Lab 1

Fundamentals of Measurement and Error Analysis

1.1 Introduction

This laboratory exercise will serve as an introduction to all of the laboratory exercises for this course. We will explore proper techniques for obtaining and analyzing data, and practice plotting and analyzing data. We will discuss a scientific methodology for conducting experiments in which we formulate a question, predict the behavior of the system based on likely solutions, acquire relevant data, and then compare our predictions with the observations. You will have a chance to plan a short experiment, make observations, and collect data for yourself, and you will also work with data sets that have been collected in advance.

Pay careful attention to the general rules that will be introduced for dealing with data, so that you can apply them every week. You will be exploring key mathematical relations and manipulating some of the same data sets that you will focus on in future weeks. By learning basic techniques now you will be able to focus on astronomy, and better understand the Universe surrounding you, when we take a second look later. Use this document as a general reference for the experimental and data analysis work that you do throughout the entire course, and be prepared to reread parts of it as you work on other exercises.

1.1.1 Goals

The primary goals of this exercise are to become comfortable planning and conducting simple observations, and to use real data to test hypotheses which relate to behavior of the natural world. We will introduce a set of online tools for this purpose and define key statistical measures which allow us to discuss trends, and you will be guided through their basic usage.

1.1.2 Materials

All online lab exercise components can be reached from the GEAS project lab URL, listed here.

http://astronomy.nmsu.edu/geas/labs/html/home.shtml

You will need the following items to perform a simple experiment:

- either a half-cup of pinto beans OR a bag of marshmallows OR several handfuls of gravel OR 30 bushes or trees OR a herd of amiable dairy cows OR access to a parking lot full of cars OR a few shelves of books OR 30 friendly people OR two bags of tortilla chips OR 30 coins of a single denomination
- appropriate tools for measurement, such as a ruler, a tape measure, string, a stopwatch, and/or a kitchen or jeweler's scale

You will not need all of these items; you will select a particular experiment based on your interests and the materials you have available to you (see page 4 for details). For certain experiments, you may find a friendly assistant (one with no particular training, but who is careful) to be very helpful.

You will also need a computer with an internet connection, to analyze your experimental data.

1.1.3 Primary Tasks

This lab is built around four activities: 1) planning and conducting a short experiment with common household items, 2) examining existing data to uncover a basic connection between seasonal changes and the height of the Sun in the sky at noon, 3) analyzing data, including error estimates, and 4) making appropriate conclusions based on evidence.

You will also be presented with an overview of our plotting and data analysis tools, and we will review the process for creating laboratory reports and sharing them with your instructors. You should find yourself reading certain sections of this chapter again during future weeks, as part of your preparation for other experiments.

1.1.4 Grading Scheme

There are 100 points available for completing the exercise and submitting the lab report perfectly. They are allotted as shown below, with set numbers of points being awarded for individual questions and tasks within each section. The 19 questions in Section 1.2 (§1.2) are each worth 1 or 4 points each, while the three data tables are worth 10 points together for a total of 38 points. The five questions in §1.3, the four questions in §1.4, and the five questions in §1.5 are each worth 2 points, for a total of 28 points. The three final (Post-Lab) questions in §1.6 are worth 3 points each, for a total of 9 points. Note that §1.8 contains 5 extra credit points.

Table 1.1: Breakdown of Points

Activity	Experiment	Errors	Fits	Trends	Questions	Summary
Section	§1.2	§1.3	§1.4	§1.5	§1.6	§1.7
Page	3	19	23	28	31	33
Points	38	10	8	10	9	25

1.1.5 Lab Reports

You will write a laboratory report for each laboratory exercise using the Google Documents application, an online tool which allows you to write reports and share your work with others. To learn how to set up a free account and work with lab report templates, return to the GEAS project lab URL and select "Getting Started > Google Documents" from the masthead. The filename for your first lab report should be defined by your last name and first initial followed by "_01". (Albert Einstein's lab report would be called einsteina_01.)

1.1.6 Timeline

Week 1: Read §1.1–§1.5, and complete activities in §1.2 through §1.4. Identify any issues that are not clear to you, so that you can receive feedback and assistance from your instructors before Week 2. Enter your preliminary results into your lab report, and make sure that your instructors have been given access to it so that they can read and comment on it.

Week 2: Complete activities in §1.5; complete §1.2.4 if you were unable to do so during Week 1. Finish final (Post-Lab) questions in §1.6, write lab summary, and submit completed lab report.

1.2 Performing a Simple Experiment

We are going to illustrate experimental procedure by performing a simple experiment, making repeated measurements, and analyzing the data. Read through all of §1.2 before beginning your experiment so that you understand each step of the task and can plan your procedure. Make sure to answer each of the 20 questions contained in this section, and to fill out all three data tables.

1.2.1 Planning Your Experiment, and Collecting Data

Begin by selecting a simple experiment, one that we can do with common household objects in a short amount of time. To keep things simple, we will measure a single property, make repeated measurements, and then analyze the distribution of those values. We will analyze the data to determine average values and see how well our measurements agree with each other. We want to become comfortable with this process so that we can apply it confidently to astronomical data sets later.

We have put together a list of simple, fun experiments, in the form of questions to answer. Select one question from the list shown below, one that you have the resources to conduct and that sounds interesting to you. (If you read through the list and think of an alternate experiment in a similar vein that you would prefer, do contact your instructor and discuss your idea with them. If your idea is a good one, it might even end up on the list of suggested experiments for next year!) You will make a series of 30 measurements in order to study the question.

List of Simple Experimental Questions

1. How much do individual pinto beans or pieces of gravel weigh?
2. How large are the circumferences of tree trunks or bushes?
3. How far can marshmallows be thrown?
4. What is the point-to-point length of tortilla chips?
5. How many pages make a book (or how wide or heavy are books)?
6. How much milk do dairy cows produce per day?
7. How wide are windshields (or how large are tires)?
8. What is the distance between the pupils of people's eyes?
9. How long does it take you to complete a practice self-review quiz for this class?
10. How heavy (or thick) are coins of a given value?

Each of these questions can be answered by conducting an experiment and making a particular measurement repeatedly. We will need to carefully define the process by which data are collected, collect the data in a uniform and non-biased fashion, and consider the precision of our measurement technique.

You will define and then follow a certain procedure when making measurements. Strive for reproducible results and keep track of the measurement precision, the numerical agreement between multiple measurements made in the same way. Minimize and quantify errors, and work to the highest level of accuracy.

We will describe the distribution of values for the measured quantity by estimating two numbers: the mean, or average value, and the spread, or width, of the distribution around the mean (how close most measured values lie to the mean).

Note that we will not be answering these questions by using archival data (looking up the answers on the internet, for example) but instead by making our own measurements. If you choose to study how tall people are, you will be measuring their heights (not just asking them how tall they think they are), for example. You will need to make precise measurements for yourself.

Once you have selected a question to answer, you need to plan your experiment. Determine that you have the materials necessary (such as a bag of marshmallows and a tape measure) and a clean, safe location in which to work. (If you decide to measure the properties of cars, make sure you have a well-lit, safe parking area to sample, and that there is no possibility of anyone accusing you of trying to damage the vehicles.)

Think carefully about how you will select the sample of objects to study and how you will make the measurements. If you want to measure tree circumferences, for example, make sure that there are enough trees in your vicinity to sample, and consider ahead of time whether having an assistant would make it easier to wrap a pliable measuring tape or a piece of string around the larger trees. Make your measurements at a particular height from the ground (typically at chest height), so that root growth does not inflate the measured lengths.

Consider how precise your measurement tool is, and whether this will provide you with an acceptable precision in making measurements. For example, if you are measuring the time it takes you to complete an online quiz you should attempt to record the lengths to the nearest tenth of a second, not just the nearest second. A good rule of thumb if you are unsure is that is that your measurement should be made to *at least* one part in one-hundred – if you are measuring a length of ten centimeters, you should record your data to at least a tenth of a centimeter (10 cm/100 = 0.1 cm, also called a millimeter), and ideally, to a smaller length.

1. In the space below, describe your experimental plan. (Remember that you can either write your answer on a printed copy of this page, or you can write directly in your online lab report template.)

Give enough details so that if another student read your notes, he or she could duplicate your efforts in a consistent fashion and obtain a data set that you could use productively in combination with your own data. Later on you will partner with another student and attempt to reproduce each other's results. If measuring the sides of tortilla chips, for example, what brand and style of chip will you use? If using three-sided chips, will you measure all three sides on each chip, or just largest side? Will you use a pliable tape measure (like a cloth one used for sewing) and lay it along each chip side, or measure the shortest distance between the two points along a straight line? What units will you use (centimeters, inches)?

Include a rough guess for the average value that you expect to measure. Keep it simple, and remember that you are simply guessing (in part, to select an appropriate measuring tool) – there are no "wrong answers" to this question. (4 points)

Now that you have a plan, it's time to go ahead and start taking data! You will start by making a few measurements, and perform a rough analysis to check that your results make sense. If necessary, you will modify your technique. You will then complete the data set.

As a first step, perform your measurement seven times, and enter the values into the second column of the table shown below (Table 1.2). In the table title, after the words "Experimental Data I," describe the quantity that you are measuring (such as "Pinto Bean Weights"), and in the column heading for column two, after the word "Value" state the units of the recorded values (such as "centimeter" or "cm") within the parentheses.

Record your values with a single set of units. For example, if you measure lengths in feet and inches convert the measurements into inches before entering them in the table. If you measured a distance of five feet and 4.25 inches, you would use the fact that there are twelve inches per foot to write this as $5 \times 12 + 4.25 = 64.25$ inches. In this expression the **value** is 64.25, and the **unit** is inches.

You will transfer this information into your online lab report later, but it is probably easiest to first record the data on paper (so that you do not need to have a computer handy).

The word “trial” tells us that we are going to perform the same measurement repeatedly. It is important to be careful, to use the same technique, and to make the measurement to the same level of precision each time. We are trying to sample the underlying, or actual, distribution of a large number of objects (such as all of the pinto beans in the world), but taking only a few measurements. It is thus important to sample as randomly as possible from the set of objects, so if you have a bag of pinto beans, don’t choose the largest (or smallest, or roundest) ones to measure, but simply work with the first seven which come to hand. Of course, if you find one that was chopped in half you should discard it, as it is not a complete pinto bean.

Once you have measured seven objects from your data set, stop and perform an initial data analysis on this sample. We want to determine the average value of the measurements, and how widely they vary over the entire sample. We will thus introduce the concept of a mean value (μ, pronounced mu or mew), and of the spread of the distribution (σ, pronounced sigma), also called the standard deviation.

For a more mathematical discussion of mean values and standard deviations, please return to the GEAS project lab URL and select “Getting Started > Mean Values” from the masthead.

Let’s assume that we are conducting the first experiment (weighing pinto beans), as the same technique will apply to any measurement. The mean value of the sample is simply the average of all seven weights. If the seven measurements are 0.124, 0.351, 0.300, 0.323, 0.377, 0.402, and 0.356 grams, then the mean value is simply

$$\mu = \frac{1}{7}\,(0.124 + 0.351 + 0.300 + 0.323 + 0.377 + 0.402 + 0.356) = 0.319 \text{ gm.}$$

2. Go ahead and enter your seven trial measurements into Table 1.2, and calculate μ. Make sure to note your measurement unit within the parentheses on the top line. (1 point)

$$\mu = \frac{1}{7}\,(\underline{\hspace{12cm}}) = \underline{\hspace{2cm}}\ \underline{\hspace{1.5cm}}.$$

Because we have only seven measurements, we want to make sure that an error in measurement does not skew our results dramatically. If you accidentally placed your thumb on the scale while making one measurement, for example, you might end up with an artificially high weight for the pinto bean under examination. We will thus perform the averaging process again, but first discard the lowest and highest measured values.

We begin by sorting the seven values in order from lowest to highest, and then remove the top and bottom values from the list. Take care to remove the *lowest* and the *highest* values, not the *first* and *last* values. (The lowest and highest values are only the first and last values after you sort your list to run from from lowest to highest.)

$$\mu = \frac{1}{5}\,(\cancel{0.124} + 0.300 + 0.323 + 0.351 + 0.356 + 0.377 + \cancel{0.402}) = 0.341 \text{ gm.}$$

Table 1.2: Experimental Data I: ____________________

Trial	Value ( )
Trial 1	
Trial 2	
Trial 3	
Trial 4	
Trial 5	
Trial 6	
Trial 7	

(2 points)

Our revised estimate of the average weight of a pinto bean is thus 0.341 grams.

3. Perform the same operation for your innermost five trials from Table 1.2 below. (1 point)

$$\mu = \frac{1}{5}\left(\underline{\hspace{10cm}}\right) = \underline{\hspace{2cm}}\ \underline{\hspace{1.5cm}}.$$

We next want to estimate the scatter in values – are most pinto beans within a tenth of a gram, or within ten grams, of the mean value?

The standard deviation tells us the average difference between the mean value and the individual measurements. It has the same units as your data points, just like the mean value. If lots of points fall much lower and much higher than the mean value, you will have a large standard deviation. If your points cluster tightly around the mean, then your standard deviation will be small.

The standard deviation is calculated by subtracting the mean value from a set of measurements, and then combining the remainders. We have five data points: 0.300, 0.323, 0.351, 0.356, and 0.377 grams. How far do these points lie from the mean value? The difference (higher or lower) between the first value (0.300 gm) and the mean value (0.341 gm) is 0.041 grams. For the other four points, we find differences of 0.018, 0.010, 0.050, and 0.036 grams.

(Note that we express each difference as a positive number, so that a point above the mean cannot "cancel out" a point below the mean.)

Sorting the differences from smallest to largest, we get values of 0.010, 0.018, 0.036 0.041, and 0.050 grams. The standard deviation for this data set is 0.030 grams, which is similar to the average value of these differences.

We can combine our values for the mean value μ and the standard deviation σ and say that our observed sample of pinto beans has a weight per bean of $\mu \pm \sigma$, or 0.341 ± 0.030 grams. In general you should find that two-thirds (67%) of all data points lie within $\pm\sigma$ of the mean value for a distribution (and 95% should lie with $\pm 2\sigma$), so we expect that two-thirds of a large sample of pinto bean weights should lie between $0.341 - 0.030 = 0.311$ and $0.341 + 0.030 = 0.371$ grams.

4. Do roughly two-thirds of our five bean weights lie between 0.311 and 0.371 grams?
(Yes / No) (1 point)

It is a bit of work to calculate standard deviations by hand, so we will go ahead and use a computer program. We will also plot our data as histograms, so that we can examine the distribution of the data points.

1.2.2 Learning about Histograms

We will begin by introducing the concept of a histogram, a plot which shows how many measurements fall near to or far from a mean value. If you measure something in the natural world a large number of times, the distribution of measured values will form what we call a normal distribution, or a bell curve. There will be a large number of measurements piled up near to the mean (average) value, and as we move further and further from the mean the number of measurements will drop to zero.

The plot shown in Figure 1.1 is a distribution of the number of pages measured for texts from a shelf of books at a public library. The x-axis runs horizontally across the page, and tells you how many pages there are in each book. The y-axis runs vertically up and down the page, and tells you how many books fell into each page range. We group the books into bins, each 15 pages wide, and then count how many books fall within each bin. The most filled bins surround the mean value, and as we move further away from this value, we find that there are fewer and fewer books in each bin.

Notice that the plot has a caption, which describes its content and the primary conclusions drawn from it, and the x- and y-axes are clearly labeled, and include units as appropriate. You should format any plots that you place within your laboratory reports similarly.

Figure 1.2 shows the distribution of book lengths again, and superimposes a normal distribution (the blue bell-shaped curve) on top of the data. Notice how the peak of the bell curve, the mean value of the distribution, lies roughly in the middle of the distribution. The

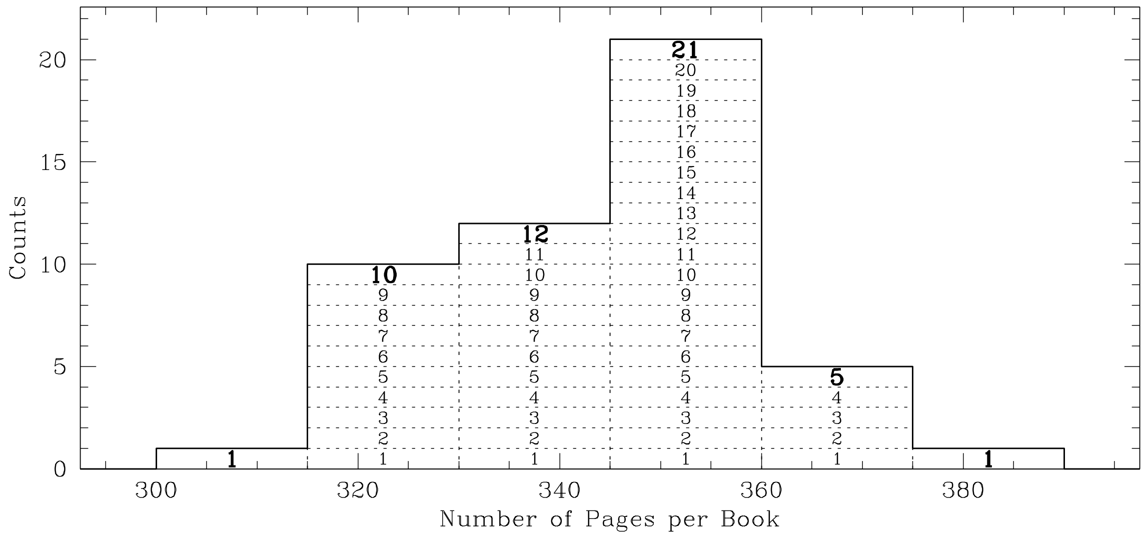


Figure 1.1: The distribution of measured book lengths (in units of pages), for a sample of 50 novels. Only one book had between 300 and 315 pages, 10 had between 315 and 330, 12 between 330 and 345, 21 between 345 and 360, five between 360 and 375, and one final text was between 375 and 390 pages long. The average book length for the sample thus lies somewhere between 340 and 350 pages.

more scattered the values are, with more book lengths lying further away from the mean, the wider the distribution, and the bell shape, will be. In the left-hand panel, the curve and the number of books per bin are only rough approximations of each other. As we increase the size of the sample, however, we can make the bins smaller and still fill each bin with a statistically significant number of points, and the distribution of book lengths will become smoother, and appear more like the underlying bell curve. Figure 1.3 displays a Gaussian drawn to fit these data, with the mean value (μ) and the size of 1σ shifts away from the mean labeled for clarity.

Now that we have an idea of the form of a histogram, let's examine a few distributions of measurements made, quite similar to those that you are making for yourself this week. Load the GEAS project lab exercise web page into a web browser (see the URL on page 2 in §1.1.2), and click on the link labeled "Web application #1 (histogram demonstration)." This will cause a new browser window to appear, containing a web application that we will use to examine a few histograms.

This web application is provided in a modern HTML5 format and an older Flash-based format. The legacy Flash-based tool is shown in the video tutorial for this lab exercise. Small cosmetic changes were made for the HTML5 version, but it operates the same way. We recommend that you use the modern HTML5 versions of all of our web applications.

When you first open the web application, look over the entire screen carefully and make sure that you understand the options available to you. There are General Options shown in

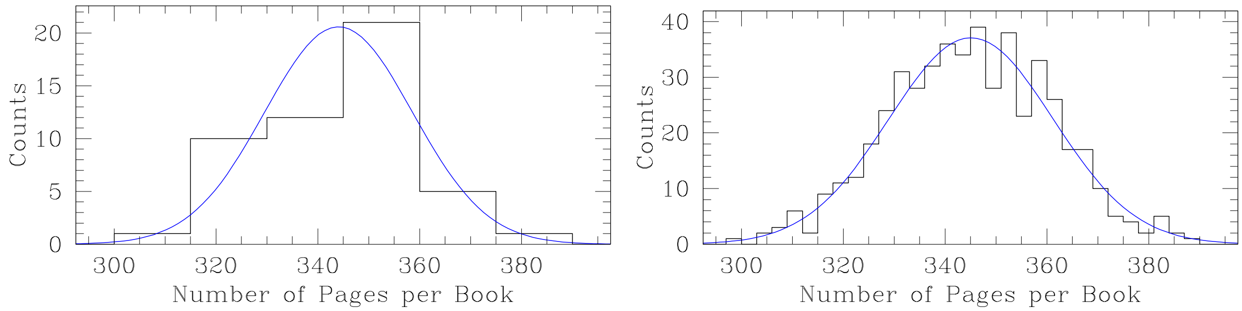


Figure 1.2: The distribution of measured book lengths for the sample of 50 books shown above in the previous figure (left), and for a larger sample of 500 books (right). The smooth bell-shaped curve shows the shape of the estimated underlying distribution in each case, with mean values (344 versus 345 pages) which match the peaks of the curves. Roughly two-thirds of the measurements fall within σ (± 15 pages) of the mean value. As the sample size increases, we can make the bins smaller, and the distribution of points shifts closer and closer to matching the smooth bell shape of a normal distribution.

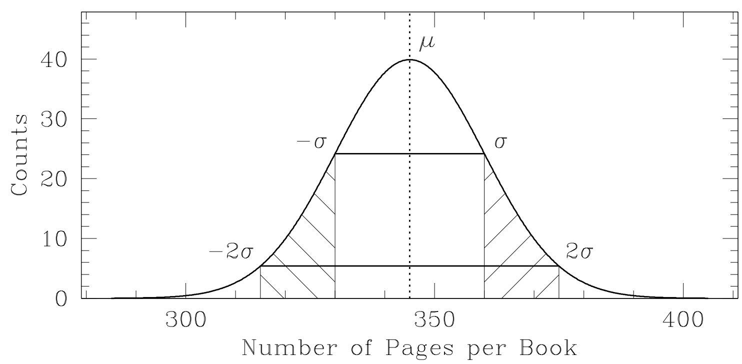


Figure 1.3: A smooth Gaussian, or bell curve, fit to the distribution of book lengths shown in the previous figure. The mean (labeled μ) is the center of the curve, and has the highest y-value (the peak of the bell shape). A line labeled $\pm\sigma$ extends from $\mu - \sigma$ to $\mu + \sigma$; roughly two-thirds of measurements will fall between these positions. A second, lower line labeled $\pm 2\sigma$ extends further, from $\mu - 2\sigma$ to $\mu + 2\sigma$; it includes the empty $\pm\sigma$ region and the two regions between -1σ and -2σ and between 1σ and 2σ (hatched for emphasis). Almost 95% of measurements fall between $\pm 2\sigma$, leaving two small tails to the far left and far right of the distribution made up of points which fall more than 2σ from the mean.

the lower right corner, which allow you to print out (make a paper copy) of your plot, or to save a copy as a PNG-format image file on your computer disk (or to attach such a file to an e-mail message that you can send to yourself). By clicking on the button labeled “Help” you will get a quick run-down on the tool options and purpose, and “Credits” will tell you a bit about the creator. You should always click on the “Help” button when you examine a new web application, as your questions may be answered there.

PNG format is simply a particular file format that stores all of the information that makes up a drawing. (You may use other image formats if that is easier for you; what matters is that the image displays properly within your lab report.) When you save a copy of a histogram (or another image from one of our web applications), your browser may attempt to attach a .txt extension to the file name, rather than letting it end in a .png extension. It is important that your file name end in .png; a name like histogram01.png is good, while a name like histogram01.png.txt is bad. If you are unsure whether your file name was defined correctly, check the name of the file on your local disk. You may need to rename it, to remove an errant .txt extension. As a double-check, try double-clicking once on the name of the saved file. If it opens up and the drawing appears, then your computer has successfully identified the file as having been written in PNG format.

This tool enables you to create histograms that show the distribution of data points for four sample data sets. Under the Experiment Options listed in the upper right corner, you can change the number of measurements that you make (see "sample size"), to observe the effect of taking lots of data, or only a little bit. You can also make your measuring tool (your ruler, or scale) very precise or very vague (see "precision"), to see how important it is to measure properties to a significant number of digits (such as measuring a length to 324.35 cm rather than just writing down 320 cm). The final variable to change allows you to "clip" the data, and discard the lowest and highest measured values from your data set before analyzing the points. (All of the N data points will still appear on the histogram, but only the innermost $N - 2$ ones will be used to determine the mean and sigma values.)

Part of doing science is being able to communicate your work clearly to others. Read the title and axis labels on the plot, so that you understand its purpose. The green columns show the number of data points in each bin, where the central bins lie closest to the mean value of the distribution. The taller the column, the more data points it contains. The red curve shows you the shape of the underlying distribution of the measured property, while the green curve shows you the approximate version estimated from the presented data set. Experiment by changing the four variables listed under Experiment Options, to see how your choices affect the level of agreement between the underlying and the observed distributions.

Let us now work through a few questions, in order to test our understanding of histograms. Start by choosing one of the four data sets, and selecting a value for the sample size and the measurement precision. Now change one of these three elements, but before you do so, predict to yourself the effect that the change will have on the plot. Repeat this process several times, until you feel that you have a good impression of how the tool works. Once you have done so, read through and answer the questions listed below.

5. Set the web application to show a distribution of seven tortilla chip lengths, with a precision of 0.05 cm. What happens if you turn data clipping on and off? When you remove the lowest and highest values from the sample, the observed value for sigma (the spread around the mean value) always (decreases / stays constant / increases). (1 point)

6. Now change the number of measurements from seven to 70,000 chip lengths. When you remove the lowest and highest values from the larger sample, the change in the observed

value for sigma is (smaller / the same / larger) then/as when this is done with only seven data points. (1 point)

7. Clipping the two outermost values from a distribution of only seven points will almost always cause the estimated value for sigma to drop to well below 100% of the underlying value. Why then do we suggest doing this in some real experiments? (1 point)

8. If a histogram of seven data points looks more like a picket fence than a bell curve, roughly how many points do you need to measure before the sample looks recognizably like a smooth bell curve?

Save a copy of your bell curve histogram to disk as an example, and include it in your Google Document lab report. Click on the button labeled "Save (or Print) Figure" to save a copy of the image to your local computer disk, making sure that the file name ends with the suffix ".png." To insert this file in your Google document lab report, open the online report, and choose "Insert > Image" from the menu bar labeled "File, Edit, View, Insert, ..." Click on the button labeled "Browse" which appears on the "Insert Image" menu, and then double-click on the name of your PNG-format file. The image will appear within the online document. You can then adjust its size to fit well within the page margins by clicking once on the image, and then shifting the small square symbols which appear at each corner of the image. Pull one of the squares toward the center of the image, and the entire image will decrease in size. (1 point)

9. As you increase the number of measurements, what happens to the relationship between the underlying and the observed values of μ and σ – do the observed values become more accurate, getting closer to the underlying values? (Yes / No) (1 point)

10. Can you ever measure an observed value for σ which is less than the measurement precision? (Yes / No) (1 point)

11. Set the web application to show a distribution of 1,000 book lengths, with a precision of 10 pages, and then change the quantity being measured from book lengths to tortilla chip lengths. What are the key differences between the two histograms, and why are they so different? What can you conclude about the way in which you should decide if your measurement precision is sufficient for a given experiment? (1 point)

1.2.3 Plotting and Analyzing your Data

Now that we have worked with a few histograms, let's build one for the seven data points that you collected in Table 1.2. Reload the GEAS project lab exercise web page (see the URL on page 2 in §1.1.2), and click on the link labeled "Web application #2 (plotting tool)." This will cause a new browser window to appear, containing a web application that we will use to plot your data as a histogram. You will be using this same plotting tool throughout the semester, so make sure to examine it carefully and clear up any questions that you have about its operations with your instructor.

First type your last name and access code into the two text boxes at the top of the form. Then carefully read through all of the information listed on the web page.

This web application can be used to plot several types of data. We can create histograms, or we can plot one variable as a function of another (we call this an xy plot, or a scatter plot). We will discuss xy plots in §1.5.2; so for now, concentrate on the histogram options.

We are going to create a histogram plot from your seven measurements. Begin by selecting "Histogram" at the top of the form from the pull-down list of three plotting options. Next, type in a title for your plot, and labels for the x- and y-axes. These labels should clearly identify the contents and purpose of the plot that you are creating. If you were measuring dairy cow output, for example, you might type a title of "Dairy Cow Daily Milk Production", and use x- and y-axis labels of "Amount of milk per cow (liters)" and "Counts." Keep each of your plot titles and axis labels to less than 40 characters (40 letters and spaces) in length. Be careful not to reverse the x- and y-axes labels – the x-axis identifies the quantity being measured, and its units, while the y-axis tells us how many data points fall within each bin along the x-axis.

Scroll down until you see the text box labeled "numbers to plot", and enter your seven measurements into the box, placing one measurement per line. Do not add any commas, parentheses, or other extra characters – just type each measurement on a separate line.

Once you have completed these steps, click on the button labeled "Create Plot" and your plot should appear in a separate browser window. Does it appear as you expected it to look?

A title should appear above the plot, and the x- and y-axes should be clearly labeled (review the example plots online if you are unsure how to label your axes). Your plot should show a histogram drawn as a series of light blue columns. Above the columns, the mean and sigma values will appear, labeled as "Mean value of $\mu \pm \sigma$."

12. Record the values below, adding units at the end of the line.

$\mu \pm \sigma =$ ____.

Now click on the back button on your browser, to return to the plotting tool. Remove your highest and lowest measured values from the set of seven, and remake your histogram. (You may need to retype your ID code to regain access to the database.)

Record the new values for μ and σ below, again with units.

$\mu \pm \sigma =$ ____. (1 point)

13. How do the two μ estimates compare (by how many σ do they differ)? Did your value for σ change in a predictable fashion? Do your two estimates of μ agree with those that you determined earlier by hand, written down on page 8? (1 point)

14. Examine your data, and decide whether your measurements were made with sufficient accuracy. (For example, did you eliminate any potential systematic errors that you identified?) Was the limiting precision good to at least one-hundredth of the mean value? Are

there any changes that you need to make to your plan, now that you have taken a small set of test data? (1 point)

You are now ready to take a larger data set. Fill in Table 1.3 with a set of 30 measurements. (If you are completely satisfied with your first seven measurements, you may repeat them here in the first seven boxes.) Make sure to again label the table with the name for the quantity being measured, and to note the units of the measurements.

15. When you are done, create a new histogram from your larger data set. Save the image file to disk (or e-mail it to yourself), and include it in your Google Document lab report. Be sure to include a plot title, label the x- and y-axes, and add a caption to go with the figure in the lab report explaining its contents and your primary conclusions. (You should do this with all of the figures that you include in your lab reports.) (1 point)

1.2.4 Reproducing Your Experiments

An important part of a successful experiment is being able to describe it clearly and accurately, so well that another person could read your description and then conduct a successful second set of trials. In order to test how well you defined your simple experiment, your instructor will give your experimental description (your answer to question 1 on page 5) to another student, and give you their description of *their* experiment in return. (*Be sure to enter your experimental description into your lab report as quickly as possible during Week 1, so that you can trade descriptions with another student and still have time to complete the entire lab assignment by the end of Week 2.*)

Once your instructor has given you the description of your partner's experiment, attempt it yourself, recording 7 data values in Table 1.4 and entering these values and your derived mean and spread for the inner 5 points in your lab report. Once you have done so, your instructor will introduce you and your partner to each other via e-mail. Together, the two of you will compare the mean and spread values that you each determined for each of your experiments.

16. For your partner's experiment, your $\mu \pm \sigma =$ ____________________________________,

and their (5-value trial) $\mu \pm \sigma =$ ____. (1 point)

Table 1.3: Experimental Data II: ________________________________

Trial	Value ( )	Trial	Value
1		16	
2		17	
3		18	
4		19	
5		20	
6		21	
7		22	
8		23	
9		24	
10		25	
11		26	
12		27	
13		28	
14		29	
15		30	

(6 points)

17. For your experiment, your (5-value trial) $\mu \pm \sigma =$ ______________________________,

and your partner's $\mu \pm \sigma =$ ____. (1 point)

18. Discuss the results that you and your partner obtained for both experiments. How close do you think that your mean values should be (in units of σ) in order for you to say that you described and conducted reproducible experiments? By how many σ do your mean

Table 1.4: Experimental Data III: ____________________________

Trial	Value ( )
Trial 1	
Trial 2	
Trial 3	
Trial 4	
Trial 5	
Trial 6	
Trial 7	

(2 points)

values actually differ? If the difference is larger than your requirement, brainstorm with your partner and identify the most likely reasons for the discrepancies. (For example, if you measured marshmallow toss lengths and one of you is the star pitcher on the baseball team, you have a probable explanation for why that person's mean toss length was larger than their partner's value.) (*You may each write answers to this question, or you may create a single answer together and place it in both of your lab reports.*) (4 points)

19. Having gone through the experience of having another person attempt to duplicate your experiment based on your written notes, how would you alter your experiment and change your experimental description, if you were to do this exercise again with a new partner? (4 points)

1.3 The Role of Errors

Read through all of §1.3 before beginning to answer the questions in the text, so that you understand the material. Make sure to answer every part of the five questions contained in this section.

1.3.1 Three Types of Error, or Variation

In scientific parlance, the word "error" often means "uncertainty" rather than "mistake," and refers to a variety of factors that can combine to blur agreement between measurements. No set of measurements is perfect, and most experimental variables have an expected spread in value (measuring scales are not infinitely precise, and quarter-pounders do not all weigh exactly one-fourth of a pound).

We are going to define three factors (natural variation, measurement error, and systematic error) which can produce changes in measurements, and learn about their effects. Measurement and systematic errors relate to the fact that we cannot design perfect experiments, where only one variable ever changes and properties are measured to infinitely small precision. Natural variation can refer to the intrinsic spread in a quantity (for example, differences in age, gender, and health produce differences in height, so all people are not the same height), or to the inevitable variation in manufactured items (even the United States Mint cannot produce truly identical pennies). Strictly speaking, natural variation is an intrinsic characteristic (how different are a set of objects, if measured perfectly?) and not a source of error, but because it can produce a similar effect we will consider it here.

Natural Variation, or Natural Error

Natural variation refers to the intrinsic width of a distribution of a measured property. If you carefully measured the widths of a large set of pennies with a vernier caliper (a measuring device with two long arms, capable of making very precise measurements of small lengths), you would find that the values clustered around a mean value, but that many tended to be slightly smaller or larger than that central value. The width of the distribution, or the amount that most measurements were above or below the mean, would tell you the natural variation in coin widths, or how much on average pennies differ from each other.

Natural variation is often a key quantity that an experiment will determine. For a well-performed experiment, it should be analogous to the measured value of σ.

Measurement Error

Measurement error refers to the precision with which a value is measured. If you carefully weighed a large set of books, but only recorded their weights to the nearest pound, then your measurements could all be off by $\pm 1/2$ pound. If you were examining books that weighed around a pound, for example, then your measurements would all tend to be 0, 1, or 2 pounds (plus a few 3-pounders, if you had a passion for Dostoevsky). If you increased the precision of your measurements by finding a scale that output values to the nearest hundredth of a pound, however, then you would be able to discern much smaller differences in weight between individual books.

Systematic Error

Systematic error refers to an aspect of an experiment which acts to distort the measurements being made, always in the same fashion (or direction). If you carefully measured the heights of a large set of people, but forgot to have your test subjects take their shoes off, then all of your measurements would be too high (they would all be biased in the same direction). People wearing high heels would have their heights overestimated more than people wearing thin-soled flip-flops, but all of the heights would be overestimates – this is why we call this type of error *systematic*. While natural error and measurement error can produce both large and small measurements, systematic errors shift the all of the measurements in one direction (all high, or all low).

In addition to the limiting precision of measurement error and the biasing effects of systematic errors, you might also have more general errors that distort measurements. For example, the wind might carry one thrown marshmallow farther away from you, but force the next to land close by; a careless experimenter might record a measured weight incorrectly; a poorly-designed experiment might ignore additional relevant factors which affect the results. In the interests of simplicity, we shall assume that as part of your experimental design process you consider and attempt to resolve such concerns before performing your experiments.

In this course, we want to identify and eliminate sources of systematic error, and reduce measurement errors to low enough levels that they do not interfere with our determinations of natural variation. As you plan your own experiments, ask yourself how systematic errors might creep into your experimental plan, and how precise your measurements need to be in order to let you measure the desired quantities accurately.

1. Select any one of the ten experimental questions listed on page 4. Identify a systematic error that could affect this experiment, and describe how to eliminate it. (2 points)

2. Compare the value of σ for your 30-point data set, and the precision P (the smallest difference that you could measure between two values).

Which of these two values represents your natural variation? (σ / P)

Which of these two values represents your measurement error? (σ / P)

Is your precision less than (or equal to) one-hundredth of your mean value? (Yes / No)

$$\frac{P}{0.01 \times \mu} = \frac{\qquad\qquad\qquad\qquad\qquad\qquad}{0.01\times} = \qquad\qquad\qquad\qquad\qquad .$$

How does your precision compare to your value for σ?

$$\frac{P}{\sigma} = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad} = \qquad\qquad\qquad\qquad\qquad .$$

In general, would you prefer that P/σ be smaller or larger than one? (Smaller / Larger) (2 points)

1.3.2 Error Bars

We have established that any measurement should have an associated error attached to it (for a single data point, this could be the precision of the measurement tool, and for the average value of a set of measurements, this could be the spread σ). How do we combine measurements when they were taken in different ways, and have different associated errors, and how do we represent the variation in associated errors?

When we have different errors associated with different points, it is often convenient to show these differences, by representing the errors as *error bars* (literally, bars of length σ drawn above and below each data point). In Figure 1.4 we display three measurements, with associated errors added on the right-hand panel. On the left, the three measurements are shown without any indication of relative error. Without any information on the relative importance of the points, we simply average them, deriving the value shown as a red line between the three values. On the right, error bars have been added to each point showing their relative precision. Realizing that the middle data point has much larger errors associated with it than the other two points have, we can minimize its impact, computing an average value which falls between the two points with relatively small error bars associated with them.

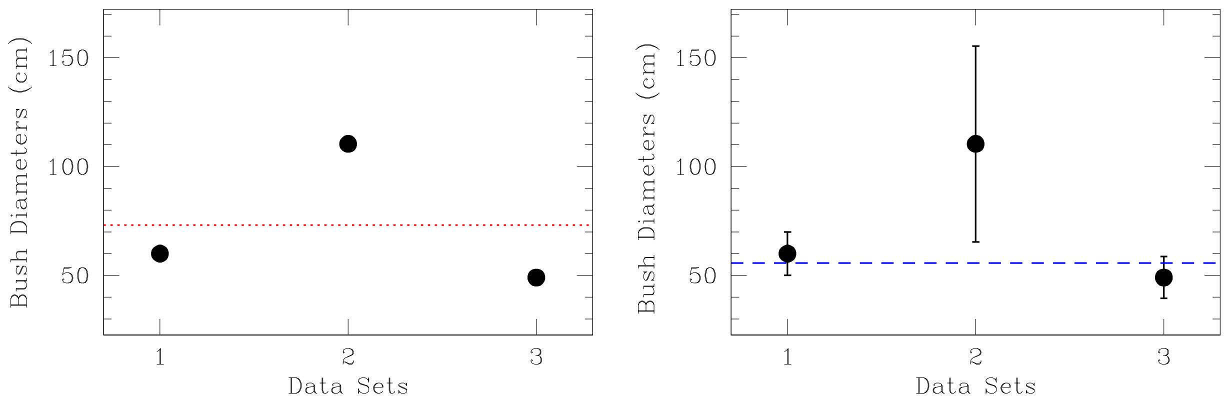


Figure 1.4: Three separate measurements of bush diameters are shown, with and without associated error bars. On the left, the three data points are averaged evenly to produce a representative value which lies in between all three points. On the right, the addition of relative errors allows us to count points 1 and 3 more than 2, and the representative value shifts down to lie between them. The knowledge that point 2 is much less constrained allows us to discount it, and thus to determine a more informed final value.

One might ask whether it would be appropriate to discard the central data point, given how far it lies from the other two, but with only three recorded data points that would be a risky action.

Error estimates, and error bars, allow us to combine data taken under different conditions. Though they cannot identify systematic errors, they are effective tools for factoring measurement errors into determinations of average properties. Consider, for example, measuring 100-meter race times accurately. In 2009, Usian Bolt of Jamaica set a world record with an amazing time of 9.58 seconds, and times for any two of the ten fastest men in the world often differ by a mere 0.01 seconds. To record such a race fairly, the race times should be recorded to the nearest 0.001 seconds (or less). Imagine that you wanted to compute an average race time for a runner, and you had some measurements recorded with Olympic track equipment, and others taken with your grandmother's antique pocket watch. To which times would you assign the most importance (and the smallest error bars)?

3. If the error bars on all of the points in Figure 1.4 were all the same length, how would the derived average values change? The red line on the left panel would (drop / stay at the same height / rise), and the blue line on the right panel would (drop / stay at the same height / rise). (2 points)

4. If the error bars on point 3 in Figure 1.4 became smaller, the blue line on the right panel would (drop toward point 3 / stay at the same height / rise away from point 3). (2 points)

5. What can error bars add to an analysis? (2 points)

1.4 Evaluating Data, and Model Fits

As we begin to study different types of astronomical data, we will need to have a toolbox of basic ways to evaluate different data sets. In one case we might need to combine a large number of repeated measurements of the same quantity (like measuring how much light is emitted from a faint star, by combining telescope observations taken on different nights). Alternatively, we might need to search for a trend between two variables (does the observed phase of the Moon depend on how long after sunset it rises?).

We will describe three different evaluation tools below, each one chosen because it will be useful to us in studying some type of astronomical data in this course. They should seem somewhat familiar to you, because you have already begun to use these concepts!

Keep in mind that our emphasis in this course will be on using these indicators *qualitatively* rather than *quantitatively*. This means that we are more interested in your being able to interpret results than in your calculating fit coefficients by hand. Our tools are designed to provide the analytical information that is most helpful to understand the meaning of your plots. (Simply put, you don't work for them – they work for you!)

Read through all of §1.4 before answering the questions, so that you have a good grasp of all four topics. Make sure to answer each part of the four questions contained in this section.

1.4.1 Mean Values and Standard Deviations

It is useful to combine repeated measurements, to determine an average value (μ) and to characterize the variation between measurements (with σ, the spread, or standard deviation). As we learned in §1.2.2, a histogram can also help in checking whether the measurements are consistent with a normal, or bell curve, distribution.

1. In Figure 1.5, how do the means of the two distributions compare (roughly)? Which one has a higher σ value? Identify another significant difference between the two sets of data. (2 points)

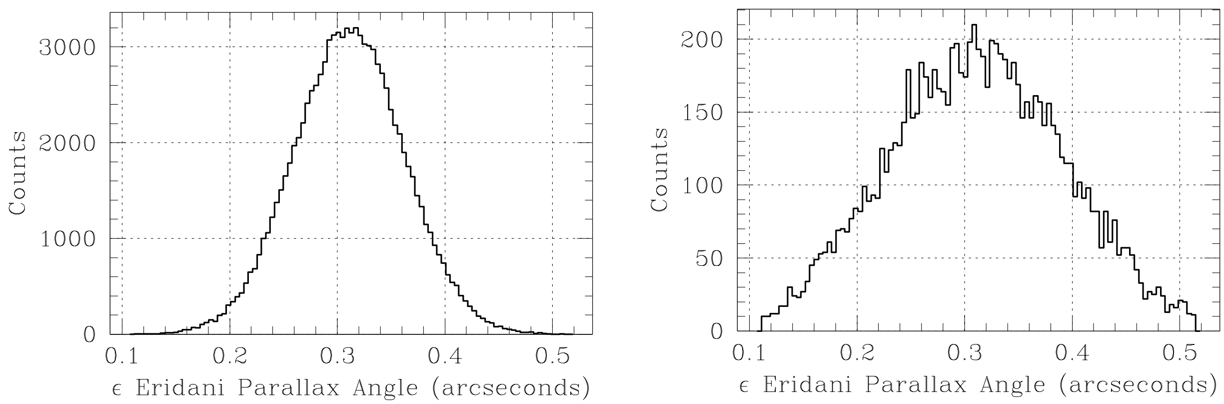


Figure 1.5: Histograms showing the distribution of measured parallax angles for the star Epsilon Eridani, in two different astronomical surveys. Estimate the values for μ for each sample, and compare the σ values. How wide is each distribution, in units of arcseconds, at the points where the number of counts has dropped by a factor of two from the peak value?

1.4.2 Slopes and y-intercepts

When measuring two variables (x and y) and trying to deduce a relationship between them, we frequently plot one against the other, creating an xy (or scatter) plot. We can then compute the slope (m) and y-intercept (b) for the combined data set (as we will explore in more detail in §1.5). For a quick review of slopes and y-intercepts, return to the GEAS project lab URL and select "Getting Started > Slopes and y-intercepts" from the masthead.

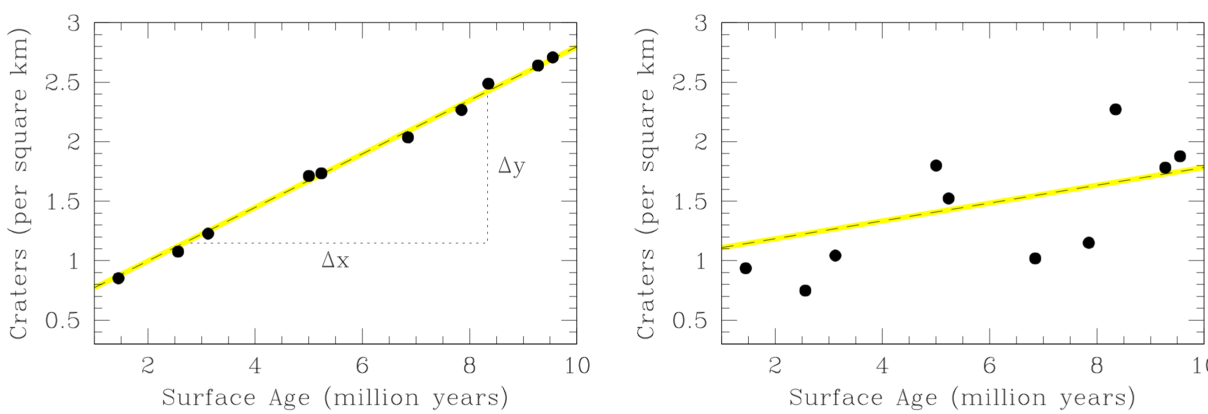


Figure 1.6: Plots showing the relationship between the surface density of craters on Mars and the age of the surface for two samples. Estimate the slope and the y-intercept from each plot for practice, and examine how scattered the points are around each line.

2. In Figure 1.6, the slope for the left panel sample is (smaller than / the same as / larger than) the slope for the right panel sample. The y-intercept is larger in the (left / right) panel sample, and the points in the (left / right) panel sample show more scatter (lie further from the line). (2 points)

Just as we add error bars to a point to indicate how confident we are in the value, we can add error measures to slopes and y-intercepts to indicate how tightly constrained they are. The larger the error bars, the more the slope or intercept could change without substantially changing the quality of the fit (and the further the points lie in general from the line).

Imagine that we want to determine the speed at which a star is moving, in units of kilometers (km) per second, so we measure its position and fit a line to the position as a function of time. The line has a slope $m = 5.00 \pm 0.05$ km sec^{-1}. A change in the slope of 0.10 (twice the assigned error) is significant, so the slope probably lies between 4.9 and 5.1 km sec^{-1}.

If the star were instead moving at a speed of $m = 5.0 \pm 0.5$ km sec^{-1}, then we could only say that the slope probably lies between 4 and 6 km sec^{-1}, a much broader range of velocities. What if the slope were $m = 5 \pm 5$ km sec^{-1}? The speed could lie anywhere between -5 and 15 km sec^{-1}. The error tells us how well we know the velocity.

3. If you determined that a star was moving at a speed of 15.0 ± 0.5 km sec^{-1} in a particular direction but I found that its speed was 14.2 ± 0.5 km sec^{-1}, could our results be consistent? Roughly how much lower would my velocity have to be for you to say that either our two studies must have observed different stars (you said Asellus Borealis, but I heard Asellus Australis), or something else was significantly different about them? You may assume that a 5σ or larger deviation is significant. (2 points)

1.4.3 Root Mean Square Deviations

When the points in a plot follow more closely to the fit line, the fit is generally a better match to the data. This tells us that there is a better agreement between the fit and the raw data points, because the line represents a more accurate physical model of the relationship between x and y, and/or because the errors on the individual data points are smaller. In Figure 1.6, for example, the points in the left panel are less scattered than those in the right panel and so we have more confidence in the derived slope and y-intercept.

When we examine repeated measurements of a single quantity (see §1.2.1), we determine a mean value (μ) and then a spread around it (σ). The smaller the value for σ, the closer we expect a set of measurements to lie to μ. In a sense, σ tells us how accurate the mean value is, and when we compare mean values from two experiments and ask if they are consistent (in agreement), we calculate their difference in terms of σ values. A difference of 2σ or less is reasonable, while a difference of 5σ indicates that something is probably different about the two data sets (or that something has gone wrong in one, or both, of the experiments).

We can use the same idea for xy plots. Rather than asking how tightly constrained a mean value is, we examine the slope and y-intercept values and attach σ-like spreads to them. When we fit a line to a set of points, we minimize the distance in y between each data point and the line. We want the combination of these offsets to be as small as possible. Figure 1.7 shows two possible lines drawn through the same set of points. Which line minimizes the offsets to the data points in y?

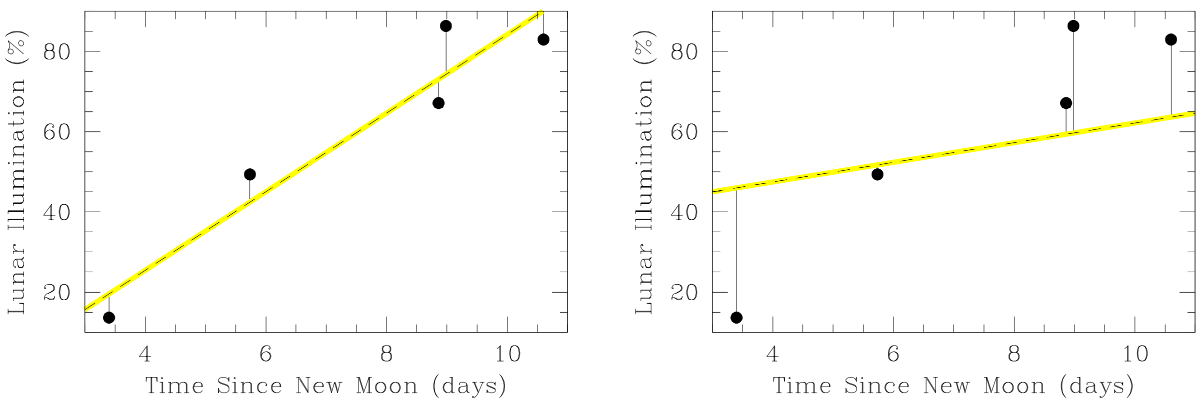


Figure 1.7: Two plots showing a relationship between the amount of time since the last new Moon and the percentage of the Moon's surface which appears illuminated from Earth. In the left panel the line has been placed to minimize the offsets from y (shown as vertical lines) from the set of points, while the offsets on the right panel are larger and the fit is less good.

We can quantify, or express as a number, the effect of these offsets in y, by adding up the net effect. We first square each offset, then sum them, and average the sum to determine what we call the "root mean square" deviation (the "rms" value). First, we need to calculate the y-position that each point would have if it lay exactly on the line (rather than a bit above or below), and find the difference between this position and that of the actual point. The offset o is defined as

$$o = y - (mx + b).$$

We square each offset so that points above and below the line will not cancel each other out (by having offsets which are equal in length but have opposing signs), and then divide by the number of points to find the average value. Finally, we take the square root to end up with a quantity which has the same units as y, and which does not increase or decrease just because we vary the size of the sample.

This rms value is a measure of how well the line fits the data. A large value indicates that there is a problem with the data set or that the model which defines the line is not a good model to completely explain the connection between the two variables (x and y). The slope and y-intercept values are specifically chosen so that the rms value is as small as possible.

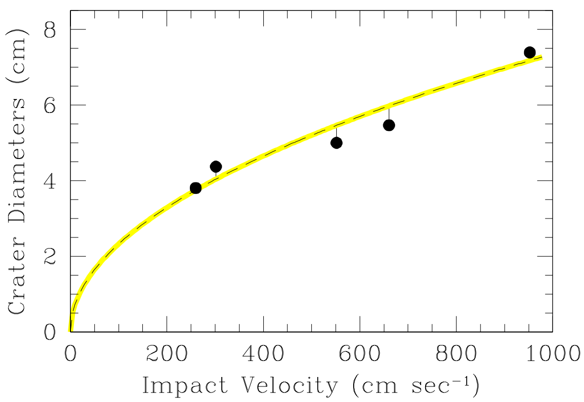
You may find rms values labeled as either "rms", "rms σ", or as just "σ" in various science classes; we'll use the rms term here, to avoid confusing rms values for (x,y) data with the spread of y values around a single measurement (which we called σ when introducing histograms in §1.2.1). These deviations are very similar quantities, albeit for two types of data, and σ is a fine general term for them. The primary difference is that a standard deviation (σ) for a repeated measurement compares each measurement of y to the mean (average) value μ, while an rms deviation compares each measurement of y to the *predicted* y-value (such as $mx + b$) generated by a fit to paired (x,y) data.

4. In Figure 1.7, the rms value for the line in the left panel is 5.7, while the value for the right panel is 10.5. Does the lower rms value belong to the line which passes most closely through the points? (Yes / No) What would it mean if a data set had an rms value of exactly zero? Does this seem very likely to happen? (2 points)

We have examined rms deviation values for a simple linear fit through a set of data points, but we don't need to limit our usage of this quantity to linear fits (straight lines). We can fit any function that we choose to reproduce y as a function of x, and then calculate the offsets between the y values and whatever shape of line our chosen model defines.

In Figure 1.8, we fit a straight line ($y = mx$) and then a curved line ($y^2 = mx$, or $y = m\sqrt{x}$) to a single set of data points. (For simplicity, we will just assume that the y-intercept is equal to zero in each case, so that all lines pass through the origin.) Which of these two functions produces a line which best matches the data points? We can often decide which function is a better match to the data by eye, just by examining the offsets in y. In a case like this, even a cursory examination shows that the offset lines are larger on the right than on the left, so we know before considering the rms values which model is better.

When comparing the two rms values, be sure to take their ratio (divide them) rather than subtracting them from each other. The fact that one is larger than the other by +1.71 is significant because both rms values are less than three, but would be much less significant if both values were larger than, for example, 1,000 (say, 1000.79 versus 1002.50). However, the fact that one is a factor of 3.16 times larger than the other is significant in any case.



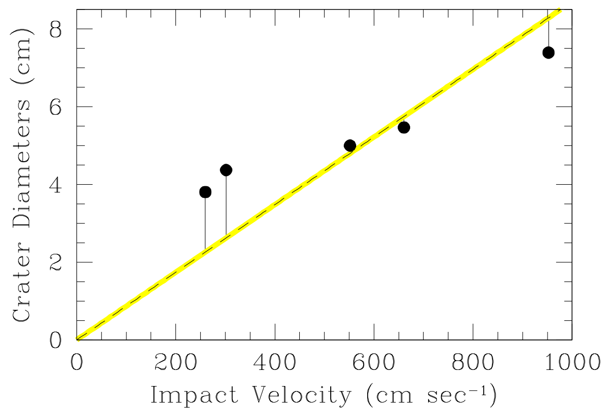


Figure 1.8: Two plots showing the relationship between the impact velocity of a small rock (x), and the size of the resulting crater (y). The points are fit with the function $y = m\sqrt{x}$ on the left panel, and $y = mx$ on the right panel. Because the points are less offset in y from the yellow model line in the plot on the left (rms = 0.79) than on the right (rms = 2.50), we observe that this function is a better match to the relationship between x and y.

1.5 Trends, or Relationships, Between Variables

We often seek to understand the relationship between two measured quantities, in order to better understand the underlying behavior of the Universe. How does the angle between the Earth, Moon, and Sun relate to the apparent lunar phase? Are the brightest stars in the sky intrinsically bright, or just very near to us? Why do the accretion disks surrounding some black holes give off high energy x-rays? Many such questions can be explored by comparing the behavior of two variables, to see whether or not they are dependent on each other and if so, how they relate.

Be sure to answer fully each of the five questions in this section.

1.5.1 Learning about xy (or Scatter) Plots

If we are studying two variables, we often label them x and y. A plot which shows the value of y for each value of x is then called an xy plot, or a scatter plot. When working with such data, make sure that you know the definition for each measured quantity, and the units, and check whether there are associated errors.

Consider how representative the data set may be. If you wanted to study the typical ages of human beings, how might your results differ if you only examined people found at a pre-school, or only those who live in a retirement home? In each of these cases, you would be drawing your data from biased samples (samples which do not represent the entire population of humans on this planet), and so your results would be biased as well. Because we typically sample a small subset of the entire set of objects under study, we need to be very careful to select representative objects.

We are going to plot the values of two measured quantities against each other and then examine their relationship. We will analyze June temperatures for various locations as a function of the Sun's peak altitude (how high it appears in the sky at noon). This exercise will get you thinking about how seasonal variations across the surface of this planet, and to help you to become comfortable making xy plots.

1.5.2 Plotting and Analyzing your xy Data

Begin by returning to the GEAS lab exercise web page (see the URL on page 2 in §1.1.2), and click on the link labeled "Seasonal table." This links to a page of data. A set of comments at the top of the page describes the listed quantities, and the three columns of numbers listed below represent the peak altitude of the Sun in the sky, in units of geometric degrees (column 1), June temperatures, in degrees (column 2), and associated errors, in degrees (column 3) for a set of northern and southern hemisphere locations across the globe.

Keep in mind that the peak altitudes for the Sun are measured in geometric degrees, where an altitude of zero means that the Sun never rose above the horizon at that location and an altitude of 90° means that Sun lay directly overhead at noon. In contrast, the temperatures are measured in terms of heat, or energy, in units of degrees Fahrenheit (where water freezes at 32° F and boils at 212° F).

Because the temperatures were recorded in June, the solar altitudes reflect the fact that the northern hemisphere of the Earth was tilted over by 23° toward the Sun, while the southern hemisphere tilted away from it. Residents of Mazatlan, Mexico, located 23° north of the Equator, thus saw the Sun pass directly overhead at local noon, and from all locations north of Mazatlan, the Sun was 23° higher at noon than it was three months earlier, when the North Pole was neither tipped toward nor away from the Sun. We will not attempt to explore the complete causes of the seasons here on Earth today. Instead, we will ask ourselves one simple question – are June temperatures at various locations on Earth tied to the observed peak altitudes of the Sun from those locations?

Read through the table, and make sure that it makes sense to you. Then copy the three columns of data (by highlighting the rows in the file which contain them and typing Control-C or Command-C) and return to the GEAS web page. Click on the link for "Web application #2 (plotting tool)" to return to the tool which you used previously to create histograms. Now, you are going to use it to plot and analyze the relationship between solar altitude and June temperature.

As before, type your last name and access code into the two text boxes at the top of the form. Re-read through all of the information listed on the web page, paying special attention to the special options for xy plots.

Begin by selecting "Y versus X plot, with errors in Y" from the pull-down list of three plotting options. Next type in a title for your plot, and labels for the x- and y-axes, keeping each label under 40 characters in length. These labels should clearly identify the contents of the plot that you are creating. You might type a title of “Northern Summer Temperatures”, and use x- and y-axes labels of “Peak Solar Altitude (degrees)” and “Temperature (degrees).” Be careful not to reverse the x- and y-axes labels – the x-axis identifies the quantity being plotted along the horizontal axis (from size to side), and the y-axis the quantity being plotted along the vertical axis (from the bottom to the top of the plot).

If you selected the option “invert” for either axis, it would run from the highest to the lowest values, rather than from the lowest to the highest (the distribution of points would be reversed, from right to left for x, or from top to bottom for y). You do not need to do this now. Go ahead and select “show linear fit to data,” to cause a line to be drawn fitting the relationship between x and y values. Do not select “force fit through origin,” however, as this would force the line to pass through the origin (where both x and y are zero).

Scroll down until you see the text box labeled “numbers to plot”, and copy your three columns of data into the box, in the order (x, y, e), or altitude, temperature, and associated error. Do not add any commas, parentheses, or other extra characters – just place each set of three measurements on a separate line. Once you have completed these steps, click on the button labeled “Create Plot” and your plot should appear in a separate browser window.

Examine the plot carefully, and make sure the the correct quantities are plotted on each axis, that the error bars are present, and that both axis labels and the title are correct. Someone reading your plot should be able to identify the quantities shown, and their units, from the title and axis labels alone. If there are any mistakes, even small misspellings, hit the “back” button on your browser, fix them, re-enter your access code, and recreate the plot.

Now examine the trend shown between x and y (Sun’s altitude and peak temperature).

1. Does y increase, or decrease, as x increases? At what altitude do the peak temperatures occur? What do you conclude about a possible connection between maximum solar altitude and the peak local temperature? (2 points)

The plot will be labeled with a slope and a y-intercept value.

2. If another survey of different locations derived a slope of 0.786 ± 0.055 for this relationship, would your results be consistent? Why, or why not? (2 points)

3. What is the physical meaning of the y-intercept for these data (what does it tell us about the relationship between the Sun's maximum altitude and the local temperature)? (2 points)

4. The plot is also labeled with a correlation coefficient, R. If R is near to 1, then x and y are positively correlated (y increases with x). If R lies between -0.5 and 0.5, x and y are fairly independent of each other (you cannot predict y from x). If R is near to -1, x and y are negatively correlated (y decreases as x increases). Is there a strong relationship between x and y here? (Yes / No) (2 points)

5. Save a copy of your plot, and include it in your lab report here. Include a figure caption describing the primary conclusions that one can draw by examining the plot. If you are unsure what to write, imagine that you showed it to a friend and explained what it meant to you. That explanation can form the bulk of your figure caption. (2 points)

1.6 Final (Post-Lab) Questions

1. Imagine that you measure parallax angles for nearby stars and find a mean value of 0.40 ± 0.04 arcseconds, while someone else finds a value of 0.41 ± 0.08 arcseconds in a similar survey. Would you say that their technique was less precise than yours, with a larger measurement error? (Yes / No)

If a third scientist found a value of 0.80 ± 0.04 arcseconds is the discrepancy more likely caused by systematic error or by measurement error?
(Systematic Error / Measurement Error) (3 points)

2. You want to study the population of dogs in your neighborhood. Since you often take an evening walk, you collect data by walking around the block every evening for a month and taking note whenever a dog barks at you through a fence. Is your survey sample biased in any way? If so, how? (3 points)

3. People often confuse "causation" (where a change in one variables *causes* a change in another) with "correlation" (where two variables vary together in a predictable fashion). Gravity causes dropped objects to fall to the ground, but purchasing flood insurance does not make your property more likely to flood. Identify an example of causation and an example of correlation, and describe the variables involved. (You get an extra pat on the back if your variables are astronomical in nature.) (3 points)

1.7 Summary

After reviewing this lab's goals (see §1.1.1), summarize the most important concepts explored in this lab and discuss what you have learned. Describe briefly the set up and implementation of your own data-taking experiment, your analysis of the resultant data, and your overall conclusions. (25 points)

Be sure to cover the following points.

- Describe what you have learned about the way to design and implement an experiment.
- Describe the primary quantities that you will use to evaluate fits to data (see §1.4), and perhaps suggest an appropriate use for each (an application based on a particular data set).

Use complete, grammatically correct sentences, and be sure to proofread your summary. It should be 300 to 500 words long.

1.8 Extra Credit

Conduct an additional survey of two related quantities of interest to you, plot the results, and derive a mathematical relationship between them. You may obtain data by either direct measurement (such as measuring the height of a bean sprout in your garden every day for a week), querying a relevant population (asking all your Facebook friends to tell you their heights and shoe sizes), or extracting data from tables (comparing the periods and peak luminosities for Cepheid variable stars in astronomy articles). Describe your experiment clearly and succinctly. Note any biases in the sample population which may limit the impact of your results.

Choose two quantities that you think are connected, so that your plot contains a clear trend (ideally a linear one that can be fit to determine a slope and a y-intercept). This is a chance for you to demonstrate how to collect and analyze data efficiently, and to display your creativity with an intriguing data set. (5 points)