ASTRONOMY 698 Homework 6: Due in class Thursday 12/2/04

1. The expression for Ω_g is

$$\Omega_g = \frac{H_o}{c} \frac{\mu m_H}{\rho_c} \langle N \rangle \mathcal{N}(X), = A \langle N \rangle \mathcal{N}(X).$$

Using, $\rho_c = 3H_o^2/8\pi G$, and for $H_o = 100$ km s^{-1} Mpc^{-1} $= 3.45 \times 10^{-18}$ s^{-1}, we have

$$A = \frac{8\pi\mu}{3} \frac{m_H G}{H_o c} = \frac{8 \cdot \pi \cdot 1.33}{3} \frac{(6.67 \times 10^{-8}) \cdot (1.67 \times 10^{-24})}{(3.45 \times 10^{-18}) \cdot (3 \times 10^{10})} = 1.2 \times 10^{-23}.$$

So, what we need are the expressions for $\langle N \rangle \mathcal{N}(X)$ for (a) the column density distribution method,

$$\langle N \rangle \mathcal{N}(X) = \frac{C}{2-\beta} \left(N_{max}^{2-\beta} - N_{min}^{2-\beta} \right),$$

and (b) the mean column density method,

$$\langle N \rangle \mathcal{N}(X) = \frac{1}{\Delta X} \sum_{i=1}^{m} N_i.$$

For the latter we also need ΔX, which is found (see below) using

$$X(z) = \frac{2}{3}(1+z)^{1.5} - 1$$

for an Einstein–de Sitter cosmology.

(a) The relevant quantities we need are $N_{max} = 5.3 \times 10^{19}$ cm^{-2} and $N_{min} = 4.0 \times 10^{12}$ cm^{-2}. Using $C = 1.4 \times 10^{10}$ and $\beta = 1.5$, we have

$$\langle N \rangle \mathcal{N}(X) = 2 \cdot 1.4 \times 10^{10} \cdot (5.3 \times 10^{19})^{0.5} = 2 \times 10^{20},$$

which gives

$$\Omega_g = A \langle N \rangle \mathcal{N}(X) = 1.2 \times 10^{-23} \cdot 2 \times 10^{20} = 0.0024$$

The uncertainty is given by

$$\sigma_{\Omega_g} = \Omega_g \left[\left(\frac{\sigma_C}{C} \right)^2 + \sigma_\beta^2 \left(R + \frac{1}{(2-\beta)} \right)^2 \right]^{1/2},$$

where

$$R = \frac{N_{max}^{2-\beta} \ln(N_{max}) - N_{min}^{2-\beta} \ln(N_{min})}{N_{max}^{2-\beta} - N_{min}^{2-\beta}} \simeq \ln(N_{max}) = 45.4,$$

and where we have uncertainties $\sigma_C/C = 0.1$ and $\sigma_\beta = 0.05$. We find

$$\sigma_{\Omega_g} = 0.0024 \left\{ (0.1)^2 + (0.05)^2 [45.4 + 2]^2 \right\}^{0.5} = 0.0024 \cdot 2.37 = 0.0057$$

Thus, we have

$$\Omega_g = 0.0024 \pm 0.0057$$

(b) The relevant quantities we need are

$$\sum_{i=1}^{m} N_i = 7.7 \times 10^{19} \quad \text{cm}^{-2},$$

where $m = 1998$, and ΔX for the redshift range $2.6 \leq z \leq 2.8$, which is computed by

$$\Delta X = X(2.8) - X(2.6) = 4.27 - 3.84 = 0.38$$

for an Einstein–de Sitter cosmology. Thus, we have

$$\Omega_g = A \langle N \rangle \mathcal{N}(X) = 1.2 \times 10^{-23} \cdot \frac{7.7 \times 10^{19}}{0.38} = 0.0024.$$

The uncertainty is given by

$$\sigma_{\Omega_g} = A \sigma_{\langle N \rangle \mathcal{N}(X)},$$

where

$$\sigma_{\langle N \rangle \mathcal{N}(X)} = \frac{1}{\Delta X} \left[\frac{m}{m-1} \sum_{i=1}^{m} (N_i - \langle N \rangle)^2 \right]^{1/2},$$

and where

$$\langle N \rangle = \frac{1}{m} \sum_{i=1}^{m} N_i = 3.9 \times 10^{16} \text{ cm}^{-2}.$$

We find

$$\sigma_{\langle N \rangle \mathcal{N}(X)} = \frac{1}{0.38} \left[\frac{1998}{(1998-1)} \cdot 2.9 \times 10^{39} \right]^{0.5} = 1.4 \times 10^{20}.$$

We have

$$\sigma_{\Omega_g} = A \sigma_{\langle N \rangle \mathcal{N}(X)} = 1.2 \times 10^{-23} \cdot 1.4 \times 10^{20} = 0.0017,$$

giving

$$\Omega_g = 0.0024 \pm 0.0017,$$

which really confirms that something is wrong with the uncertainty from method (a)!

The lesson to learn here, is that the value of N_{max} dominates the results for the computation of Ω_g. Comparing to $\Omega_b = 0.02$, we see that $\Omega_g/\Omega_b = 0.1$, or 10% of the total baryon content. In truth, the Ly α forest is more like 90% of the baryon content at $\langle z \rangle = 2.7$. The key is that high column density lines are very rare, and the method is very sensitive to the highest column density in a survey. Many lines of sight are needed to sample the high column density regime, otherwise Ω_g will be an underestimate.

2. (a) The Schaye formula is

$$N(\text{H}\,\text{I}) = 2.7 \times 10^{13}\,(1+\delta)^{1.5}\,T_4^{-0.26}\,\Gamma_{12}^{-1}\left(\frac{1+z}{4}\right)^{4.5},$$

where $(\Omega_b h^2/0.02)^{1.5}(f_g/0.16)^{0.5} = 1$ for our assumptions. (i) A plot of this formula is given in Fig. 1 for $\delta = 1$, 10, 100, and 1000 for $0 \leq z \leq 4$ as the solid curves. (ii) The *approximate* ranges of $\log N(\text{H}\,\text{I})$ for $50 \leq \delta \leq 100$ for $z = 4$, 3, 2, 1, and 0 are 16.3–16.8, 15.9–16.4, 15.4–15.9, 14.6–15.1, 13.3–13.8 cm^{-2}, respectively. (iii) The important qualitative trend is that the neutral hydrogen column density probed for a given overdensity increases with increasing redshift.

(b) Using the formula

$$\Gamma(z) = 6.7 \times 10^{-13}\,(1+z)^{0.73}\,\exp\left[-\frac{(z-2.3)^2}{1.9}\right],$$

which can be rewritten

$$\Gamma_{12} = 1.816\left(\frac{1+z}{4}\right)^{0.73}\exp\left[-\frac{(z-2.3)^2}{1.9}\right].$$

Subsituting Γ_{12}^{-1}, we have

$$N(\text{H}\,\text{I}) = 1.31 \times 10^{13}\,(1+\delta)^{1.5}\,T_4^{-0.26}\left(\frac{1+z}{4}\right)^{3.77}\exp\left[\frac{(z-2.3)^2}{1.9}\right],$$

A plot of this formula is given in Fig. 1 for $\delta = 1$, 10, 100, and 1000 as the dash curves. (ii) The *approximate* ranges of $\log N(\text{H}\,\text{I})$ for $50 \leq \delta \leq 100$ for $z = 4$, 3, 2, 1, and 0 are 16.6–17.2, 15.7–16.3, 15.1–15.7, 14.7–15.3, 14.4–15.0 cm^{-2}, respectively. (iii) The important qualitative trend is that the neutral hydrogen column density probed for a given overdensity is flatter with increasing redshift. The clouds are more neutral at lower redshifts. This would result in evolution in surveys with a fixed sensitivity. There is also a slight increase for $z = 4$, and it is apparent that this increases at even higher redshift as the QSO luminosity function drops in this regime.

It is apparent that the declining photoionization rate at low redshift makes a significant difference in the column density associated with a given overdensity at $z < 1.5$. This also appears to begin to make an effect at $z > 4$. Note that the break in the evolution in the N/dz is at $z = 1.7$, which is where the curve flattens.

See Fig. 1 on the following page...

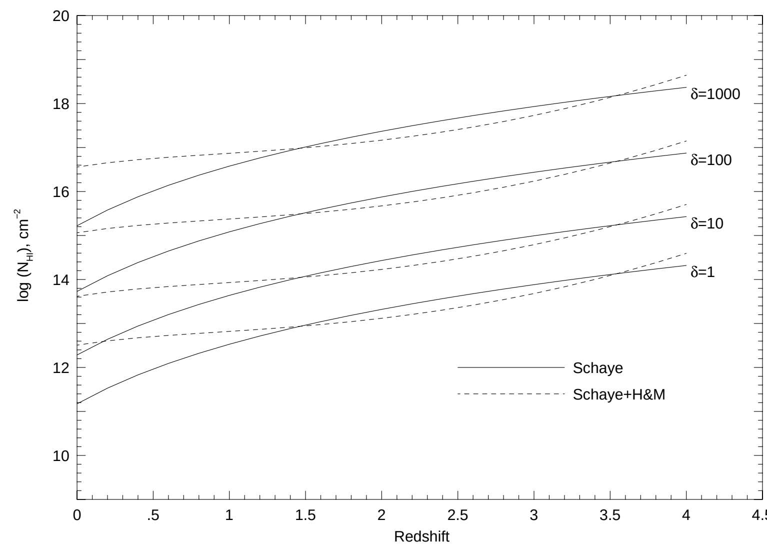

Figure 1. — The Schaye and Schaye+H&M formulae.