where λ_{air} is the air wavelength for an environment of 15° C at atmospheric pressure. Further discussion on air to vacuum conversions for astronomical spectroscopic applications have been reviewed in Murphy et al. (2001).

7.4.2 Heliocentric correction

The heliocentric correction is quite easily made from the formula

$$\lambda_h = \left(1 + \frac{v_h}{c}\right)\lambda_{vac}, \tag{7.26}$$

where v_h is the telescope line of sight velocity in the direction of the source in the heliocentric reference frame (accounting for Earth's orbital velocity and rotation velocity at the time the source spectrum is obtained). The calculation requires the altitude, longitude, and latitude of the observatory, the observation date and time, the sky coordinates of the source, and a model of the ephemerae of the solar system (especially the Earth and moon). The computation of v_h is quite involved and beyond the scope of this discussion. For further details, see Stumpff (1977, 1980).

7.5 Flux calibration

Flux calibration is the process of determining the observed continuum flux, F^0_λ from the measured continuum counts, I^c_λ, which are related via Eq. 6.70,

$$I^c_\lambda = \epsilon^{\rm QE}_\lambda B_\lambda \, \epsilon^{\rm O}_\lambda \, \epsilon^{\rm G}_\lambda \, \epsilon^{\rm C}_\lambda \, \epsilon^{\rm T}_\lambda \, \epsilon^{\rm A}_\lambda \, F^0_\lambda \, \frac{\lambda}{hc} \left(A_{\rm T}\Delta t\right), \tag{7.27}$$

where the various attenuation efficiencies are summarized in § 6.13. Writing (see Eq. 6.71)

$$\epsilon(\lambda) = \epsilon^{\rm QE}_\lambda \, \epsilon^{\rm O}_\lambda \, \epsilon^{\rm G}_\lambda \, \epsilon^{\rm C}_\lambda \, \epsilon^{\rm S}_\lambda \, \epsilon^{\rm T}_\lambda \,, \tag{7.28}$$

which we call the telescope throughput, and writing

$$S_\lambda = B_\lambda \, \epsilon(\lambda) \, \epsilon^{\rm A}_\lambda(z) \frac{\lambda}{hc} \left(A_{\rm T}\Delta t\right), \tag{7.29}$$

we have a shortend notation of Eq. 7.27,

$$I^c_\lambda = S_\lambda F^0_\lambda, \tag{7.30}$$

where S_λ [counts Å cm^2 sec erg^{-1}] is known as the sensitivity function. Note that the azimuth angle, z, has been explicitly included in the atmospheric attenuation efficiency appearing in Eq. 7.29 in order to emphasize that the sensitivity function is airmass dependent.

As can be seen from the terms appearing in Eq. 7.29, the sensitivity function depends upon several particulars of the observatory and its location, including the telescope throughput, airmass dependent atmospheric attenuation, and collecting area of the telescope. The spectrograph design is also important, in that

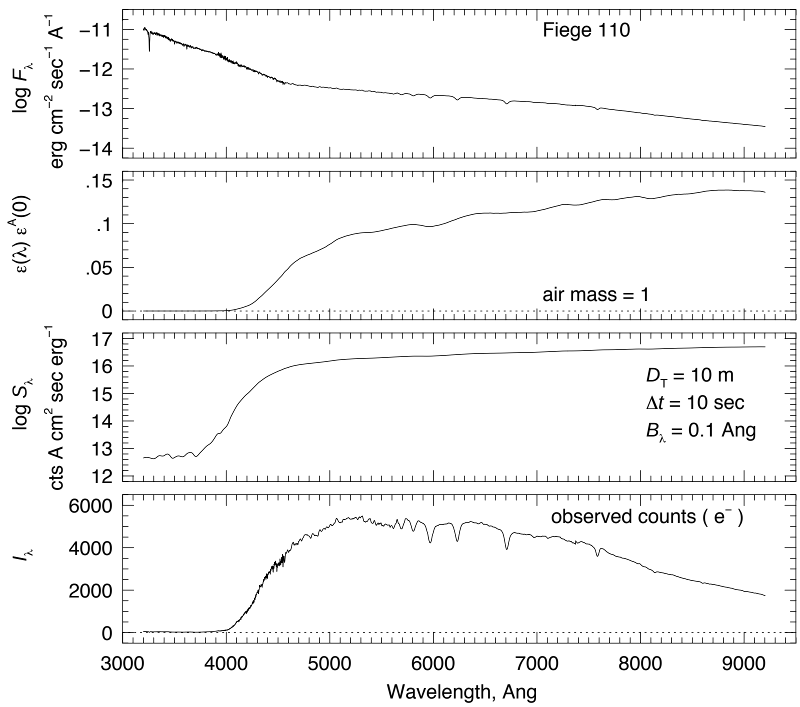

Figure 7.4: (a) The oberved flux, F_λ, of the star Feige 110. (b) The total throughput, $\epsilon(\lambda)\,\epsilon^{\rm A}_\lambda$, based upon the hypothetical telescope and optical spectrograph throughput shown in Figure 6.12a and the atmospheric throughput curve presented in Figure 6.12b for unit aimass. (c) The sensitivty curve, S_λ, for a $D_{\rm T} = 10$ [meter] diameter telescope for a $\Delta t = 10$ [sec] exposure for $B_\lambda = 0.1$ Å pixels. (d). The observed counts, I_λ per pixel.

the wavelength interval per pixel, $B_\lambda = \Delta\lambda_{\rm pix}$, is a factor. From the stand point of the observation itself, the exposure time is an additional factor.

In Figure 7.4, the relationship between the observed flux, total throughput, sensitivity function, and observed counts is illustrated for a hypothetical telescope/spectrograph facility. For this illustration, we have assumed a telescope diameter of $D_{\rm T} = 10$ [meter], an exposure time of $\Delta t = 10$ [sec], and a spectrograph with a pixel sampling rate of $B_\lambda = 0.1$ [Å pixel^{-1}]. The observed flux of the star Feige 110 (Oke, 1990)[3] is presented in Figure 7.4a. The total throughput, plotted in Figure 7.4b, is the product of the telescope throughput and atmospheric attenuation (for unit airmass) shown in Figures 6.12a and 6.12b, respectively.

The sensitivity function, S_λ, for the assumed observational particulars, is given in Figure 7.4c. The total observed counts per pixel, given by Eq. 7.30, is shown in in Figure 7.4d. Note how the drastic reduction in throughput below

[3]The electronic data were obtained from the on–line archive of optical and UV spectrophotmetric flux standard stars made available by the European Southern Observatory. (*http://www.eso.org/sci/observing/tools/standards/spectra/*)

4000 Å for this example results in negligible counts even though in this wavelength regime the observed flux is 1.5–2 orders of magnitude higher than where the total throughput is at its highest, roughly 10%.

Note that S_λ scales linearly with $B_\lambda\, A_{\rm T}\Delta t$, so that the counts per pixel are also in direct proportion (an order magnitude change in any one of these quanties results in a 1 dex shift of the function illustrated in Figure 7.4*c*). Often, the sensitivity function is expressed in magnitude units, $m(S_\lambda) = -2.5\log S_\lambda$, which is on the order of -32 to -41 for our example function. If the source flux density is coverted to magnitude units, $m(F^0_\lambda)$, then the counts in magnitude units is simpy $m(I^c_\lambda) = m(S_\lambda) + m(F^0_\lambda)$, from which it follows,

$$I^c_\lambda = 10^{-0.4m(I^c_\lambda)} = 10^{-0.4\left[m(S_\lambda)+m(F^0_\lambda)\right]} \,. \qquad (7.31)$$

Note that the above magnitudes are of the flux density. Fror AB mangitudes, the frequency fux density is utilized, $m(\mathrm{AB}) = -2.5\log(F_\nu) - 48.59$ (see Eq. 5.40). Note that the sensitivity function would also need to be converted to or measured in frequency units, i.e., S_ν, before being converted to AB magnitudes.

In practice, flux calibrating a source is a set of procedures undertaken in order to accurately determine the sensitivity function for the particular telescope/spectrograph facility and observational conditions, following which Eq. 7.30 or Eq. 7.31 is applied to compute the observed continuum flux. The sensitivity function is determined by observing so–called flux standard objects, usually bright stars. Flux standard stars are observed under ideal conditions for telescope/spectragraphs for which the sensitivity functions are well calibrated. Public lists of these stars and their spectral flux densities are often made available by the observatory (there are countless papers of published flux standard stars, but a few good resources include Massey *et al.*, 1988; Oke, 1990; Turnshek *et al.*, 1990; Bohlin *et al.*, 2001, and references therein).

Flux calibration require photometric conditions, i.e., no added atmospheric optical depth due to clouds, dust, etc., under excellent seeing conditions. The steps for flux calibration consist of: (1) observing flux standards stars at similar airmass to the program objects (the idea is to have $\epsilon^{\rm A}_\lambda(z)$ be as identical as possible for the standard stars and program objects; (2) reducing the standard stars and program objects in identical fashion; (3) taking the ratio of observed counts to the published observed flux of the standard stars and computing the sensitivity function (often this result is heavily smoothed); and (4) applying Eq. 7.30 to the program objects.

Since the quantities $B_\lambda A_{\rm T}$ are fixed for the observations of both standard stars and program objects, and because the exposure time, Δt, can be properly scaled for the program objects, it is only in the cases where the total throughput, $\epsilon(\lambda)\epsilon^{\rm A}_\lambda(z)$, is identical for both the standard stars and program objects that the flux calibration can be accurate. One of the major concern is that light loss (the factor $\epsilon^{\rm S}_\lambda$ appearing in $\epsilon(\lambda)$), can be variable from observation to observation if the telescope pointing and tracking is not ideal.

If the science objectives are simply to measure the properties of absorption lines, then flux calibration is not required because absorption is quantified relative to the continuum; the flux calibration divides out. However, if properties of the source spectrum itself are part of the science goals, then flux calibration may be necessary.

Flux calibration with echelle spectra is more challenging than calibration of lower dispersion spectra. Due to the echelle format and the free spectral range of the different orders of the grating, there can be duplicate coverage of the bluer wavelength regions. Since these dupilications are from different orders off the grating, they have separate blaze functions (Eq. 6.14). For a given duplicated wavelength on orders n and $n+1$, the value efficiencies, $\epsilon^{\mathrm{G}}_{\lambda}$, may differ by a factor of a few so that flux calibration may yield unmatching flux values without careful treatment (e.g. Suzuki *et al.*, 2003)

7.6 Continuum fitting

Virtually all measurements of absorption features will involve the ratio $I_{\lambda}/I^{\mathrm{c}}_{\lambda}$. Thus, for the analysis of absorption lines, the continuum level, I^{c}_{λ}, must be estimated for wavelength regions spanning absorption features.

The estimation of the continumm is functional model, usually a smooth curve fit through the data employing an objective statistical treatment (such as least squares fitting, see § 3.5). Often, the smooth curve is a polynomial or a series of splined polynomials and is known as the "continuum fit" or the "fitted continum". We will denote the fitted continuum values by $\bar{I}^{\mathrm{c}}_{\lambda}$.

It is important to realize that a continuum fit (which may have $\bar{I}^{\mathrm{c}}_{\lambda}$ either slightly systematically larger or smaller than the actual I^{c}_{λ}) introduces systematic errors in the measured quantities. Uncertainty in $\bar{I}^{\mathrm{c}}_{\lambda}$, denoted $\sigma_{\bar{I}^{\mathrm{c}}_{\lambda}}$, should be properly accounted for the uncertainty estimates of quantified absorption properties. Whereas the uncertainty in the counts, $\sigma_{I_{\lambda}}$, are independent quantities for each wavelength, the uncertainties in the fitted continumm must acount for that fact that they depend upon the I_{λ} over a *range* of wavelengths. Thus, the uncertanties in $\bar{I}^{\mathrm{c}}_{\lambda}$ are dependent upon the I_{λ} and $\sigma_{I_{\lambda}}$ used to model the continuum.

7.6.1 Using orthogonal functions

Sembach & Savage (1992) showed that using normalized orthogonal (orthonormal) functions for continuum modeling provide an ideal approach to minimizing the $\sigma_{\bar{I}_{\lambda}}$. Here, we review their approach to continuum fitting, which is most appropriate for wavelength regions of the spectrum where the counts are not varying rapidly or for small regions where scientific analysis is to be performed.

In general, orthonormal functions are defined by the result of their inner product

$$\int_a^b P_j(x)P_k(x)\,dx = \begin{cases} 0 & : \quad j \neq k \\ 1 & : \quad j = k. \end{cases} \tag{7.32}$$

where a and b are the lower and upper limits of the interval over which orthogonality holds, and j and k are the orders of the functions. For purposes