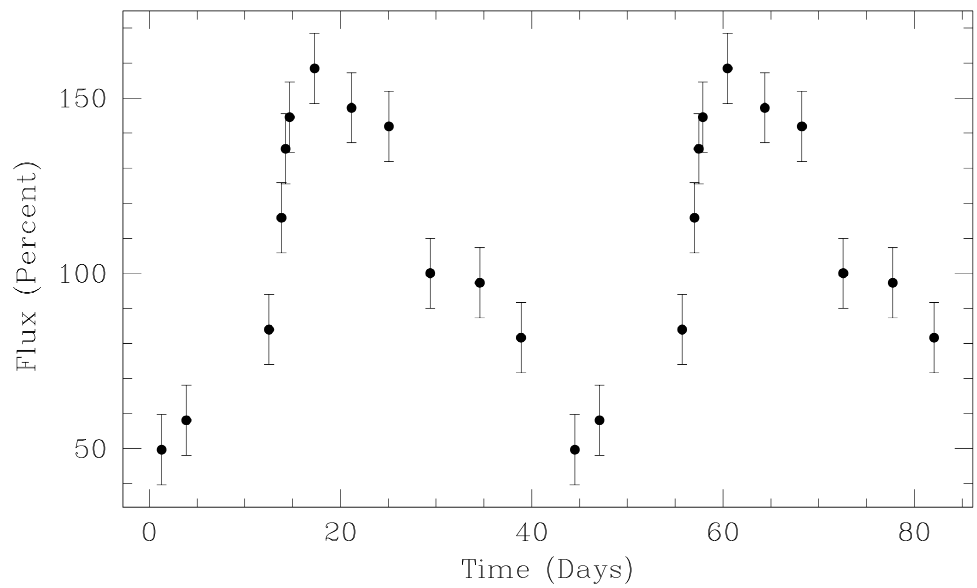

Figure 7.7: The light curve for a second variable star in the galaxy M100. What is the period of variability for this star? Is it intrinsically brighter or fainter than the star shown in Figure 7.5?

2. Figure 7.7 shows the light curve for a second Cepheid variable star in the M100 galaxy. (2 points)

(a) The period of variability for this star is ____________ (to the nearest day or two).

(b) This star is intrinsically (brighter / fainter) than the one shown in Figure 7.5.

7.3.2 A Hubble Diagram for More Distant Galaxies with Type Ia Supernovae

Variable stars are too faint to be observed beyond distances of 30 Mpc, but supernovae explosions can be detected out to 1,700 Mpc (over a considerable fraction of the known Universe). Figure 7.8 is a Hubble diagram constructed from galaxy distances based on individual supernovae events. When we compare it to Figure 7.6, we notice immediately that it extends out more than ten times further in distance and has a much higher correlation coefficient (the relationship between d and v is less noisy). Given the increased range (supernovae can be detected from much farther away, because they are so much brighter) and higher correlation coefficient, why don't we just use the supernovae technique to find distances to all galaxies?

The key point to consider is that the brighter supernovae are much easier to detect than Cepheid variable stars, *when they are present.* Astronomers have found more than fifty Cepheid variable stars in M100 alone, and they are even easier to find in galaxies which lie closer to the Milky Way. Once a Cepheid star has been identified, we can derive its distance by observing it as few as ten times over a several-month period to determine its period of variability and apparent brightness. If we later discovered an error in the observations, we could go back at any time to repeat them.

A supernovae explosion occurs once in every hundred years per galaxy, however, and lasts for only a handful of days. They are rare events, and so we have observed very few of them nearby. Imagine the space surrounding the Milky Way galaxy as a series of nested concentric spherical shells, each one larger than the next. Because each successive shell has a larger radius and lies farther from the Milky Way, it contains a larger volume and holds more galaxies. The odds of our detecting a supernovae in our lifetimes actually increase if we look further away from the Milky Way, even though the supernovae become fainter, just because there are more galaxies available at larger distances to host them.

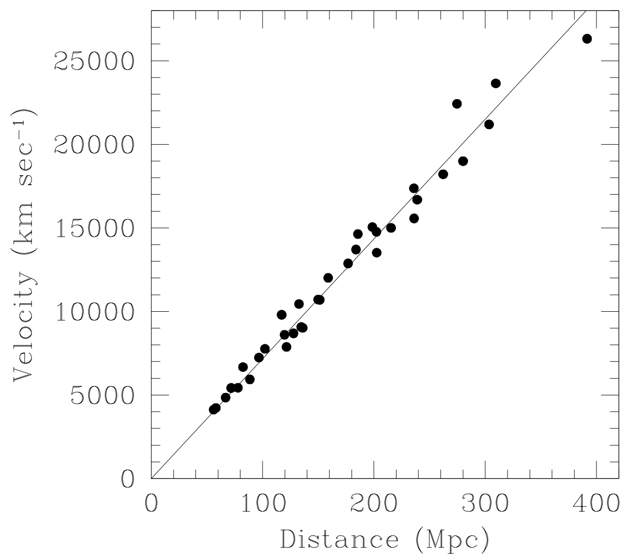

Figure 7.8: A Hubble diagram showing the relationship between galaxy distance and recessional velocity, where distances were derived from the observed peak luminosities for supernovae observed within each galaxy. The slope of the relationship is 71.7 ± 0.8 km sec^{-1} per Mpc and the correlation coefficient R is 99%, indicating that there is a very strong positive relationship between velocity and distance.

1. What is the redshift z for the most distant galaxy shown in Figure 7.8? (1 point)

2. Figures 7.6 and 7.8 illustrate the process of deriving values for the Hubble constant (H_0) from samples of galaxies containing Cepheid variable stars and supernovae. In each case, is the derived value for H_0 consistent with the accepted value of 72 km sec^{-1} per Mpc? (Are the differences more or less than 2σ?) Please show your work. (3 points)

3. How many galaxies appear in Figure 7.6 and also appear in Figure 7.8? Why might even a small number of galaxies in which we had detected both Cepheid variable stars and a supernovae be of particular interest? (Hint: How do we know that these distance estimation techniques are accurate?) (1 point)

4. How many supernovae could have been seen in the nearest 10 galaxies since the telescope was invented in 1600? How many could have been observed with modern telescopes and recording devices (over the last 30 years)? (3 points)

5. There is an unwritten rule of courtesy among astronomers that if a supernovae occurs nearby, anyone with time on a major telescope in the next few days observes it and circulates the images worldwide. Why do you think we do this? (1 point)

7.4 Brightest Cluster Galaxies as Distance Indicators

We refer to the brightest galaxy in a cluster of hundreds (or even thousands) of galaxies as a BCG. For example, the giant elliptical galaxy known as M87 is the BCG in the nearest large cluster of galaxies, a dense grouping of over 1,300 galaxies known as the Virgo Cluster. You will examine a series of images of BCGs taken by the Sloan Digital Sky Survey (SDSS). The BCG will appear at the center of each image, surrounded by other fuzzy amber-colored galaxies within the same cluster.

We will determine the distances to BCGs three different ways, determining a "D-size" distance based on the angular size of each galaxy, a "D-flux" distance based on the measured brightness of each galaxy, and a "D-z" distance based on the galaxy redshift. The following three examples will help us to better understand how each distance is determined.

Example 7.5

We can use the apparent (observed) sizes of BCGs to determine how far away they are, by assuming that these galaxies are all the same actual size. Galaxies which appear larger must then lie closer to us, and those which appear smaller must lie at greater distances. We say that the relationship between the apparent size θ and distance D-size is

$$\theta \propto \frac{1}{\text{D-size}}. \tag{7.8}$$

For nearby galaxies, a galaxy which is twice as far away looks half as big, while one which is half as far away appears twice as large. Let's assume that all BCGs are 20 kiloparsecs (kpc) in size, 130% the size of the Milky Way. A BCG at a distance of 50 Mpc would then have an angular radius of $82''$ (82 arcseconds, where there are 3,600 arcseconds in a degree), while one at 100 Mpc would have a size of $41''$.

$$\frac{\text{D-size}}{50} = \frac{82}{\theta} \tag{7.9}$$

where D-size is measured in units of megaparsecs and θ in units of arcseconds, so that

$$\text{D-size (Mpc)} = \frac{4,100}{\theta('')}. \tag{7.10}$$

As we examine galaxies at larger distances (above 400 Mpc), the relationship between angular size and distance becomes slightly more complex, because we have to take into effect cosmological factors (space is curved, and the Universe was smaller in the past). These are small corrections, however. Of more importance is the fact that all BCGs are not the exact same size. If a particular galaxy is smaller than 20 kpc in size, for example, we will overestimate the distance to it with this technique.

Example 7.6

We can also use the apparent (observed) brightnesses of BCGs to determine how far away they are, by assuming that these galaxies all emit the same amount of light. Galaxies which appear brighter must then lie closer to us, and those which appear fainter must lie at greater distances. We say that the relationship between the apparent brightness f and distance D-flux is

$$f \propto \frac{1}{\text{D-flux}^2}, \quad \text{or} \; \sqrt{f} = \frac{1}{\text{D-flux}}. \tag{7.11}$$

For nearby galaxies, a galaxy which is twice as far away looks four times fainter, while one which is half as far away appears one-fourth as bright. Let's assume that all BCGs are three times brighter than the Milky Way. A BCG at a distance of 50 Mpc would then be observed to add 2,120,000 counts to our calibrated SDSS images, while one at 100 Mpc would add only 530,000 counts. (Each "count" in the calibrated images corresponds to a number of photons detected at the telescope.)

$$\frac{\text{D-flux}}{50} = \sqrt{\frac{2,120,000}{f}} \tag{7.12}$$

where D-flux is measured in units of megaparsecs and f in units of counts, so that

$$\text{D-flux (Mpc)} = \sqrt{\frac{5.30 \times 10^9}{f(\text{counts})}}. \tag{7.13}$$

As we examine galaxies at larger distances, the relationship between flux and distance also becomes slightly more complex due to cosmological effects, but these are still small corrections. The fact that all BCGs are not equally bright is much more important. If a particular galaxy gives off more light than expected, for example, we will underestimate the distance to it with this technique.

<u>Example 7.7</u>

We can use the observed redshifts of BCGs to determine how far away they are, by assuming that the Universe is expanding. Galaxies with smaller redshifts must then lie closer to us, and those with larger redshifts must lie at greater distances. We say that the relationship between the redshift z and distance D-z is

$$z \propto \text{D-z}. \tag{7.14}$$

For nearby galaxies, a galaxy which is twice as far away has twice as large a redshift, while one which is half as far away has half as large a redshift. The critical difference between this technique and those for D-size and D-flux, however, is that we don't need to make any assumptions about the physical properties of the galaxy in order to determine its distance from its redshift. From Hubble's Law, we know that

$$v = H_0 \times \text{D-z}, \quad \text{and as } z = \frac{v}{c}, \; z = \frac{H_0}{c} \times \text{D-z}. \tag{7.15}$$

A BCG at a distance of 50 Mpc would then have a redshift of 0.012, while one at 100 Mpc would have a redshift of 0.024.

$$\frac{\text{D-z}}{50} = \frac{z}{0.012} \tag{7.16}$$

where D-z is measured in units of megaparsecs and z is dimensionless, so that

$$\text{D-z (Mpc)} = \frac{c}{H_0} z = 4,200\,z. \tag{7.17}$$

As we examine galaxies at larger distances ($z > 0.10$, beyond 400 Mpc), the relationship between redshift and distance becomes slightly more complex, because we again have to factor in cosmological effects.

7.4.1 Finding Distances to Brightest Cluster Galaxies

We are going to estimate the distances to 34 BCGs, using images and spectra from the Sloan Digital Sky Survey (SDSS). You will estimate D-size, D-flux, and D-z for ten galaxies, and then combine your results with measurements that we have already made for another 24 galaxies.

Measuring D-size and D-flux from Galaxy Images

Reload the GEAS project lab exercise web page (see the URL on page 4 in §7.1.2), and click on the link for this exercise labeled "Web application #1 (galaxy distances I)."

We will now measure the angular sizes of our BCGs, by fitting an ellipse of appropriate size and orientation to each one of them. As we do so, we'll also measure the amount of light contained within this elliptical aperture to see how bright they are. We'll assume that each one of the galaxies has a radius of 20 kpc and would produce 2,120,000 counts in our images at a distance of 50 Mpc, and calculate distances accordingly. (This calculation will differ slightly from the simplified models presented in Examples 7.6 through 7.7 to take in to account cosmological effects, but the basic idea will be the same.)

Imaging Tool Tips

The image tool interface contains five primary panels, as well as a set of key options across the top of the screen. Start by clicking on the button labeled "Help" to learn about the basic properties of the tool. The three top panels all have to do with the image of the galaxy. The left panel contains the controls to adjust the position, size, and orientation of the green ellipse on top of the galaxy image shown in the middle panel. Each image is $100''$ arcseconds wide, so if a particular central BCG pictured appears smaller than the others that means that it would appear smaller on the sky as well (the images are all the same size). The right panel shows the distribution of light within the ellipse. It is split in half along its shortest dimension, called the minor axis, so if you have centered it properly on the galaxy the radial plot should appear symmetric about the point $x = 0$.

The bottom two panels contain a table of galaxy properties, including the position of the ellipse on the image, its position angle (0° for a tall ellipse and 90° for a fat one), its axial ratio (0 for a skinny ellipse, 1 for a circular one), the length of the semi-major axis (the longest radius you can fit into the ellipse), D-size based on the semi-major axis, the amount of light contained within the ellipse, and D-flux based on the light. On the right we have the spectrum of each galaxy. As you work with each image, examine the spectrum as well to see if there are any correlations – can you make any predictions about the spectra based on the images, or vice versa?

1. Once in a great while, a network error will interfere with the loading of one of the galaxy images in this tool. A warning message will appear when you first load the tool if this occurs. *If this happens to you, force your browser to reload the application (most will do so if you hold down the control key or the shift key while reloading the page).* If you do not do this, then at least one of your radial plots will not contain any data and you will not be able to complete this exercise.

2. Examine the presence of the green ellipse on top of the image of Galaxy #1. We can vary its eccentricity by changing the axial ratio in the Control Panel, and rotate it in place by changing the position angle. The four arrows shift the ellipse left or right and up or down, and the two buttons to their right make the ellipse grow bigger or smaller. Try changing all of the setting in the Control Panel until you are confident that you understand how they work.

3. Select "hide ellipse" from the list of options at the top of the screen and then "zoom in" to inspect the galaxy image more closely. Then select "show ellipse" to draw the elliptical aperture again.

4. Decide whether the galaxy is more oval or more round, and set the axial ratio for the ellipse accordingly (0 for very elongated, 1 for round). Identify the longest axis of the galaxy, and rotate the ellipse to match by varying its position angle.

5. Use the four position arrows to move the aperture around so that its center coincides with the center of the galaxy (the bright core).

6. Use the two remaining arrows on the Control Panel to increase or decrease the size of the ellipse. Your goal is to have the aperture contain all of the light from the galaxy, including its faint (nearly-invisible) outer regions, and a minimum of light from foreground Milky Way stars, other galaxies, and the background sky.

7. Use the radial distribution of counts plot to finalize the aperture. Make the aperture large enough that the counts are distributed symmetrically on either side of the central position (where $x = 0$). Set the size so that the boundaries (shown as two green vertical lines on the counts plot) are placed where the number of counts starts to rise noticeably upward relative to the background level. You may sometimes need to decrease the aperture size to keep a foreground star or another galaxy outside of the ellipse; sometime you can avoid doing this by instead changing its position angle.

8. Steps 6 and 7 above involve making difficult decisions, and compromising your desire to include all of the galaxy flux with your need to include as little as possible of the light from other stars and galaxies. The key is to be consistent in your approach and to treat all ten

galaxies uniformly. Try to place the ellipse to mark the same brightness level on all images, as best you can. For some galaxies there will be no "perfect" solution; we have given you some difficult cases to consider to illustrate the type of decisions that are often made when dealing with real astronomical data. Read through the following questions before you fit the galaxy images, and keep them in mind as you work. When finished, you'll have two different distance determinations (D-size and D-flux) for all ten galaxies, based on their apparent sizes and apparent brightness.

1. Save a copy of the final galaxy data table and include it in your lab report. (7 points)

2. Save a copy of the image for Galaxy #10 and include it in your lab report. (1 point)

3. As you decrease the size of the elliptical aperture on a galaxy image, D-size (increases / decreases / remains constant) and D-flux (increases / decreases / remains constant). (1 point)

4. If the size of a galaxy is over-estimated (the aperture is larger than the galaxy), D-size will be (too large / too small). (1 point)

5. If the size of a galaxy is under-estimated (the aperture is smaller than the galaxy), D-flux will be (too large / too small). (1 point)

6. Most of the light from the galaxies is uniformly distributed in smooth ellipses around the cores. Why is there a second peak in the radial counts plot for Galaxy #6? (1 point)

7. Which of the ten galaxies posed the largest challenges to fit? Which was the easiest to analyze? Explain your choices. (2 points)

Measuring D-z from Galaxy Spectra

Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled

"Web application #2 (galaxy distances II)."

We will now measure the redshifts of our BCGs, by taking a rest-frame spectrum of a typical BCG (the spectrum as it would appear for a galaxy at rest with respect to the Milky Way) and redshifting it ourselves to match the spectrum of each of our ten BCGs. (The calculation of D-z will differ slightly from the simplified model presented in Example 7.8, to take in to account cosmological effects, but the basic idea will be the same.)

Spectral Tool Tips

The spectral tool interface contains four primary panels, as well as a set of key options across the top of the screen. Start by clicking on the button labeled "Help" to learn about the basic properties of the tool. The largest panel (on the left) shows the spectrum of each galaxy (in pale green) and the rest-frame spectrum of a typical BCG (in pale blue). The controls at the bottom of the panel allow you to shift the rest-frame spectrum left and right (by defining a redshift for it) until it matches up with the observed spectrum.

Use the slider bar to set an approximate value, and then "touch up" the redshift with the more delicate button controls on either side. Note that several strong emission and absorption features have been marked by element in the rest-frame spectrum (Fe for iron, Ca for calcium, Hα through $H\delta$ for hydrogen, Mg for magnesium, and Na for sodium). If these features are strong, then these elements are present in the galaxy.

You can position the rest-frame spectrum two ways: either pay attention to the broad, underlying shape of the continuum (the overall shape of the spectrum), or select the deepest absorption lines in the two spectra and match them up. To fine-tune your redshift value, use the outermost redshift button controls to make small shifts until the correlation coefficient (R) is maximized. When this value, printed next to the redshift on the bottom of the panel, is as close to 100% as possible, you will have found the best redshift for the galaxy. The second panel (from left to right) is a visual representation of the correlation coefficient – try to raise the value of the "correlation thermometer" all the way (to 100%) in each case.

A data table on the right records the galaxy redshifts and derived distances D-z, as well as the final correlation coefficients. In the lower right corner you will find an image of each galaxy. Keep an eye on the images as you fit the spectra, and try to identify trends that you see between the two types of data.

Read through the following questions before you fit the galaxy spectra, and keep them in mind as you work. When finished, you'll have a third distance determination (D-z) for all ten galaxies, based on their redshifts.

1. Save a copy of the final galaxy data table and include it in your lab report. (7 points)

2. Save a copy of the spectrum for Galaxy #9 and include it in your lab report. Are you more confident in your estimates of D-z or of D-size and D-flux for this galaxy? Why? (2 points)

3. As you increase the redshift for a galaxy spectrum, D-z (increases / decreases / remains constant). (1 point)

4. More distant galaxies appear (smaller / larger) on the images, and their spectra shift to the (left / right) to (shorter / longer) wavelengths. (1 point)

5. Which of the ten galaxies posed the largest challenges in determining redshifts? Which was the easiest to fit? Explain your choices. (2 points)

7.5 How Reliable Are Your Distance Determinations?

Which of the three distance estimates for BCGs, D-size based on angular size, D-flux based on apparent brightness, and D-z based on redshift, is the most accurate? Review Examples 7.5–7.7 on pages 15–16, and complete the questions below to explore this topic.

7.5.1 Evaluating Distances Based on Apparent Sizes of BCGs

As Example 7.5 explains, to determine the distances to BCGs from their angular sizes (their apparent sizes on the sky) we must assume that they are all the same actual size. (If viewed from the same distance, they would all appear equally large.) Is this a good assumption? Figure 7.9 shows the distribution of galaxy sizes throughout the sample.

1. Two-thirds of the BCGs will lie within 1σ of the mean size, between ____________ kpc and ____________ kpc. For the remaining third, our D-size distance estimates would be more than 30% too low or more than 66% too high (see Example 7.5). (1 point)

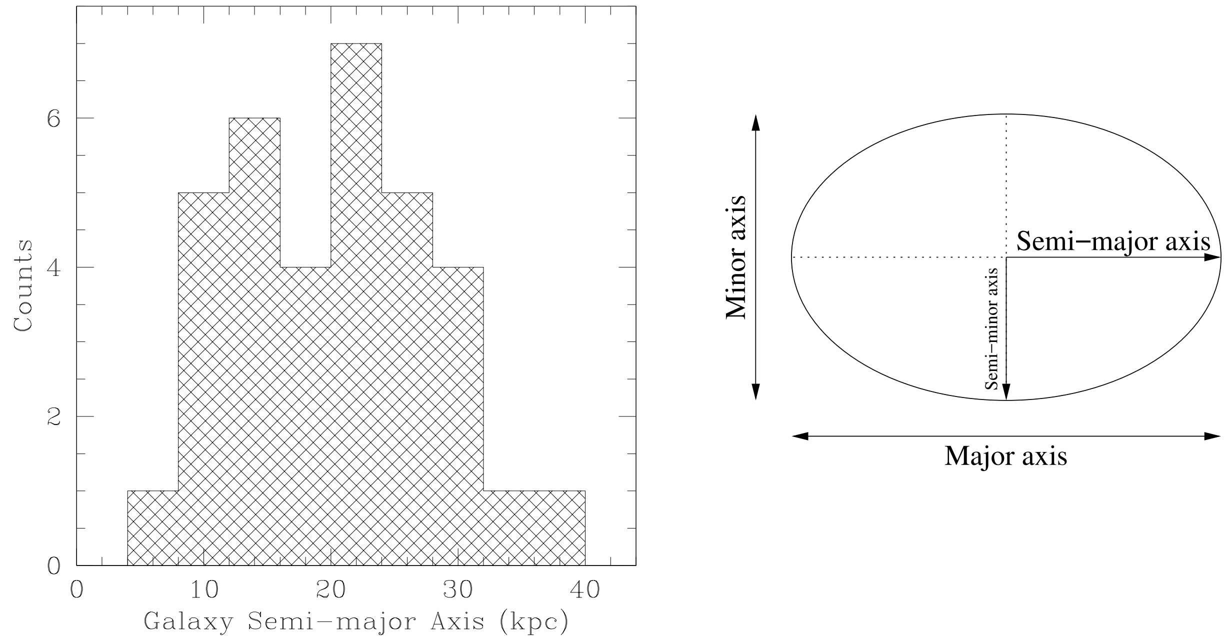

Figure 7.9: This histogram (left) shows the distribution of physical sizes within our sample of 34 BCGs. Semi-major axes are plotted in units of kiloparsecs (kpc), with an average value of 20 ± 8 kpc. The figure on the right illustrates the definition of the semi-major axis for an ellipse (the largest radial length you can fit inside it).

7.5.2 Evaluating Distances Based on Apparent Brightnesses of BCGs

As Example 7.6 explains, to determine the distances to BCGs from their apparent brightnesses (how bright they appear from Earth) we must assume that they all emit the same amount of light. (If viewed from the same distance, they would all appear equally bright.) Is this a good assumption? Figure 7.10 shows the distribution of galaxy luminosities throughout the sample.

1. Two-thirds of the BCGs will lie within 1σ of the mean luminosity, between ____________

and ____________ times as bright as the Milky Way. For the remaining third, D-flux would be more than 16% too low or more than 33% too high (see Example 7.6). (1 point)

2. (D-size / D-flux) is more accurate, on average. (1 point)

7.5.3 Comparing Distance Estimators

We have estimated the distance to each BCG in three ways, from their sizes, their fluxes, and their redshifts. How do these techniques compare? To compare all three methods, we will plot each type of distance estimate against a “gold standard” based on measurements

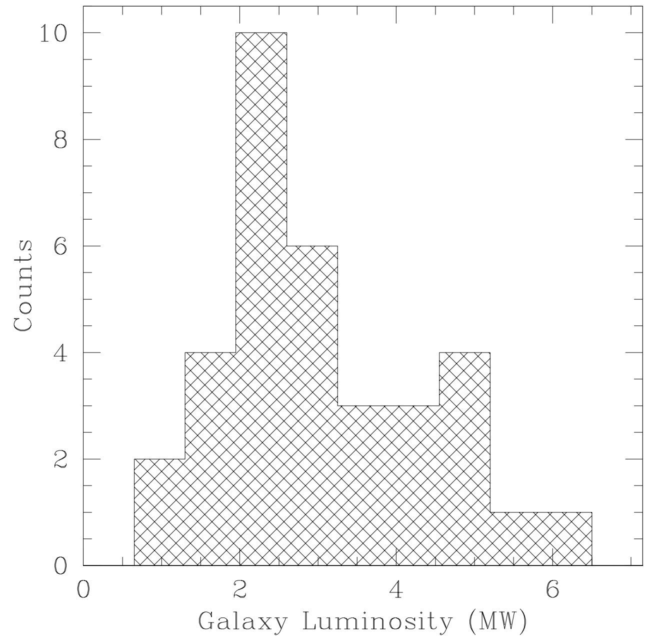

Figure 7.10: This histogram shows the distribution of luminosities within our sample of 34 BCGs. A BCG with a luminosity of unity would emit as much light as the Milky Way galaxy. The average value is 3.0 ± 1.3, relative to the Milky Way.

of the distances to each galaxy cluster. Each of these measurements are based on multiple galaxies within each cluster, so they can leverage multiple measuring techniques and benefit from combining data taken for all of the galaxies within each cluster.

Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled "Web application #3 (distances plotting tool)." Enter your distance measurements for each BCG in the box labeled "Distances to plot: D-size, D-flux, and D-z," placing three measurements on a line for each of your ten BCGs.

Once you have done so you will create a plot comparing D-size, D-flux, and D-z in turn against the distance to each cluster. Each plot window will contain points for 34 BCGs, the ten that you fit and 24 more that we analyzed using the same techniques. Your data will be shown as large magenta points, while the other 24 BCGs will appear as smaller red points.

Let's examine these plots, and see how well our distance estimators worked.

1. Be sure to save a JPG- or PNG-format copy of the plot to your computer, to include in your lab report. (7 points)

2. Do the ten points from your data set appear to follow the same trend as the 24 additional points? If not, how do they differ, and why do you think this might have occurred? (1 point)

3. If all of the distance estimators were "perfect", what values would you expect for a slope and a y-intercept for each window? (1 point)

4. The two distance estimators (D-size / D-flux / D-z) had the worst rms deviations (highest σ values) when plotted against the cluster distances. (1 point)

5. Distance estimator (D-size / D-flux / D-z) had the best rms deviation when plotted against the cluster distances. Do the points in this plot also appear to follow the best-fit line most closely, just looking by eye? (yes / no) (1 point)

6. Why do you think that this technique produced the best results? (2 points)

7.5.4 The Range of BCG Properties

Figures 7.9 and 7.10 illustrate the fact the BCGs exist over a significant range of sizes and luminosities. By assuming that they were all the same size and emitted the same amount of light, we were able to make only rough approximations of their distances. Let's now plot size against luminosity in Figure 7.11, and learn a bit more about these galaxies.

1. Is the most luminous BCG also the largest one? (yes / no) (1 point)

2. Is the least luminous BCG also the smallest one? (yes / no) (1 point)

3. What percentage of the 34 galaxies are either brighter and smaller than average, or fainter and larger than average? (2 points)

4. In general, BCGs which are intrinsically brighter than average are also (larger / smaller) than average, while those which are intrinsically fainter than average are also (larger / smaller) than average. Explain this result, assuming that the luminosity of a galaxy is just the total amount of light emitted by all of its stars. (2 points)

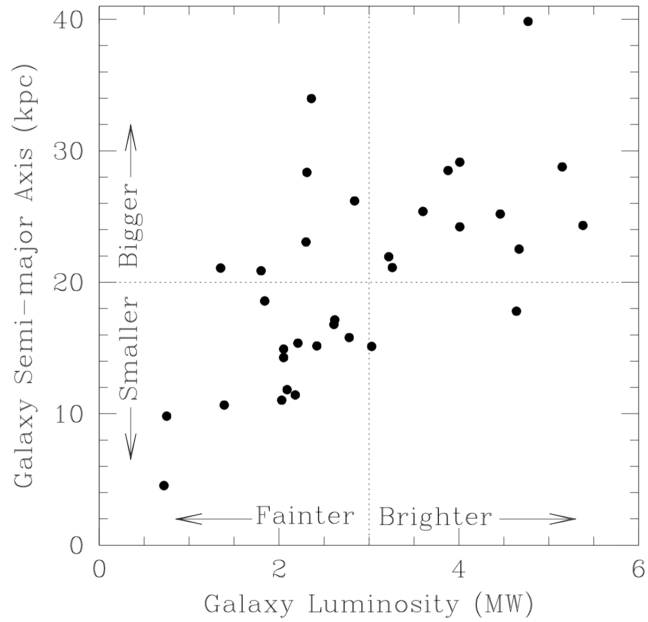

Figure 7.11: The distribution of physical sizes and luminosities within our sample of 34 BCGs, with the same units as those used in Figures 7.9 and 7.10. The horizontal dotted line indicates the semi-major axis of the average BCG (20 kpc), so galaxies above this line are larger than average and galaxies below this line are smaller than average. The vertical dotted line indicates the luminosity of the average BCG (three times that of the Milky Way), so galaxies to the right of this line are brighter than average and galaxies to the left of this line are fainter than average.

7.6 Telescopes Are Time Machines

We've been working with observational data that support one of the greatest discoveries of all time, the fact that the Universe is expanding outward. Galaxies are moving further and further away from us with every passing second, and also moving further and further away from each other. Not only are most galaxies rushing away from us – as shown by all those redshifted spectra – those farther away are moving proportionally faster.

Example 7.8
Though you may not have realized it, as you worked with images and spectra of distant galaxies you were traveling through time. It takes light a whole second to travel just 300,000 kilometers. When we examine the light emitted by distant galaxies which lie hundreds or thousands of megaparsecs away from us, we see them not as they are today, but as they were billions of years in the past.

The light emitted by a galaxy with a redshift of 0.10 was emitted more than a billion years ago (1.1 billion years), and the photons from one at a redshift of 0.30 left their home more than three (3.3) billion years in the past. Imagine an astronomy student living in a galaxy with a redshift of 0.40 examining images of our own Milky Way galaxy – from their point of view, our solar system would still be in the throes of formation, and life on Earth would be an unknown possibility of the far-flung future!

7.7 Final (Post-Lab) Questions

1. In the 1930–1950 era the Andromeda Galaxy was thought to lie a mere 750,000 light years away, but this distance was increased to two million light years in later years. What happened? The Andromeda Galaxy did not leap and skip across the cosmos! The calibration for the Cepheid period-luminosity relationship improved, however, changing all distances which depended on it. Similar corrections occurred in the 1970–2000 era, producing large changes in cosmologically-based distances determined from measured redshifts, as the value of the important constant known as ____________ also became more accurate. (1 point)

2. How do the BCGs differ qualitatively and quantitatively from our Milky Way galaxy? (Consider galaxy sizes, luminosities, colors, types, and environments.) (2 points)

3. What is the redshift of the most distant BCG that you examined? Roughly how long ago was the light captured in the image that you analyzed emitted by the galaxy? (2 points)

4. If it takes light a full second to travel 300,000 kilometers, how long does it take to travel one parsec, or even 400 Mpc, in units of years? (A parsec is equal to 3×10^{13} kilometers, and there are $\pi \times 10^{7}$ seconds in a year.) (3 points)

5. We can use the Hubble constant H_0 to estimate the age of the Universe. We observe that the Universe is expanding; clusters of galaxies are all moving apart from one another. In the past the galaxies were closer together, and if we go back far enough in time we get to a point where this expansion started. We call that event – the start of the expansion of the Universe – the "Big Bang." As the Universe expands, more distant galaxies are observed to be moving away faster since there is more expanding space between us and them. There is thus a correlation between the distance and the time. Let's see how that works!

If a friend says that they drove at a velocity v of 60 miles per hour and covered a distance d of 120 miles, you instantly know their travel time t (two hours). Velocity equals distance per unit time, so

$$v = \frac{d}{t}, \quad \text{and } t = \frac{d}{v}. \tag{7.18}$$

In this case,

$$t = \frac{d}{v} = \frac{120\,\text{miles}}{60\,\text{miles per hour}} = \frac{120}{60}\text{ hours} = 2\text{ hours}. \tag{7.19}$$

We can perform the exact same calculation for galaxies, using the fact that $H_0 = v/d$.

$$t = \frac{d}{v} = \frac{1}{H_0}. \tag{7.20}$$

Until now we have expressed the units of H_0 as km sec^{-1} per Mpc. We need to cancel the units of length (kilometers and megaparsecs), leaving seconds (a unit of time).

$$t = \frac{d}{v} = \frac{1}{H_0} = \frac{1}{72\text{ km sec}^{-1}\text{ per Mpc}} = \frac{1}{72}\frac{\text{Mpc-sec}}{\text{km}}. \tag{7.21}$$

(a) One parsec is equal to 3×10^{13} kilometers, so how many kilometers are there in a megaparsec (in 10^6 parsecs)? (1 point)

Let's call this value L. We can now replace the Mpc in our expression for t with "L kilometers".

$$t = \frac{1}{72}\frac{\text{Mpc-sec}}{\text{km}} = \frac{1}{72}\frac{L\text{ km-sec}}{\text{km}} = \frac{L}{72}\frac{\cancel{\text{km}}\text{-sec}}{\cancel{\text{km}}} = \frac{L}{72}\text{ seconds}. \tag{7.22}$$

(b) There are $\pi \times 10^7$ seconds in a year, so what is the age of the Universe, in units of billions of years? (3 points)

$$t = \frac{L}{72}\text{ seconds} \times \frac{\text{year}}{\pi \times 10^7\text{ seconds}} = \frac{L}{72\pi \times 10^7}\text{ years}. \tag{7.23}$$

(c) If the Hubble constant H_0 were twice as large, the age of the Universe would be (twice as large / half as large / unchanged). (1 point)

7.8 Summary

After reviewing this lab's goals (see §7.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. (24 points)

Be sure to cover the following points.

- Describe various methods astronomers employ to obtain distances to celestial objects.
- Define "redshift", and discuss how it is determined from a galaxy spectrum.
- Explain what Hubble diagrams, Hubble's Law, and the Hubble constant are, how they relate to determining distances to other galaxies, and why they are important.
- Discuss how the distance to the brightest galaxy in a cluster (BCG) can be determined several ways (by assuming that all BCGs are the same size, or that they emit the same amount of energy, or by measuring their redshifts), and how reliable the derived distances are in each case.

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

7.9 Extra Credit

The Hubble constant H_0 determines both the size of the Universe and its age. For example, the extent of the observable Universe can be described in terms of the distance to the galaxy with the greatest measured redshift. This redshift tells us how far away the galaxy lies, and how much time has passed since the light that we see from it was emitted (the "look-back" time). Both values depend on H_0.

Research this topic briefly, and provide the latest numbers associated with the most distant galaxy ever observed and the age of our Universe. Describe how these values would change if the value of H_0 was revised to be greater or smaller. (5 points)