Measuring Angular Extents

We can use angles to define the *sizes* of various objects on the sky. Both the Sun and the Moon span an angle of $0.5°$ (half a degree, or 30 arcminutes), which means that they appear to be roughly the same size. (This is why total solar eclipses, in which the Moon briefly blocks out the Sun, are both spectacular and rare.) We know that the Sun is 400 times larger than the Moon; it only looks as if it is the same size because it is also 400 time further away from us than the Moon is.

Figure 1 describes the geometric relationship between the diameter D of an object, the distance d it lies away from an observer, and the angle 2α that it spans on the sky. We can form a right triangle with height $D/2$ and width d, for angle α.

If the distance d to an object increases, does its angular extent (2α) become larger or smaller? If the diameter of an object D increases, does α become larger or smaller?

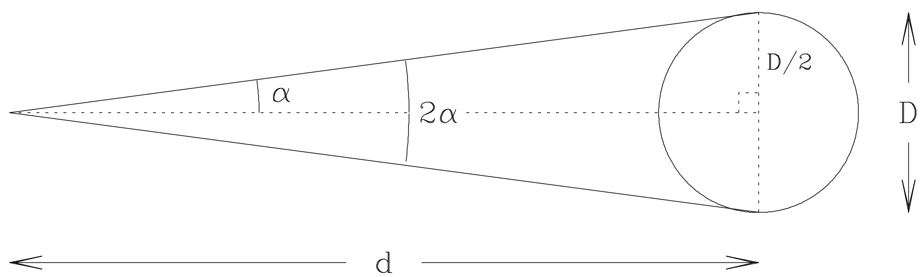

Figure 1: This figure illustrates the relationship between the distance d to an object, its linear diameter D, and the angle 2α that it takes up on the sky (its apparent angular size). We can bisect the angle 2α into two equal parts, two angles each with value α. Each one then forms a right triangle with height $D/2$ and width d.

Angular Size of the Moon

Try the following activity when the Moon is rising or setting, located near to the horizon rather than high in the sky. It may be easier to do when the Moon is gibbous (almost fully illuminated), but you can work with even the crescent phases if you must.

Determine the position of the Moon in the sky, and find a window through which you can observe it. Remember that the Moon rises in the east and sets in the west, so select a window that will provide a good view of this horizon at the time at which you choose to work. For example, the crescent moon sets in the west after sunset, while the waxing gibbous or full moon rises in the east just before or around sunset.

Make sure to observe from directly in front of the window, with the Moon well-centered inside it, rather than shifting to one side or the other. If you look straight into the window while standing directly in front of it, you should be looking right at the Moon.

Stand about five feet in front of the window, and find the Moon. Now tape a dime to the window near to the position of the Moon. Try to fold the tape over on itself or use doublesided tape, so that you can hide it all behind the dime (this will help you to see the dime more clearly). Then back away from the window, until the Moon and the dime appear to be the same size (have the same diameter).

Measure the distance between your observing location (your eyes) and the dime. (This will be easier if you can recruit a friendly assistant to help!) Repeat this process twice more, for a total of three measurements. Be sure to walk all the way up to the window and then back away (slowly) each time, so that each measurement is independent.

Use the GEAS plotting tool (at http://astronomy.nmsu.edu/geas/labs/html/plotter.shtml) to average the three measurements, and find the standard deviation. Be sure to note your units!

1. The three measurements of the distance at which the Moon and a dime appeared to be

the same size are ___________________, ___________________, and ___________________ .

2. The average distance is thus _________________________________ $\pm$ ___________________.

We can see that as the distance d to an object increases, its angular size α decreases (things look smaller when they are further away). If an object increases in size (its diameter D gets larger), its angular size will increase as well.

We can thus predict at what distance an object of a certain size will *appear* to span a particular angle. For the case of the Moon, where $2\alpha = 0.5°$ and $\alpha = 0.25°$,

$$\frac{d}{D} = \frac{1}{\tan(2\alpha)} = \frac{1}{\tan(0.5°)} = \frac{1}{0.00873} = 115.$$

We have made use of the fact that $\tan(2\alpha) = D/d$, but you do not need to be familiar with the tangent function to use the result. The key point is that if you know the angular size of an object, you know the ratio d/D.

3. If a dime has a diameter D of 1.79 centimeters, from what distance would you predict that it would appear to span an angle of 0.5° on the sky (appearing the same size as the Moon)?

$d = 115 \times D =$

4. Is this value within 2σ of your measured distance? If not, speculate as to what might have caused the disparity. *(If you measured the distance in feet and inches, remember that there are twelve inches in a foot, and 2.54 centimeters in an inch.)*

5. If you had taped a quarter instead of a dime to the window, would the distance d at which the coin appeared to span 0.5° have increased or decreased? Explain your answer.

6. The Moon has a diameter of 3,478 kilometers and lies 384,000 kilometers from Earth, while the Sun has a diameter of 1,392,000 kilometers and lies 149,700,000 kilometers from Earth. Which one should appear larger on the sky? Just a bit larger, or significantly larger (more than 20%)?

Parallax Angles

Let's introduce a related concept, one that will form the basis for determining distances to stars. The term "parallax" refers to an angular shift in the position of a distant object on the sky, apparent when it is observed from two separate locations. To experience it directly for yourself, hold your index finger out at arm's length. Close your right eye and position your finger so that, as you view it through your left eye, it appears to line up with a distant object (a tree, or a door frame). Now hold your index finger still, and close your left eye and open your right one. When viewed from this slightly different position, your finger should no longer line up with the distant object. This shift in position is called a "parallax shift." If you switch back and forth between eyes, while keeping your finger perfectly still, you can make your finger "dance" back and forth across your field of view repeatedly.

Next stand an arm's length away from the dime that you have taped on the window, and perform the same maneuver (observing the dime in place of your index finger). Does the dime shift position as you observe it with first one eye and then the other? If you step back away from the window, does the amount that the dime appears to move grow or shrink?

Now observe the Moon rather than the dime. Does it seem to shift position when viewed from first one and then the other eye, or does it stay still?

7. Describe the outcome of your parallax observations of the dime and the Moon. What can you conclude about the distance to the Moon, relative to the distance to the dime?