Lab 2

Observing the Sky

2.1 Introduction

Early humans were fascinated by the order and harmony they observed in the celestial realm, the regular, predictable motions of Sun, Moon, planets, and "fixed" stars (organized into patterns called constellations). They watched stars rise in the east, their height (or altitude) above the horizon steadily increasing with each passing hour of the night. After they reached a high point, at culmination, they were observed to slowly set in the west. Other stars never rose or set, but rotated once a day around a fixed point in the sky (the celestial pole).

The ancients recognized seven objects that moved slowly with respect to the fixed stars. These were the Sun, the Moon, and five bright planets (from the Greek word for "wandering star"). The Sun and Moon moved steadily eastward through a band of twelve constellations located above the Earth's equator called the zodiac. As people spent a lot of time outdoors without artificial lighting, changes in the appearance of the Moon were obvious to them. They tracked the Moon's phase (fractional illumination) from night to night, as it grew from new to full and back to new each month (or "moonth").

The Sun moves more slowly through the sky. Its journey once around the zodiac takes a full year, and its changing position heralds seasonal changes. The ancient Babylonians, Egyptians, Mayans, Chinese and other civilizations designed calendars based on their observations of the Sun and Moon, and carefully watched the skies. In New Mexico, ancient pueblo peoples used celestial patterns to determine when to plant crops in spring, when to prepare irrigation ditches for summer rains, and how to orient buildings to maximize solar heating in winter. Buildings and roads constructed between the 10^{th} and 12^{th} centuries in the Chaco Canyon region (between present-day Albuquerque and Farmington) follow geometric patterns and orientations related to lunar and solar cycles based on astronomical knowledge that must have been gained over generations.

Celestial studies contributed to the development of science, if only through the realization that there is order in the natural world. Scientists attempt to make sense of observations of natural phenomena. Their conceptual models enable predictions of future events, leading in time to the development of useful tools and technologies.

Consider models of the night sky. For a long time, people held to a geocentric model in which all celestial objects moved in circles around a stationary Earth. This idea was rejected only a few hundred years ago, as we began to appreciate that "things are not always as they appear." Despite what our eyes tell us, few celestial objects actually circle the Earth. Their apparent daily motions from east to west are caused by the Earth spinning around once on its axis each day in the opposite direction. The Sun's steady advance eastward (with respect to the stars) is not due to its circling the Earth once a year, but rather to the Earth revolving westward around the Sun, taking a year to make a complete circuit.

This more sophisticated understanding allowed humans to construct more accurate calendars, and to use the stars to navigate reliably across the Earth's surface. The ancient Polynesians used the stars to steer small vessels for thousands of miles to the small island we call Hawaii. By the 19^{th} century, ship navigators were able to establish latitudes and longitudes using a sextant, a handheld instrument for measuring angles, and a marine chronometer, a clock for keeping time precisely at sea. Modern global positioning system (GPS) technology is even more reliable and precise.

In this lab, you will use a simple sextant to determine your latitude (how far you lie above the equator) by measuring the altitude of the North Star. You'll also find it another way, as one might have needed to do when at sea if clouds to the north obscured the North Star: you'll measure the altitude of a first quarter Moon appearing due south around sunset.

The Moon is the one celestial object that actually does circle the Earth. You'll track its changing appearance and estimate the fraction of the visible lunar disk that is illuminated by sunlight at various times over an eight-day period. During this time, you'll also use a sextant to measure the angle between the direction to the Moon and the direction to the Sun. After considering the rate at which the Moon moves, you'll plot its fractional illumination versus time and look for a mathematical correlation.

2.1.1 Goals

The primary goals of this laboratory exercise are to understand the monthly orbit of the Moon, relate its changing appearance and location in the sky to its phase, illumination and elongation angle, and to visualize the positions of the Moon, Earth, and Sun throughout this cycle. We will learn about the daily and yearly motions of the Earth and Moon, and how they affect the appearance of our night and day skies, and the passage of the Sun, Moon, and stars through them. We will also develop our skills at reading figures, conducting observations, recording data accurately, and analyzing the results.

2.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed here.

http://astronomy.nmsu.edu/geas/labs/labs.html

You will need the following items to perform a simple experiment:

- several sextants (provided on pages 33 through 37)
- two pieces of 10.5 inch by 6.5 inch thick cardboard
- the cardboard tube from the center of a roll of paper towels or toilet paper
- a needle, a pin, and two 9-inch pieces of brightly colored thread
- a $2^1/_2$ foot piece of string
- a pair of scissors, and a roll of tape
- a ruler, yardstick, or measuring tape
- a straight drinking straw
- a paper clip
- a pencil or pen
- a calculator (or calculator app)
- a flashlight (or flashlight app; with a red filter or red cellophane cover, if possible)
- a friendly assistant (helpful when making observations)

Your assistant could be an adult or an older child, and needs no special knowledge of astronomy. S/he can assist you in measuring angles on a protractor, and holding your measuring apparatus still, at a particular angle while you use it.

You will also need a computer with an internet connection, to analyze the data you collect from your experiment.

2.1.3 Primary Tasks

This lab is built on one major observing activity (tracking the position and appearance of the Moon over an eight-day period), and two short sky observations that can be completed in one sitting. Most observations will be done during the day, and you will have quite a bit of flexibility in scheduling them. One will take place near to sunset, and one in the evening.

Make sure that you have a safe place to observe the sky outdoors. A flat location with some free space around it will help to ensure that no buildings block your view of the Moon or Sun in the sky. When you work at dusk or in the dark, make certain that any cars which might come along can see you clearly.

Because we cannot schedule the rising and setting of the Sun or Moon for our convenience, you may need to begin your observations for this lab exercise a couple of days early, or to end them a day or two late (before or after the beginning or the end of the two-week period associated with this exercise). Be sure that you understand which days you will be observing the Moon, and discuss any questions that you have with your instructors well ahead of time.

It is critical that you are ready to observe the Moon at dusk three days after the new moon phase, and near sunset when the first quarter moon culminates 3 or 4 days later. Make a note of the dates of these two events as soon as you begin this exercise, so that you are prepared. Your other seven daily observations of the Moon can then occur with some flexibility as long as they are completed over the next ten evenings, by the full moon.

2.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. The 29 questions for this exercise (including the final questions), are worth 2 points each, for a total of 58 points. The data table in Section 2.2.3 (§2.2.3) is worth 12 points. Note that §2.5 contains 5 extra credit points.

Table 2.1: Breakdown of Points

Activity	Moon	First Quarter	North Star	Questions	Summary
Section	§2.2.3	§2.2.5	§2.2.6	§2.3	§2.4
Page	8	17	24	28	29
Points	30	18	14	8	30

2.1.5 Timeline

You may need to make your first observations of the elongation and illumination of the Moon a few days before Week 1 officially begins, as we cannot alter the timing of the lunar month. Check with your instructor ahead of time to determine if this is the case.

Week 1: Read §2.1–§2.2, begin activities in §2.2.3, §2.2.5, and §2.2.6, and begin final (Post-Lab) questions in §2.3. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report template, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Complete observations, finish final (Post-Lab) questions in §2.3, write lab summary, and submit completed lab report.

2.2 Observing the Sky

Pay close attention to the figures in this section, and use your visualization skills. You will learn the most from the figures if you first examine them, reading the captions and making sure to understand each aspect, then read the relevant accompanying text, and then review the figures to make sure that you understand them. If you are confused by a drawing, try recreating it piece by piece for yourself in your notes, adding each component as you determine its purpose. There is a real skill to reading figures accurately, one that we are developing in this course.

2.2.1 The Position of the Sun in the Sky over a Year

The Sun is the brightest celestial object that we can observe. Its position above or below the horizon determines whether it is daytime or nighttime here on Earth. We see no other stars during the day, because the Sun outshines them all. Before discussing the apparent motion of the Sun over the course of a day and throughout the year, let's define a few terms.

The Sun lies below the horizon at night – so the ground (the Earth) blocks our view of it. The horizon forms a circle around us, where the ground and sky appear to meet. Imagine yourself at the center of this circle, looking eastward. As you turn your head to the right and look south, you will shift through one-fourth of the circle, or 90°. The angle between east and south is thus 90°. If you continue turning your head rightward until you've seen the horizon in all directions, you will have rotated once around a complete circle – through 360°. Now imagine moving your head up and down rather than to the side. As you direct your eyes up from the horizon to an imaginary point directly overhead (the zenith), your view will again move through one-fourth of a circle, or another 90°.

We can depict the apparent motion of celestial objects by tracing their changing positions on the "celestial sphere," an imaginary dome which extends across the entire sky, pictured in Figure 2.1. Place yourself in the position of the observer on the left, drawn in blue at the center of this celestial sphere. Imagine you are looking eastward toward the horizon on the first day of Spring (March 21, for the northern hemisphere). At sunrise, the Sun will appear on the horizon directly east of you. It rises every morning in the east and sets hours later in the west. On the first day of Spring, however, it rises exactly due east of the observer and sets exactly due west, twelve hours later. As the figure illustrates, it will reach its highest elevation (altitude) above the horizon at local noon. When an object stops rising through the sky and begins to set, we say that it is "culminating" (or transiting).

Using Figure 2.1, orient yourself due south and imagine raising your eyes up from the horizon toward the zenith (directly overhead), and then continuing to bend backwards (careful!), tracing a line all the way across the sky to the northern horizon. You will have traced a half-circle across the sky, one starting due south of you, passing overhead, and then descending to the north. This arc is called your "meridian" (a line of constant longitude). As celestial objects culminate, they intersect the observer's meridian. An object's altitude at culmination is the angle between the direction to the southern (or northern, if nearer) horizon (marked "S" and "N"), and the direction to the object (labeled culmination). Note that the largest altitude possible is 90° (for an object lying directly overhead, at the zenith.)

The Sun culminates every day at local noon. From the continental United States, it will

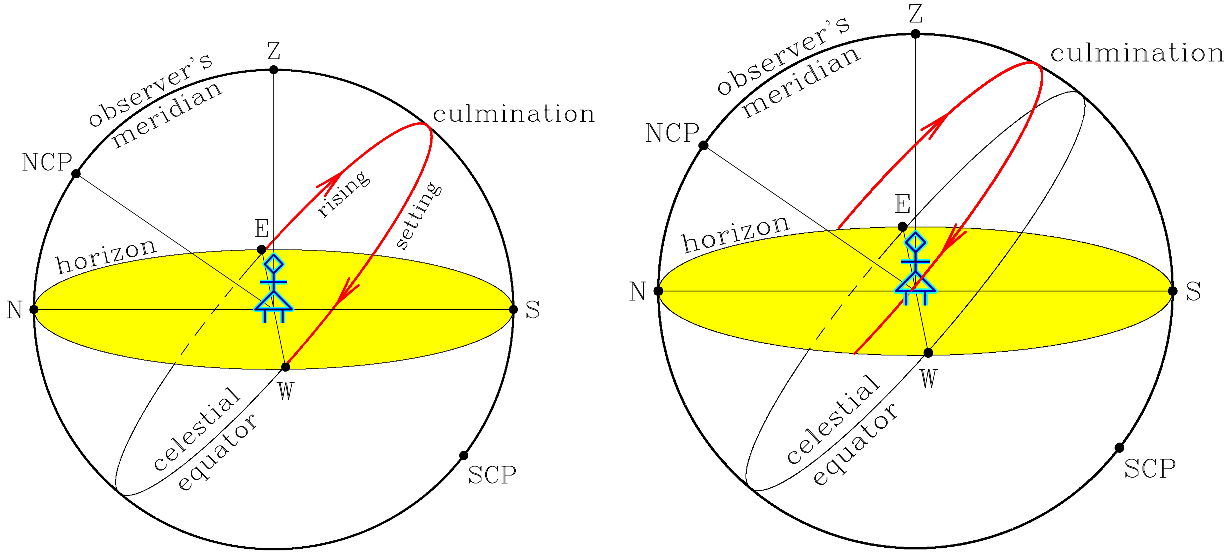


Figure 2.1: This figure identifies landmarks used when observing, both in an observer's frame of reference (their local horizon, stretching off to the north and south, and east and west, and the sky above them), and along the celestial sphere that surrounds the Earth. The red line indicates the path of the Sun through the sky over the course of a day on the Spring Equinox (March 21, on the left), where the Earth is tipped neither toward nor away from the Sun, and three months later on the Summer Solstice (June 21, on the right), where the Earth's North Pole is tipped over by 23° toward the Sun and the Sun thus appears to rise and set in the north, and to culminate higher in the sky.

always lie to the south (it appears in the north when viewed from much of the southern hemisphere). Its altitude at culmination will depend upon the time of year, or how much the Earth's North Pole is currently tipped over toward (or away from) the Sun.

Consider the Sun's position on March 21, when the Earth is tipped neither toward nor away from the Sun. With the Earth effectively upright, the Sun will be located in the same plane as the Earth's equator, along the celestial equator (the projection of the Earth's equator onto the celestial sphere). Its path through the sky on that day is shown in red on the left-hand side of Figure 2.1.

From March 21 to June 21, the Sun will shift steadily north of the celestial equator. As the Earth's North Pole tips over toward the Sun, the Sun will appear higher and higher in the sky when viewed from the continental United States. Its altitude at culmination will increase with each passing day. The right-hand side of Figure 2.1 shows its path (drawn again in red) through the sky on June 21, appearing north of the celestial equator at all times. For the next three months the North Pole will slowly tip back away from the Sun, causing the Sun to descend back to the celestial equator. On December 21, the first day of northern winter, it will lie 23° south of the celestial equator, culminating with its lowest altitude of the year.

2.2.2 The Position of the Moon in the Sky over a Month

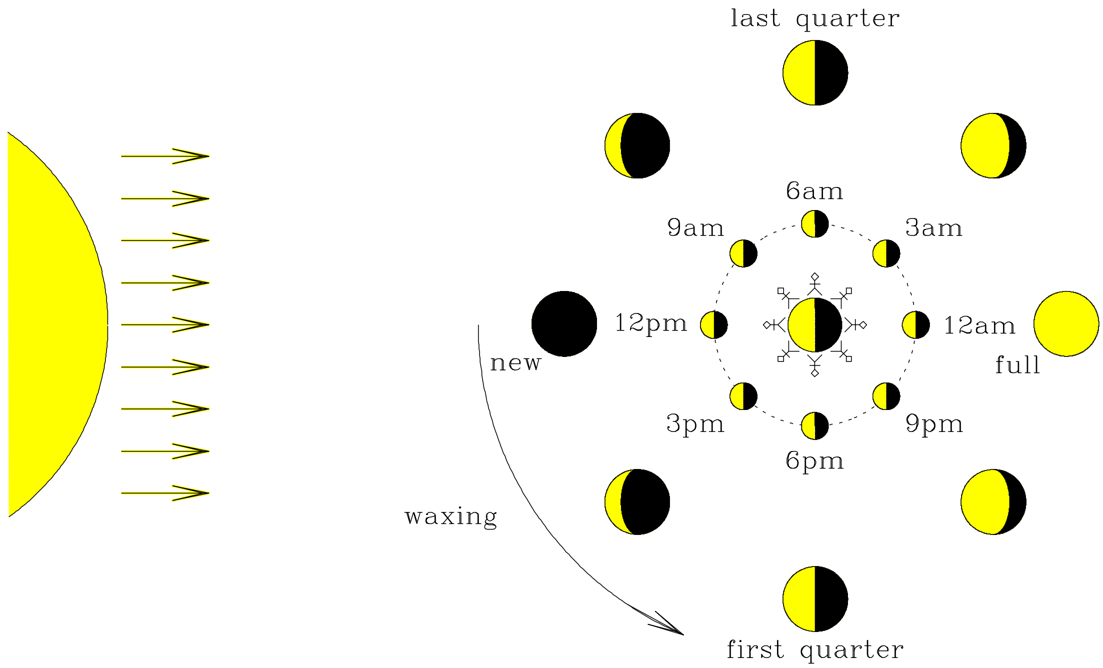


Figure 2.2: This figure illustrates how the light from the Sun (left) always illuminates the half of the Earth and the Moon that face toward it, and how the Moon's orbit around the Earth then makes it appear to pass through a sequence of phases (partially illuminated states) every lunar month. The small people on the Earth's surface are spaced out to cover eight locations around the equator, and the small moons indicate the movement of the Moon around the Earth every 29.5 days. The larger moon sketches surrounding the Earth and Moon indicate *the appearance of the Moon as viewed from Earth* at these times. The Moon begins in the new phase, dark in the sky and culminating around noon, and then its perceived illumination waxes (increases) for two weeks as the Moon moves around the Earth through the waxing crescent, first quarter, and waxing gibbous phases, until it appears on the opposite side of the Earth than the Sun, as a "full" moon, culminating round midnight. It then takes another two weeks to swing back around in front of the Earth, now appearing to wane (decrease) in illumination each night.

Having examined the yearly apparent movement of the Sun through the sky, let us now turn our attention to the Moon. To understand its movement and changing appearance, we will need to take into account the relative positions of the Moon, Earth, and Sun.

Just as the Earth takes a year to orbit once around the Sun, the Moon takes a month (29.5 days) to orbit once around the Earth. Figure 2.2 shows the Moon in eight evenly spaced positions around the Earth; it orbits the Earth in a counter-clockwise direction when viewed from above the North Pole. The inner ring of Moons, and the Earth that they surround, are all illuminated on the side that faces the Sun, and dark on the side which is cast in shadow. Eight stick figures below them represent observers looking up at the sky; in each case, *the phase of the Moon is determined by the fraction of the illuminated portion of the*

Moon which can be seen from Earth. The outer ring of Moons shows its appearance, or phase. Walk through each of the eight observer's positions in turn. Your impression of how much each of them can see of the illuminated portion of the Moon above them should match the phase shown along the outer, larger ring of Moons.

The Sun defines the time of day: it rises in the east at dawn, ascends high in the sky at local noon (when it casts the shortest shadows), and then sets at dusk in the west. We can similarly tell what time of the month it is from the Moon.

At local noon, the Sun culminates high in the sky. If the Moon appears near to it, the illuminated portion will face away from the Earth, toward the Sun. We call this a "new" moon, and it is very difficult to even find in the sky. Over the next few days of the month the Moon slowly orbits around the Earth, shifting away from the Sun. As a small portion of its illuminated side begins to peek into view from Earth, it takes on the shape of a thin, waxing (increasing) crescent. This crescent grows with time, over a week. The Sun, Earth, and Moon now make up a right angle, and so we see a half-illuminated lunar disk from Earth (overhead as the Sun sets at dusk). We call this the "first quarter" moon, because one-fourth of the lunar month has passed since the new phase. As the Moon continues to move around the Earth, more and more of the illuminated portion comes into view. The Moon is more than half-illuminated, and so we call it a "gibbous" (or bulging) moon. After two full weeks have passed, the Sun and Moon will lie on opposite sides of the Earth. The Moon will rise as the Sun sets, and its entire disk will be illuminated (a "full" moon).

The Moon now comes around to the other side of the Earth, and over the next two weeks dwindles from the full back to the new phase. It spends almost a week in the waning gibbous phase, each night rising later in the evening and becoming less illuminated. After three weeks, the Moon again makes a right angle with the Earth and Sun, and we call it the "last quarter" moon. Over the fourth, final week of the month, the Moon slowly sneaks around back in front of the Sun, becoming harder to see by eye and taking on a thin crescent appearance again.

If we know the phase of the Moon, we know roughly how many days have passed since the last new phase. We can predict when it will rise, pass overhead, and set, as long as we know the angle created by the Sun, Earth, and Moon (the lunar elongation angle).

2.2.3 The Elongation and Illumination of the Moon

Our primary observing project is to measure the elongation of the Moon, the angle formed by the Moon, Earth, and Sun. We will begin by constructing a simple sextant, to measure the angle between two positions on the sky. True sextants are mechanical devices made of metal and glass, combining the properties of a small telescope with the ability to measure small angles precisely. Our devices will be much simpler, but we'll be able to construct them quickly out of common household materials.

Cut out the sextant template found on page 33, being careful to form straight edges and cut along the *outer dotted line* on the page. You will have a large rectangle with a protractor drawn on it, with a small tab attached to the lower-left corner. Fold the lowest 20% of the tab up by 90°, along the dashed line. Then fold the rest of the tab to the right by 90°, making a sharp crease so that the "lollipop" shape drawn on the tab stands upright. You should have formed a small, three-sided square corner on the lower-left corner of your sextant. Tape the raised edges into place, so that this corner is stable (and as straight as possible). Tape the whole sextant securely to a piece of cardboard now, so that it will stay flat and be rigid.

Thread a needle and stick it through the center of the sextant (the dot centered between the "0" and the "180"). Pull just enough thread through that you can tape it securely to the back of the cardboard, and do so. Pull the other end of the thread tight, and tape it loosely to the edge of the sextant (beyond the ring of numbers) so that it will stay out of the way for now. Take the pin and stick it through the center of the sextant, leaving just enough of it sticking up to form a second lollipop shape, with a 1/3-inch base. (You want the head of the pin and the center of the lollipop symbol on the tab to both be at the same height above the protractor, roughly 1/3 of an inch.) Tape over the sharp end of the pin with a small piece of tape, so that it cannot scratch anyone.

Your sextant is now ready for use! You are going to use it to find the angle between a line pointing to the Sun and a line pointing to the Moon on the sky. You want to align the Sun to fall at zero degrees along the edge of the sextant, and then place the sextant in the plane that contains both the Sun and the Moon, and read off the angle of the Moon. You will place the sextant so that the Sun lies along the zero degree position, but **do not look directly at the Sun**. Instead, observe the shadow the pin casts – line it up so that the shadow of the pin head appears in the center of the lollipop mark on the raised tab.

Once you have the sextant pointing at the Sun, rotate the protractor so that the Moon lies along it as well, at some angle between 0° and 180°. Then adjust your position so that the pin shadow is again centered within the lollipop symbol. (You may have to go back and forth a few times, but with practice this will become easier to do.) With the Sun and the Moon in place, adjust the thread to run from the pin to the Moon. As the Sun lies at zero degrees, the thread will lie along the angle between the Sun and the Moon!

If the Moon lies quite close to the Sun (in the crescent phase), you will find it helpful to have an assistant. S/he can look over your shoulder and check that the shadow of the pin stays in place as you focus on moving the thread to mark the angle to the Moon, or vice versa.

Measuring the crescent Moon elongation without an assistant

If you cannot scrounge up an assistant at dusk during the crescent moon phase or keep your eye on both the pin shadow and the Moon simultaneously, we have an alternative plan. (The sextant works very well for the first quarter through full phases, but can be tricky to use when the Moon is close to the Sun.) This technique will be simpler, but less accurate. Wait until near sunset (when the Sun will be less bright), and then estimate the angle between the Sun and the crescent moon by measuring it with the knuckles of your clenched hand, held outstretched at arm's length.

First determine the distance between the setting Sun and the moon in units of the width of the knuckles (all four fingers) as your fist is clenched at arm's length. You might find that you can place your fist 2.6 times in the gap, for example. Measure the gap three times.

Next, convert the width of your knuckles into an angle on the sky (see Figure 2.3). First measure r, the width of your fist, with a ruler. Then measure d, the distance between your eyes and your outstretched fist held at shoulder height. (You can stretch a piece of string out from your fist to your eyes, and then measure the length of the string.) Divide r by d, and find this ratio in the r/d ratios listed in Table 2.2. Read off the value for the angle α to the *left* (not right) of the r/d value closest to your value. This number is the width of your outstretched fist, in units of degrees, and should be roughly eight or ten degrees.

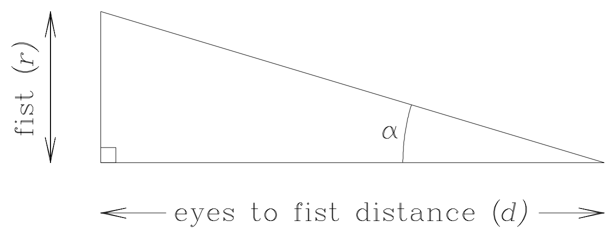


Figure 2.3: You can form a right triangle with your fist at arm's length, and then determine the angular width of your knuckles. The height of the triangle r is equal to the width of your fist, and its side d is equal to the distance between your fist and your eyes (roughly the length of your outstretched arm). When you estimate the size of an angle on the sky with your fist, each fist-width is equal to α degrees.

To determine α, we formed a right triangle that is as long as your arm and as tall as your fist. The ratio of these values determines a unique angle that corresponds to the angular width of your fist on the sky. If you found that your knuckles were 10° wide, for example, then you should be able to walk your arms through a right angle by placing each fist in turn on top of the other, and counting out nine fists, from the horizon to the zenith (overhead).

Table 2.2: Height-to-Side Ratios (r/d) for Right Triangles

α (°)	r/d	α (°)	r/d	α (°)	r/d	α (°)	r/d	α (°)	r/d
4.0	0.0699	6.0	0.1051	8.0	0.1405	10.0	0.1763	12.0	0.2126
4.5	0.0787	6.5	0.1139	8.5	0.1495	10.5	0.1853	12.5	0.2217
5.0	0.0875	7.0	0.1228	9.0	0.1584	11.0	0.1944	13.0	0.2309
5.5	0.0963	7.5	0.1317	9.5	0.1673	11.5	0.2035	13.5	0.2401

Now multiply your three values for the number of fist-widths between the Moon and the Sun by α, and you will have converted your angular measurements from fists into degrees!

If you use this technique for observing the crescent moon, record your values for r, d, and α below so that your instructor can check your work. Don't forget to include units.

r = ____________________, d = ____________________, α = ____________________.

We will be measuring the Moon–Earth–Sun (MES) angle over an eight-day period between the new and full moon phases, and observing the apparent phase and fractional illumination of the moon as well. You may work whenever both the Sun and the Moon are above the horizon. Use your lunar phase wheel to work out good observing times that work best with your schedule. We recommend waiting until three days after the new moon to begin, so that enough of the Moon is illuminated to make it clearly visible in the sky.

As you make all lunar observations, be careful to never stare at the Sun! Doing so would cause headaches, and could damage your eyes permanently.

Close to the new moon, the Moon and Sun will be close together, both above the horizon for most of the day. As the lunar month progresses, however, the Moon will lag behind the Sun more and more, until by the full moon it will be just rising as the Sun sets. By this point, your daily observing window will have narrowed to sunset. In addition, within several days of the new moon you will find it easiest to observe at sunset, because the Sun, lying so close to the Moon, will be less bright at dusk. Make sets of three independent observations at a time, once a day if possible, starting three days after the new moon. If you have to skip a day here and there due to cloudy weather or a busy schedule, that is understandable.

Because the Moon takes 29.5 days to orbit once around the Earth, it travels more than ten degrees every 24 hours. You could take observations twice over the course of a day and expect to see a small change over a few hours, if your measurements were accurate enough.

Record your MES measurements in Table 2.3. Each day, record the date and the time of day (to the nearest minute) in the first two columns, and then place your three measurements of the MES angle in the fourth through sixth columns (to the nearest degree). Record the time of the middle observation, as this should be a reasonable "average" time.

Don't worry if your three measurements differ by more than a few degrees on some days. Though you should try to be as careful as possible, realize that the primary purpose of taking three measurements is to estimate how difficult it is to use the sextant apparatus accurately.

The third column stores the number of days between your observations and the nearest new moon, to the nearest thousandth of a day (0.001 days is roughly one minute). Look up the time of the nearest new moon on the GEAS project lab exercise web page (see the URL on page 3 in §2.1.2). New moon times are listed there in a table, accessed from the link labeled "Table #1 (lunar phase dates)". Add the date and time to the title of your table, and below.

1. The nearest new moon is on _________________________, at _____________________.

<u>Example 2.1</u>
Let's practice finding the amount of time between an observation and the time of the nearest new moon. Imagine that you made your first observation on April 6, 2011 at 7:20 pm. We would inspect the table of lunar phases, and see that the nearest new moon date to April 6 is a few days earlier, on April 3 at 8:32 am.

Table 2.3: Lunar Observations near to New Moon on ______________________________

Date	Time[1]	ΔTime[2] (days)	Moon–Earth–Sun Angle[3] (°)			$\mu \pm \sigma$ (°)	Illum.[4] (%)	Phase[5]

(12 points)

[1]Time of day of the observations to the nearest minute (such as 6:52 pm).
[2]Time since or till the nearest new moon phase in units of days, to the nearest thousandth of a day (such as 2.456 days).
[3]Elongation angle formed by the Moon, Earth, and Sun. Measure this angle three times in a row each day, and then enter the average value and standard deviation as $\mu \pm \sigma$.
[4]The fractional illumination of the Moon, estimated by comparison to the pictures in Figure 2.4.
[5]The approximate phase of the Moon (such as waxing thin crescent, or waning gibbous).

We first count the number of full days between April 3 and April 6 – 3 days. We then calculate the number of hours between 8:32 am and 7:20 pm: 3:28 hours between 8:32 am and noon plus 7:20 hours in the afternoon, or 10 hours and 48 minutes. Three days plus 10 hours and 48 minutes sums to $3 + {}^{10}/_{24} + {}^{48}/_{1440} = 3.450$ days, as there are 24 hours in a day and 1440 (24×60) minutes in a day. We enter this value in the third column of Table 2.3.

We next combine our three measurements of the MES angle, and find the mean value and standard deviation. Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled "Web application #1 (plotting tool)." Scroll down until you see the text box labeled "numbers to plot", and enter your three MES measurements into the box. Do not add any commas, parentheses, or other extra characters – just type the three numbers. The mean (average) value of your three measurements will appear above the box, followed by the standard deviation. Record these values in the seventh column of Table 2.3.

As you make your observations, compare the appearance of the Moon with the images in Figure 2.4. Estimate the fraction of the lunar disk which is illuminated, by taking into account the values listed for each image on the figure, and place this value in the eighth table column. (This should be a fraction, a number between 0 and 1.) In the final column, make a note as to the phase of the Moon (thin crescent, just past first quarter, ...). Your table row should now be complete!

Once you have filled Table 2.3 with with eight sets of measurements, you will be ready to analyze the results. Your first task is to explore the relationship between the observed elongation of the Moon and the amount of time since the new moon. Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled "Web application #1 (plotting tool)." Select "Y versus X plot, with errors in Y" from the pull-down menu, to plot the MES angle and associated errors (σ) against time (t).

Place t on the x-axis (the horizontal axis, running from left to right) and the MES angle on the y-axis (the vertical axis, running up and down), so when you enter your data under "numbers to plot" on the form, place t first on each line, followed by the average MES angle and then σ, the standard deviation measured for each set of three averaged MES angle values. Give your plot a title – something like "Lunar Elongation Data" is fine. Label the x-axis "Time (days)", and label the y-axis "MES Angle (degrees)" or something similar.

Trace the points on the plot by eye, and ask yourself whether a straight line is a good fit to the data (do the points follow a straight line?). The correlation coefficient R should be within a few percent of 100%, indicating a strong positive correlation (as t gets larger, the MES angle increases as well).

2. Be sure to save a JPG- or PNG-format copy of the plot to your computer, to include in your lab report.

Let's now inspect the slope and y-intercept for our line fit. Remember that the slope (m) is defined as the change in y divided by the change in x, and the y-intercept (b) is the y value for which x is zero.

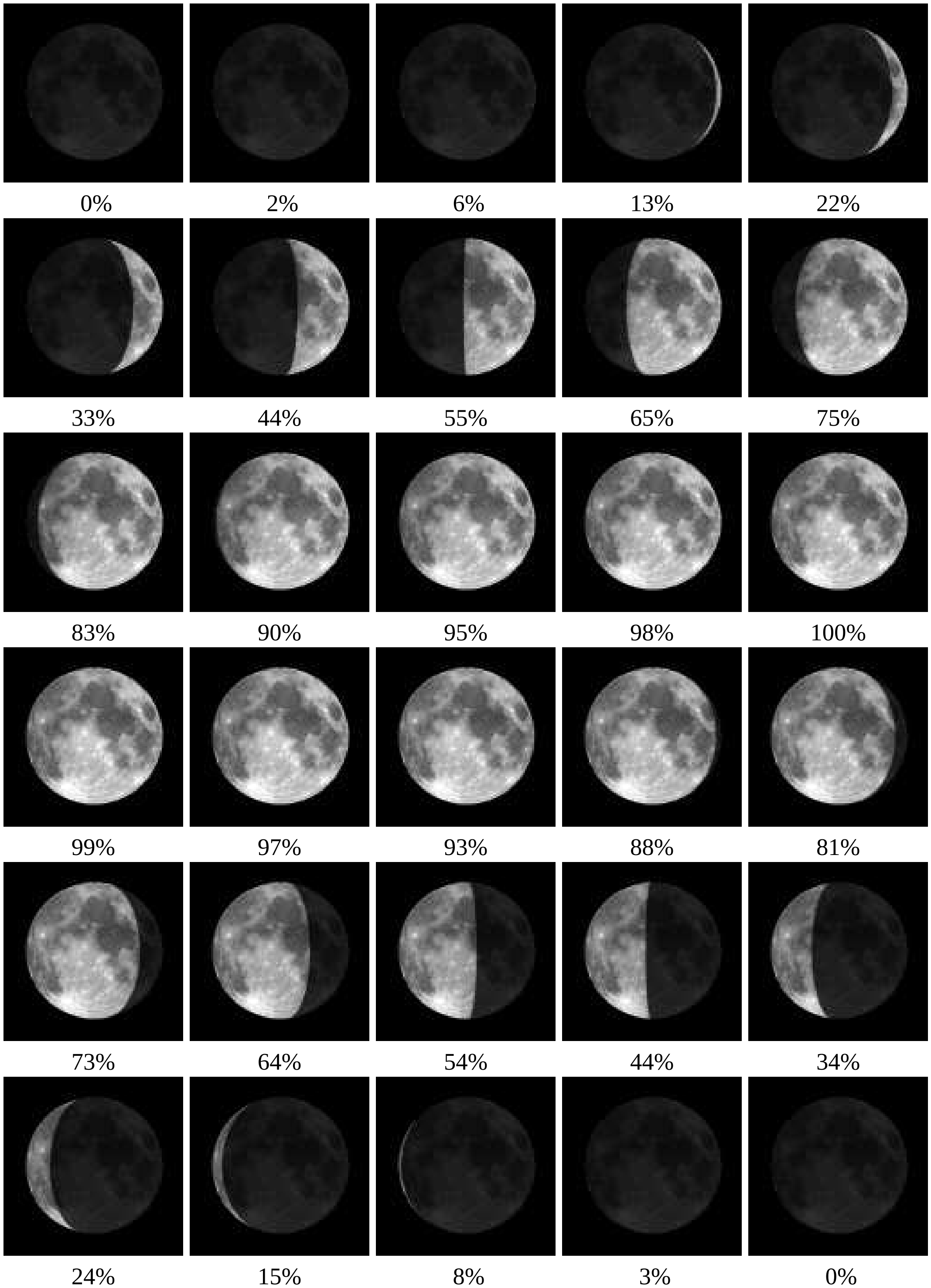


Figure 2.4: This set of images connects the phase of the Moon with its fractional illumination (listed below each image), over the course of a lunar month. When observing, either select an image that matches the appearance of your Moon or interpolate (choose an intermediate stage between two images). Make a note of the associated illumination (or average the values for the two images that surround yours in appearance).

3. For this plot, the units of the slope are ____,

and the units of the y-intercept are _______________________.

4. It is important to be able to connect this plot to its physical basis. What is the meaning of the slope and y-intercept, in terms of the connection between the Sun and the Moon? (How do they help us to explain the movement of the Moon around the Earth?)

5. In a more "perfect" (read "boring") solar system, the Moon's orbit would be a perfect circle around the Earth, lying exactly within the plane of the ecliptic (the plane in which the Earth orbited around the Sun), and all measurements would be free of error. The Moon would still take 29.5 days to orbit once around the Earth. In this case, what exact values of m and b would you expect to measure?

6. You have plotted points covering a substantial fraction of the first half of the lunar month, and fit a straight line to them. If you took data for another two weeks, describe what a plot of all of the data would look like (over the entire month).

We will next plot the fractional illumination of the Moon as a function of the time since the new moon. Reload the GEAS project lab exercise web page, and click on the link for this exercise labeled "Web application #2 (illumination versus time modeling tool)." Place t on the x-axis again, and fractional illumination on the y-axis, so when you enter your data under "numbers to plot", place t first on each line, followed by the illumination estimate. Give your plot a title, and label the x and y axes. Then click on the button labeled "Create Plot" and a plot of your data should appear in a separate browser window.

The relationship between the illumination and time can be fit well with a sine function, an up-and-down wave pattern which varies between 0 and 1 over each month. If you have calculated your times since the new moon phase correctly, the green line on the plot should pass near to or through the origin (0, 0).

7. Be sure to save a JPG- or PNG-format copy of this plot as well, to include in your lab report.

8. (a) Based on this plot, the illuminated fraction of the Moon 27.5 days after a new Moon

is equal to the fraction ____________ days after a new Moon.

(b) On the tenth day of the lunar month, the illuminated portion of the Moon is (more / less) than five times that of the second day.

9. Based on this plot and your understanding of the geometry of the Sun, Earth, Moon system, what illumination values would you expect for the new, first quarter, full, and last quarter moon? Explain your answer, describing the physical positions of these three bodies and the patterns of illumination and shadow in each case.

2.2.4 Rescheduling Observations due to Cloudy Weather

If you have very cloudy weather for both weeks of this exercise, you may need to swap the order in which you complete exercises and observe the Moon in the second half of the month. (If this were to happen, your instructor would discuss the new schedule with you.) In this case, use the second sextant (found on page 35). During the second half of the lunar month, the Moon will rise ahead of the Sun, and proceed ahead of it across the sky. During the waning gibbous phase, you will catch the Moon setting as the Sun rises at dawn. Over the next two weeks, the Moon will set later and later, and you will be able to observe later and later in the morning, until by the waning crescent moon phase the Moon will have almost rejoined the Sun in the sky.

You will construct this sextant according to the same directions, and again measure the angle formed by the Moon, Earth, and Sun. Calculate the amount of time between your observation and the nearest new moon date that follows (roughly two weeks for the waning gibbous phase, and a few days for the waning crescent phase).

2.2.5 Measuring the Moon's Altitude, Finding Our Latitude

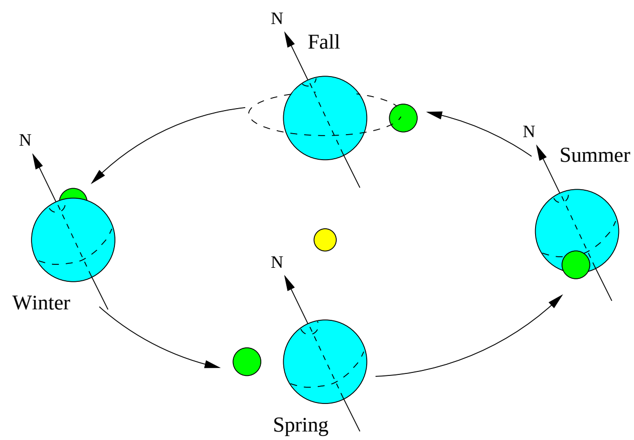


Figure 2.5: This figure is centered around the Sun (yellow globe), and shows the positions of the Earth (light blue globes) and the first quarter phase Moon (green globes) during each of the four cardinal seasons. The Earth is tilted over by 23°, indicated here by showing the tilt of the rotational axis running through the North (N) and South Poles. A small dashed circle indicates the position of the northern polar cap, and a longer dashed arc traces the Earth's equator. The northern hemisphere of the Earth experiences a warm Summer when the North Pole tips it over toward the Sun, and a cold Winter six months later when it is tipped away from the Sun. During the transitional Fall and Spring seasons, the Earth is not tipped toward or away from the Sun. Because the Sun, Earth and first quarter Moon form a right angle, the Earth-Sun "tip pattern" is shifted by three months for the Earth-Moon system. (If the Earth is currently tipped toward the first quarter Moon, as shown on the bottom, then three months later it will tip toward the Sun, as shown to the right.)

We can measure the altitude of the Moon, its height above the horizon, very easily. Like the Sun, the Moon rises in the east, ascends through the sky, then descends, and finally sets in the west. We will determine its maximum altitude, by observing it *culminate*, when it transits, or passes as close as it can to directly overhead. (If you are located more then 29° north of the equator, then the culminating Moon will always pass south of you.)

This will lead us to estimate our own latitude, how far north or south of the equator we are located. The basic idea is quite simple. Because the Moon orbits around the Earth and stays within 29° of the equator, if we are on the equator, the Moon will pass close to overhead. If we shift our position up to the North Pole, however, the Moon will stay close to the horizon at all times, never rising very high in the sky. By charting its maximum altitude above the horizon, we can determine if we are nearer to the equator or to the poles.

The Moon orbits the Earth in the same plane as the Earth orbits the Sun, rather than staying above the Earth's equator! Because the Earth is tipped over by 23°, the altitude of the Moon will thus appear to shift north and south by ±23° over the course of a month, plus an additional ±5° of wobble as it dips in and out of the ecliptic (the plane in which the Earth orbits around the Sun).

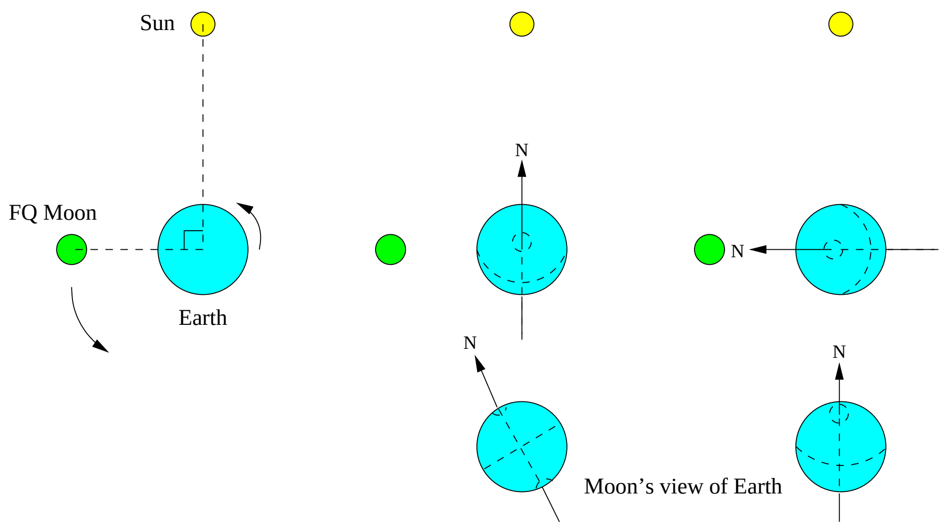


Figure 2.6: This three-panel figure displays the right angle formed by the Sun, Earth, and first quarter Moon (left). The two right panels show the Earth tipped toward the Sun during Summer (middle panel), and tipped toward the first quarter Moon three months earlier (right panel). The additional Moon images in the two right panels show *how the Earth looks when viewed from the Moon at these times.* In the middle panel the Moon sees the Earth's equator as a tilted line, with both northern and southern polar caps peeking over the edge of the globe. In the right panel the Earth has tipped over toward the Moon, so the entire northern polar cap is visible and the South Pole is hidden from view. The Moon is now orbiting somewhat north of the Earth's equator, above the northern hemisphere.

Let's review the movements of the Moon and the Earth around the Sun over a year, to understand where the Moon will appear in the sky. Figure 2.5 shows the path of the Earth around the Sun, with the northern hemisphere seasons labeled. As the Earth orbits around the Sun, the North Pole tips alternatively toward, and then away, from the Sun. This causes seasonal variations in temperature due to changes in the length of the day (the amount of time during which the Sun lies above the horizon) and the height of the Sun in the sky. The higher the Sun gets, the more directly, and intensely, the sunlight is experienced below.

As the Earth makes its yearly orbit, the Moon is zipping around the Earth once per lunar month. Over the course of each month the Moon makes a complete circle around the Earth, so at one point the Earth will be tipped toward it, and then two weeks later it will be tipped away from it. Because the phase of the Moon depends upon the position of the Moon relative to the Sun, the Earth will tip toward the Moon during different lunar phases during each of the four seasons.

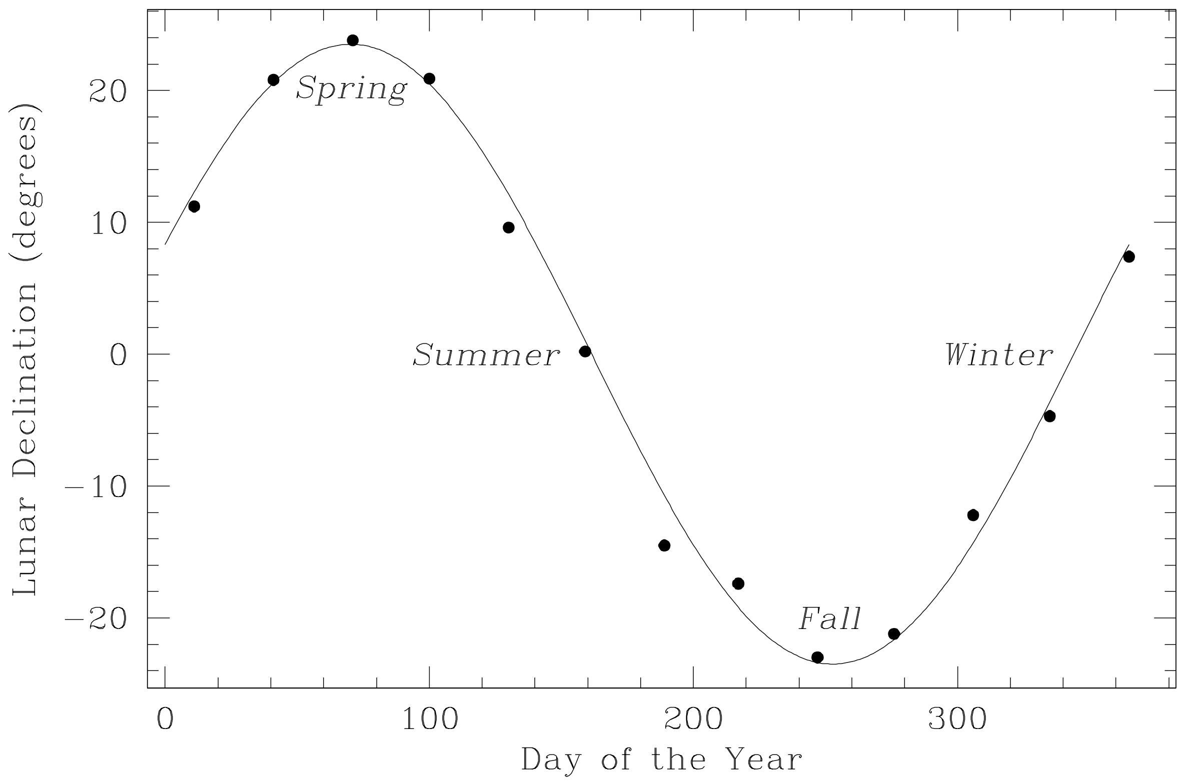


Figure 2.7: This plot shows the variation in the declination of the first quarter Moon (how far it lies north or south of the equator) over the course of a year. It can be fit well by a $\pm 23°$ sine wave, though a secondary trend pushes most of the points slightly off of the curve. We can see that in the Spring the Moon has a large positive declination, and so it lies north of the equator. This is because the Earth has tipped its North Pole over *toward* the first quarter moon; as the North Pole tips over, the equator dips below the Moon. Three months later, in Summer, the Earth is tipped over toward the Sun, and so the Moon has a declination of roughly zero (because the Earth is no longer tipped over toward it). The pattern continues into the Fall, where the Earth tips *away* from the first quarter moon and the Moon lies south of the equator and takes on a negative declination. In Winter, the Earth is again upright relative to the first quarter moon, and so the Moon has a declination near zero again. Because the Sun, Earth, and first quarter moon form a right angle, the Earth's rotational axis is either tipped toward or away from the Sun (causing Summer and Winter) and the Moon lies on the equator with a declination of roughly zero, or the Earth is tipped toward or away from the Moon rather than the Sun (during the neutral Spring and Fall seasons), causing the Moon to appear north or south of the equator.

1. During which season is the North Pole tipped over toward the first quarter moon? During which season is it tipped away from the first quarter moon?

2. Explain the physical cause of the shape of the curve shown in Figure 2.7, justifying both the general form and the fact that the points do not lie exactly on the curve.

We're going to make a short observation of the first quarter moon, and measure its altitude above the horizon as it culminates. This observations needs to be made at a particular time, so check the data listed at the GEAS project lab exercise web page. First quarter times are listed there in a table, accessed from the link labeled "Table #1 (lunar phase dates)". To the right of the times of the first quarter moons for the year, we have listed the times at which those first quarter moons culminate (attain their highest elevation in the sky). This is the time of your observation. The next column in the table lists the declination of the Moon at this time (lunar *dec*), or how far it lies to the north (+) or south (−) of the equator.

3. (a) The nearest first quarter moon culmination is on ________________, at ___________.

(b) The lunar declination (*dec*) at this time is ______________________ degrees, which is (north / south) of the equator.

Our goal is to measure the altitude of the culminating first quarter moon, how far (at what angle) it lies above the horizon. We begin by constructing another simple sextant.

Cut out the sextant found on page 37, a large rectangle with a protractor drawn on it. Tape the sextant securely to a piece of cardboard so that it will stay flat. Thread the needle and stick it through the center of the sextant. Pull just enough thread through that you can tape it securely to the back of the sextant, and do so. Attach a paper clip or other fairly light object to the other end of the thread, so that gravity will pull it down along a straight, vertical line (without a weight, a light breeze could shift it easily).

Take a straw and tape it carefully along the line between the two "90" labels, making every attempt to keep it straight along this line. This is your sighting tube.

Your altitude-finding sextant is now ready for use! You are going to use it to find the angle of the Moon above the horizon. First verify that if you stare at the horizon through the straw the thread points to zero degrees, and if you stare straight up the thread points to 90°. As you now tilt the straw to point up at the Moon, the sextant will tilt with you, and the vertical thread will indicate the tilt angle. Peer through the straw, and place the Moon at its center. Have an assistant read off the angle marked by the thread as you do so. This is the altitude of the Moon, the angle at which it lies above the horizon.

Schedule your observations to take place with a few minutes of the listed culmination time, to increase your accuracy. If the Moon is still rising in the east, or has begun to descend to the west, then you will systematically underestimate its culmination altitude.

Take three independent observations so that you are less sensitive to small random shifts in the equipment as you read the angle to the nearest degree. Use the GEAS plotting tool to average the three measurements and find the standard deviation.

4. The three observed values for the lunar altitude are _______________, _______________,

and _______________ degrees.

5. The altitude of the culminating first quarter moon was _______________ ± ____________ degrees.

Once you are done with your third observation, turn around and face in the opposite direction (due north). Take a note of the direction and any local landmarks, as you will need to find due north for your next observation (of the North Star). You might even draw an arrow pointing north on the ground with a piece of chalk.

You have measured the altitude of the culminating Moon, how far it lies above the southern horizon. We will combine this number with the declination of the Moon, how far it lies above or below the equator, to estimate our latitude (how far we are located above the equator).

We will assume that you are located at least 29° above the equator as we do this (though not so far north that the Moon lies below the horizon), so that the Moon always culminates *south* of your position. (If you are at a lower latitude, work with your instructor to understand how to modify this procedure for your location, where the Moon could appear to the north.)

In Figure 2.8, we explain the definition of an observer's latitude (*lat*) and the declination (*dec*) of the Moon. We chart positions on the surface of the Earth according to two coordinates, latitude and longitude. Latitude tells you how far north or south you are from the equator, while longitude tells you how far east or west you are from the Royal Observatory in Greenwich, England. A positive latitude lies in the northern hemisphere, while a negative value indicates a location south of the equator. We define the declination of an astronomical object very similarly to latitude; declination tells you how far north or south an object lies of the plane containing the equator.

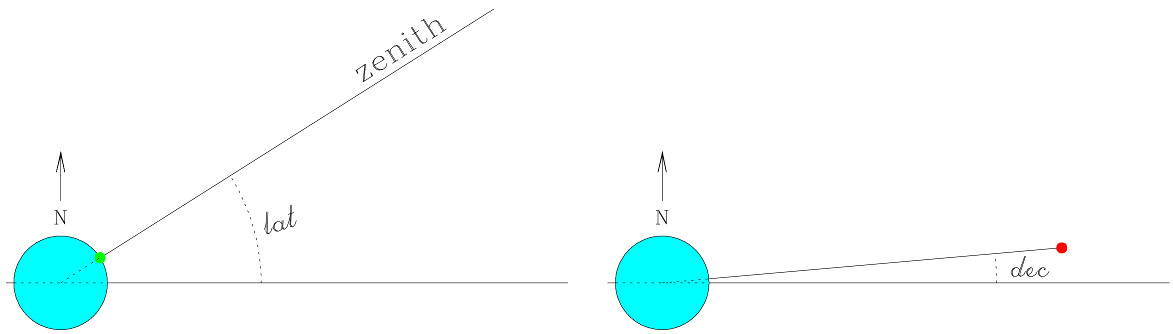


Figure 2.8: In this figure, the large light blue globe represents the Earth, with the North Pole facing upward, the South Pole facing downward, and the equator running horizontally across the page. On the left, we define the latitude (*lat*) of the observer (marked with a green dot) as the angle between the equator and the observer's position. The zenith points up into the sky above the observer's head. On the right, we illustrate that the declination (*dec*) of the Moon (red dot) is its angular position above the equator. Both angles are measured upward from the equator.

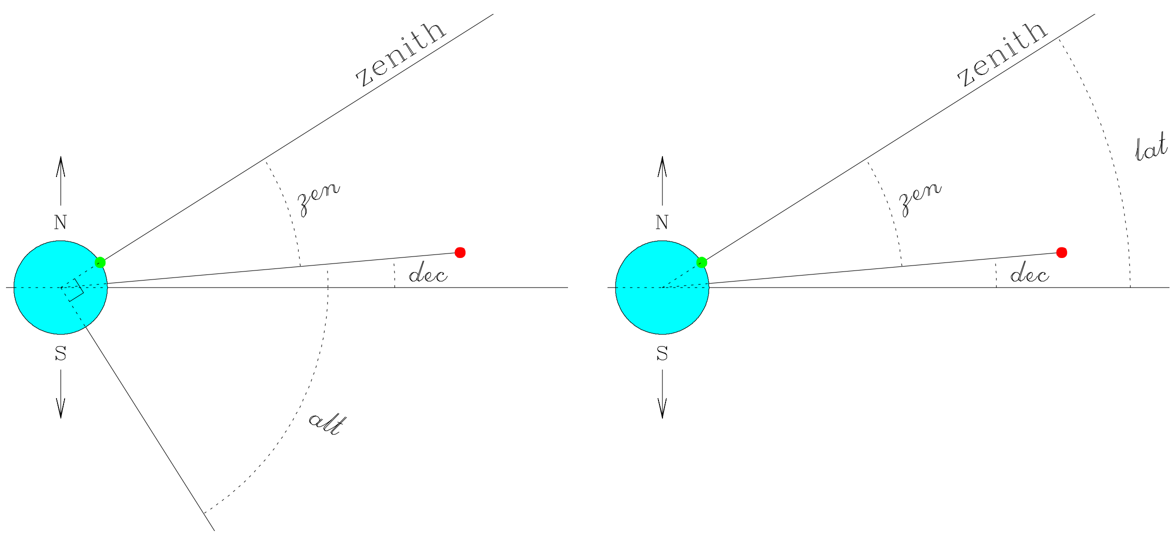


Figure 2.9: In this figure, as the previous one, the light blue globe represents the Earth, with the North Pole facing upward, the South Pole facing downward, and the equator running horizontally across the page. On the left, we again illustrate the declination of the Moon (red dot). We add to this figure the zenith distance (*zen*) and the altitude (*alt*). The altitude of the Moon is the angle it makes with the observer's horizon, which sums to 90° when combined with the zenith distance (*zen*). On the right, we see that we can define the latitude (*lat*) as the sum of the zenith (*zen*) and the lunar declination (*dec*).

In Figure 2.9, we add the definitions of the altitude (*alt*) and the zenith distance (*zen*) of an object such as the Moon. While the declination of an object is the same for all observers, the altitude of an object in the sky is a local measurement. (The Moon can lie almost directly overhead in Florida at the same time that an observer in Hawaii will see it rising above the eastern horizon.) Altitude is tied to the observer's location; the closer an object lies to directly overhead (near to the zenith), the larger its altitude must be. An object rising on the horizon has an altitude of zero, while an object directly overhead has an altitude of 90°. If you put one arm out in front of you and point at the horizon, and point the other arm directly overhead, your arms will form a right angle (90°). As the altitude of an object is the angle between it and the horizon and the zenith distance is the angle between it an the zenith, the sum of an object's altitude and zenith distance is always equal to 90°.

$$zen = 90^\circ - alt$$

For simplicity, you may assume that the standard deviation σ of the zenith distance and latitude are the same as that estimated for the lunar altitude. (If you measured the Moon's altitude to within 2°, you can estimate your zenith and latitude to the same precision.)

6. The zenith distance of the culminating first quarter moon was ______________ ± ________ degrees.

In the second panel of Figure 2.9, we can combine these concepts to deduce a relationship between the zenith distance of the Moon and its declination, for an observer at a given latitude.

$$lat = zen + dec$$

7. The latitude of my observing location, ____,

is ______________________ ± ____________ degrees.

Now look up the latitude of your observing location, and compare this value to your estimate.

8. The tabulated latitude of my observing location is __________________________ degrees.

9. By how many σ do the two latitude values differ? If the difference is more then 2σ, discuss possible sources of error in your measurements that could account for the difference.

2.2.6 Measuring the North Star's Altitude, Finding Our Latitude

We can estimate our latitude another way, without needing to look up any numbers. We will take advantage of a lucky feature of the northern hemisphere, one that has saved countless sailors and land-based travelers from losing their way at night. (If you live in the southern hemisphere, speak to your instructor about an alternate activity or a multi-day field trip.)

We know that the Earth spins around in place once a day. Its rotational axis is an imaginary line which extends above the South Pole down through the Earth and out above the North Pole. If you continue on up into the sky high above the North Pole (along what we call the North Celestial Pole, or the NCP), you will eventually run into a star – Polaris, or the North Star! (By chance, there is no star visible from Earth in the patch of sky above the South Pole along the SCP, which is why penguins make such poor navigators and should not be entrusted with the maps on long journeys.)

Because the North Star lies almost directly above the North Pole, and the Earth rotates once a day around its rotational axis, the northern sky appears to rotate counter-clockwise around the North Star, completing a full circle once every 24 hours. This star neither rises nor sets (nor do its nearest neighbors)! Figure 2.10 shows the orientation of the constellations surrounding it, to help you find the North Star on any night of the year.

You are going to find the North Star and measure its altitude above the horizon. This will lead directly to an estimate of your local latitude. Refer to Figure 2.10 to find the North Star in the early evening after dark. Work away from street lights, house lights, and other sources of light pollution, but make sure that your location is safe. If you have never gone sky-watching at night, you might ask a family member or good friend who has done so to join you. The North Star is so easy to find, however, that you should not worry if you end up looking for it on your own.

Try to perform this observation with a new or crescent phase Moon; if you wait until the first quarter or gibbous Moon, the moonlight will make the sky quite bright, and Polaris will fade into the background and become harder to find.

Give your eyes at least five (ideally twenty) minutes to adjust to the dark once you go outside; as they do so, fainter stars will slowly appear. Face in the opposite direction than you stood when viewing the Moon culminating due south. Review Figure 2.10 and examine the sky. Look first for the Big Dipper, the brightest constellation near to the North Star. The stars of the Little Dipper are fainter, so don't worry if they are less visible – the North Star is the brightest star in the Little Dipper, so if you find one star in it, you have succeeded.

Once you have identified the North Star, attempt to measure its altitude. This will be somewhat more challenging than your earlier work measuring the altitude of the Moon, because the stars are so much fainter than the Moon. If you have good eyesight and are observing in a location with a very dark sky (with no street lamps or porch lights), try to sight it through the straw of the sighting tube on your sextant. If this proves to be too difficult, do not despair. Replace the straw with the larger tube from the center of a roll of

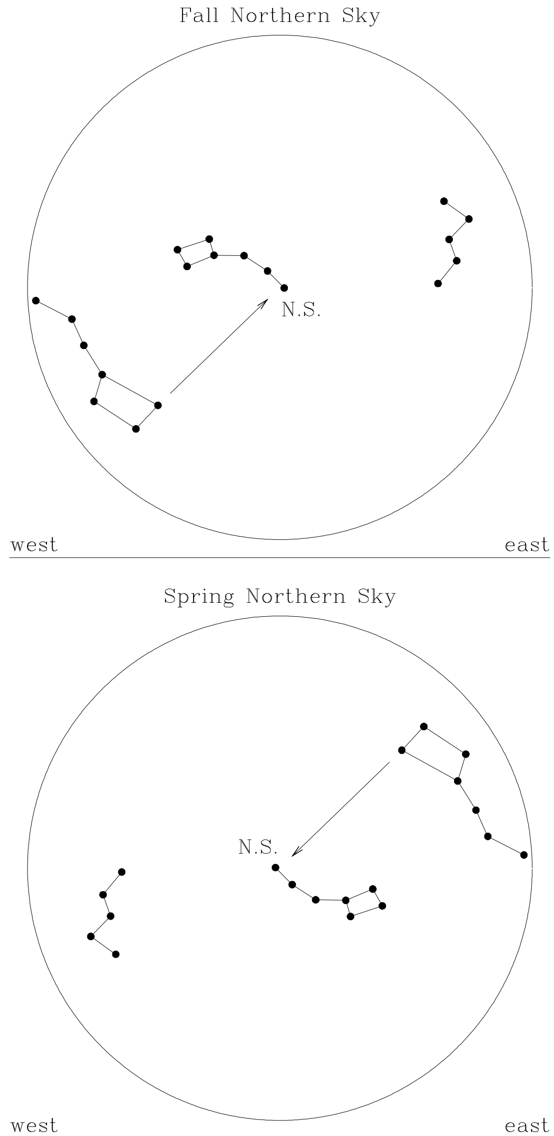


Figure 2.10: This set of images shows the brightest constellations surrounding the North Star (Polaris, labeled N.S.), throughout the year in the early evening. The classic technique for finding the North Star is to face northward, identify the bowl of the Big Dipper in Ursae Major, and then follow the line formed by the two stars which make up the lip due North (see arrows on plots). This will lead you to Polaris, a star in the handle of the Little Dipper (Ursae Minor) which happens to lie above the Earth's rotational axis and so forms a handy reference for navigation. When the Big Dipper is low on the horizon, it may be helpful to orient yourself by first finding the characteristic "W" shape of Cassiopeia, on the other side of the North Star from the Big Dipper. Over the course of each night these constellations will rotate counter-clockwise, mapping out a full circle in the northern sky around Polaris once every 24 hours.

paper towel or toilet paper (attaching it along the same line connecting the 0- and 180-degree labels), and try to center the star in the larger field. If this too proves to be difficult, then either hold the sextant in your hand, placing the 0–180 degree line firmly between and along your index and middle fingers, or tape the paper lightly to your arm. Now point your index finger toward the North Star.

With each of these techniques, you will have pointed the 0–180 degree line toward the North Star. Now ask your assistant to read off the altitude measurement from the sextant. (Tell them not to shine their flashlight beam into your eyes as they do so.) If you have the sextant in your hand, sit down so that your assistant can easily reach the sextant. (You may want to add a red filter to the flashlight if you have one, or to wrap a small piece of red cellophane around the lit end. This will help your eyes to stay dark-adapted.)

Make your measurement three times, taking care to drop your arm down to your side between measurements so that you re-find the North Star each time. If you have to sight the star along your arm rather than finding it through a sighting tube you will probably end up with a larger standard deviation for your three measurements, but that's an acceptable compromise.

Jot the numbers down to the nearest degree, and then combine them to form a mean value and standard deviation.

1. The three measurements of the North Star altitude are ________________, ________________,

and ________________ degrees.

2. The altitude of the North Star was ________________ $\pm$ __________ degrees.

How can we deduce our latitude from the observed altitude of the North Star? Examine the two examples shown in Figure 2.11, where the observer is located near to the equator and then at a higher northern latitude. Keep in mind the the angle between the observer's zenith and horizon is always 90°, as is the angle between the Earth's equator and North Pole.

3. Bearing in mind that the North Star is located high in the sky above the North Pole, at what altitude (how far above the horizon) would it appear if viewed from the North Pole, with a latitude of 90°? *(Remember that the angle between the horizon and the zenith, directly overhead, is 90°.)*

4. At what altitude (how far above the horizon) would the North Star appear if viewed from the equator, with a latitude of 0°?

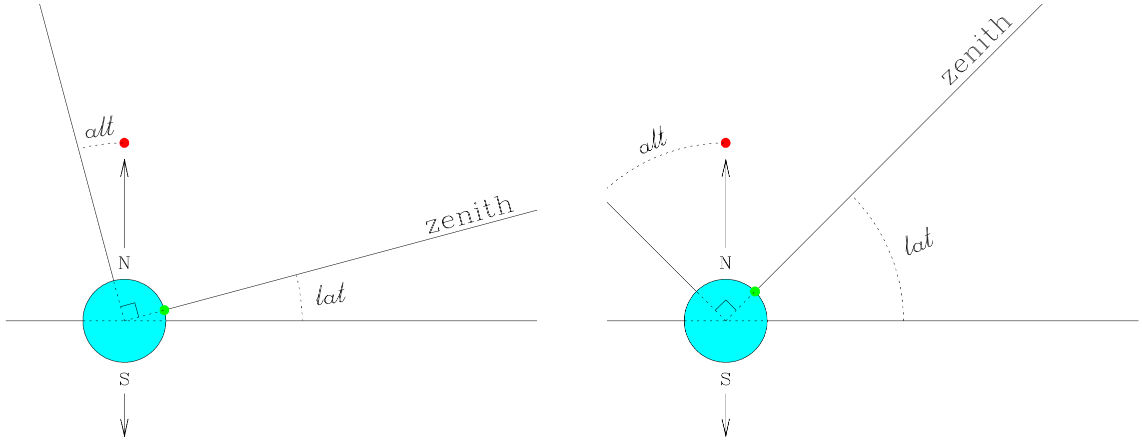


Figure 2.11: In this figure, as in previous ones, the light blue globe represents the Earth, with the North Pole facing upward and the equator running horizontally across the page. We mark the altitude of the North Star (red dot), and the latitude of an observer (green dot), for two observers who lie near to (left) and far from (right) the equator. What is the relationship between the altitude of the North Star and the latitude of the observer?

5. Explain the simple connection between the altitude of the North Star and an observer's latitude.

6. The latitude of my observing location is ___________________ $\pm$ ____________ degrees.

7. By how many σ do the latitude estimate based on the North Star and the tabulated value you looked up earlier (see §2.2.5) differ? If the difference is more then 2σ, discuss possible sources of error in your measurements that could account for the difference.

2.3 Final (Post-Lab) Questions

1. Fill in the following 12 blanks with the words sunrise, noon, sunset, or midnight.

The new moon rises at (roughly) ____________, culminates at ____________, and sets at ____________. The first quarter moon rises at ____________, culminates at ____________, and sets at ____________. The full moon rises at ____________, culminates at ____________, and sets at ____________. The last quarter moon rises at ____________, culminates at ____________, and sets at ____________.

2. Describe one way in which you could introduce a systematic error into the measurement of your latitude, while estimating the altitude of the culminating first quarter moon.

3. For an observer at a latitude of +32.3° (north of the equator), what should a first quarter moon's altitude be at culmination on June 21? Show your work.

4. If the culminating first quarter moon has a declination of -23° in the Fall (appears 23° south of the equator), what will the declination be of the last quarter moon two weeks later?

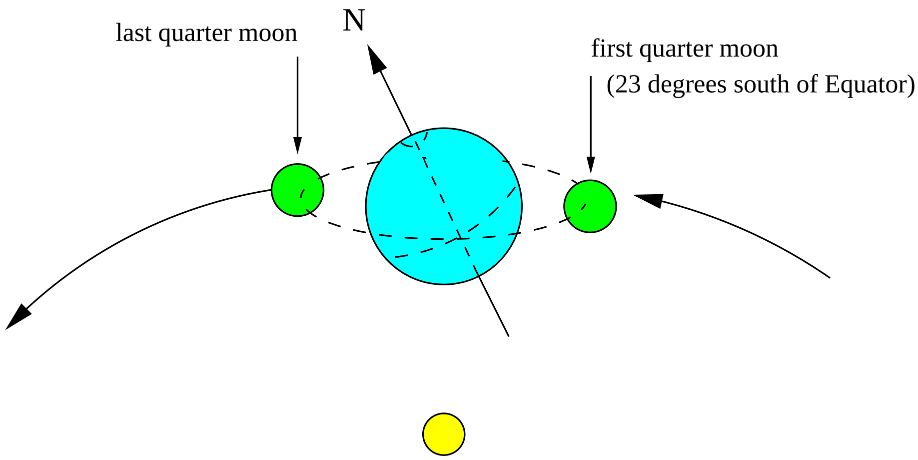


Figure 2.12: This figure shows the Sun (yellow globe) and Earth (light blue globe), and the Moon in its first and last quarter phases (green globes) during the Fall season. The Earth is tilted over by 23° to the left, as shown by the tilt of the rotational axis running through the North (N) and South Poles. A small dashed circle indicates the position of the northern polar cap, and a longer dashed arc traces the Earth's equator. The orbital path of the Moon is also drawn, as a dashed ellipse.

2.4 Summary

After reviewing this lab's goals (see §2.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. Describe your observations of the sky, your analysis of the resultant data, and your overall conclusions. (30 points)

Be sure to cover the following points.

- Describe the movement of the Moon over the course of a lunar month, and explain how its appearance in the sky changes accordingly.
- Tell us what you have learned about techniques for measuring angles on the sky, and how to observe basic properties (location, size, appearance) of the Sun, Moon, and stars in the sky.
- Report on any observational, measurement-related, and/or logistical challenges that you faced while conducting this lab, and explain how you dealt with them. *(If you came up with a clever idea to solve a problem, we'll pass it on to students in future classes.)*

- Explain the relationship between an observer's latitude and observables like the altitude of the culminating first quarter moon and the North Star, and tell us why it was important that travelers determine their latitudes in the past.

- If you thought of any particular ideas or visualization tricks that helped you to understand the movement of the stars or Moon through the sky that aided you in completing this exercise, please tell us about them.

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

2.5 Extra Credit

Estimate the radius of the Earth, by peering around the edge of the horizon at sunset or sunrise.

Figure 2.13 illustrates two positions for an observer watching the Sun set at dusk, first lying down and seeing it set at ground-level (R), and then standing and watching it appear to set again from an elevation of $R + h$, where h is the height of the observer. The Earth rotates counterclockwise through a small angle α between the mini-sunsets.

You can time the interval Δt between the two sunsets, and then compare this time to the full 24 hours (or $24 \times 60 \times 60 = 86,400$ seconds) needed for a complete rotation of the Earth.

$$\frac{\Delta t}{24\,\text{hours}} = \frac{\Delta t}{86,400\,\text{seconds}} = \frac{\alpha}{360^\circ}$$

so if Δt is measured in units of seconds and α is measured in degrees,

$$\alpha = \Delta t \times \frac{360}{86,400} = \frac{\Delta t}{240}.$$

Your time interval Δt should lie somewhere between 2 and 20 seconds, and be measured to the nearest tenth of a second.

Once we have measured Δt and h (your height), we can combine them to form an estimate of the radius of the Earth. There is a right triangle formed in Figure 2.13, with width R and hypotenuse (longest side) $R + h$. The ratio of the width to the hypotenuse for angle α is called the cosine of α, and can be calculated by scientific calculators. If we know α and h, we can thus deduce a value for R.

$$\cos(\alpha) \equiv \frac{R}{R+h}$$

and so

$$R = R\cos(\alpha) + h\cos(\alpha)$$

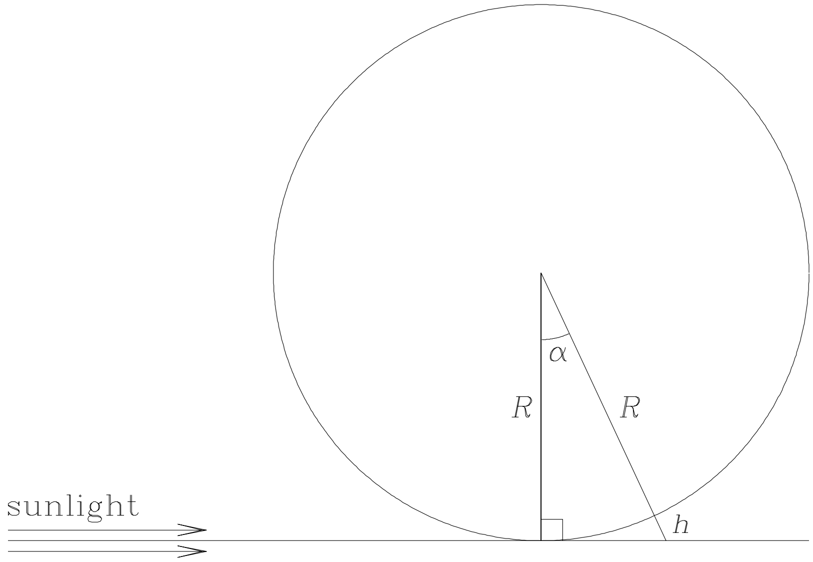


Figure 2.13: You can see two sunsets for the price of one, by first lying down and watching the Sun set below the horizon and then quickly leaping up and watching it set again (your increase in height will allow you to see "around" the Earth a bit further. At sunrise, you can also double your view by first watching the Sun rise while standing, and then lying down and seeing it rise again. This figure indicates the viewing geometry at sunset, with the Earth's radius labeled as R, the observer's height as h, and the angular rotation of the Earth between mini-sunsets as α (magnified for clarity). Note that the Earth's rotates in a counter-clockwise fashion in this figure.

and

$$R\,[1 - \cos(\alpha)] \;=\; h\cos(\alpha)$$

and solving for R,

$$R \;=\; h\left[\frac{\cos(\alpha)}{1\;-\;\cos(\alpha)}\right].$$

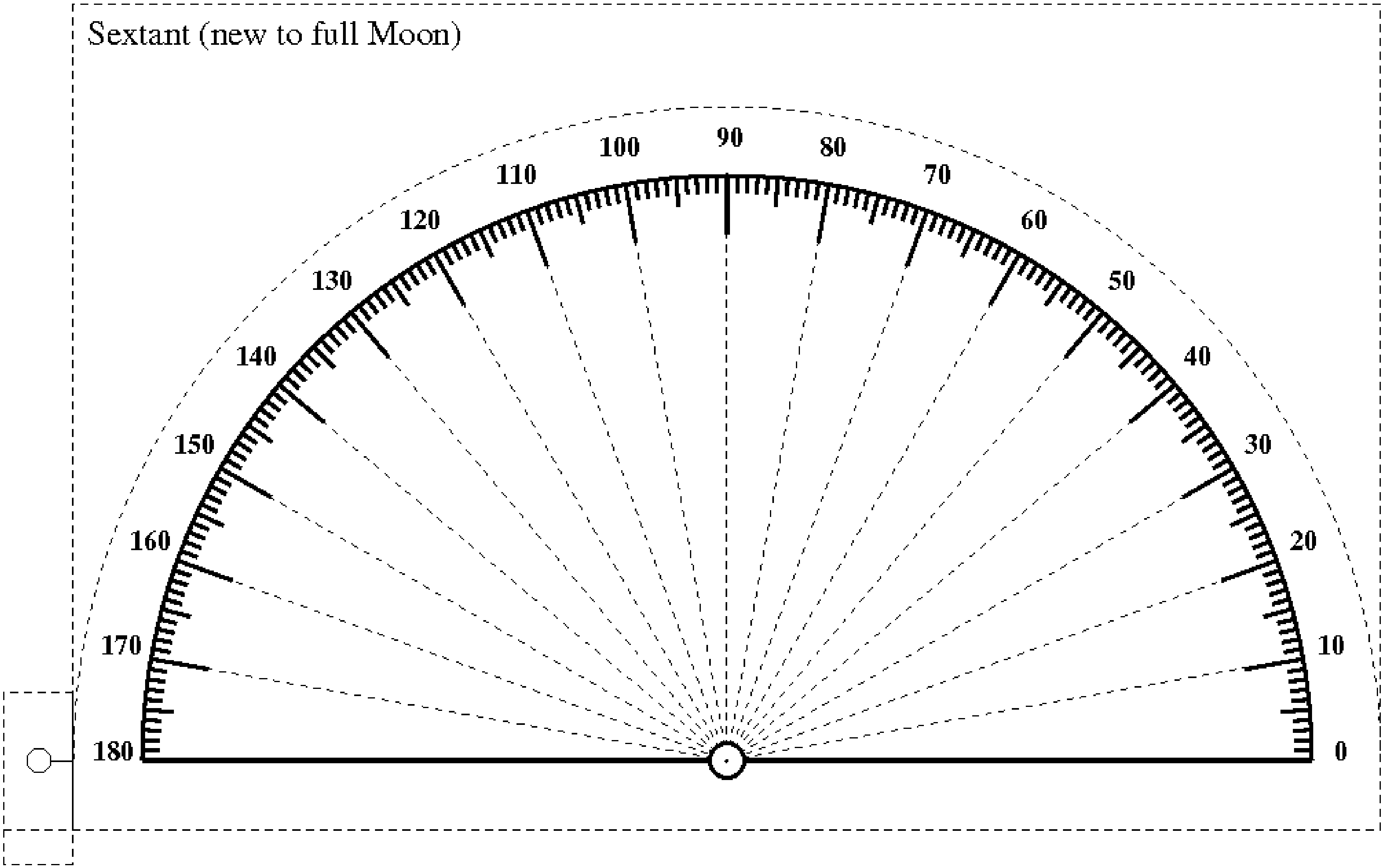
Check that your calculator is set up to receive angles in units of degrees, by calculating $\cos(60°)$, which is equal to $1/2$. (If you calculate a value of 0.9998, you need to switch your calculator units from radians to degrees.)

Now recall that $\alpha = \frac{\Delta t}{240}$, and calculate a value for R.

The Earth's radius has been measured to be $6,371\pm8$ kilometers. How does your home-grown estimate compare?

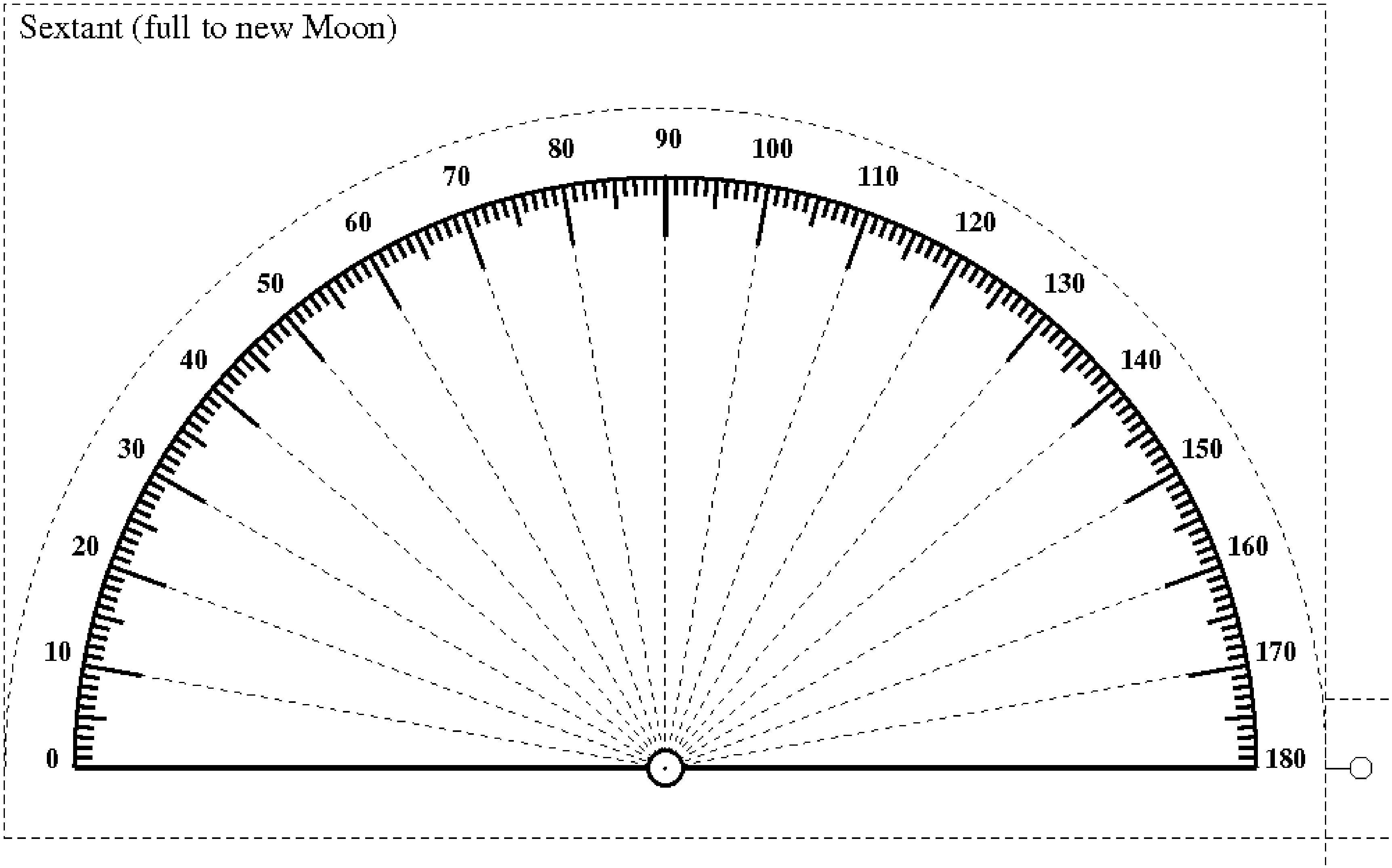
Cut along outer dotted line.

Sextant (new to full Moon)



Cut along outer dotted line.

Sextant (full to new Moon)



Cut along outer dotted line.

Sextant (for altitudes)