ASTR 605 NEW Homework 3

- 1. We have seen in class that the line absorption coefficient is fully expressed as $\kappa_{\lambda}(s) = \sum n_{ijk}(s)\alpha_{ijk}(\lambda)$, where $\alpha_{ijk}(\lambda)$ is the absorption cross section. The line emission coefficient can similarly be expressed as $\eta_{\lambda}(s) = \sum n_{ijk}(s)j_{ijk}(\lambda, T)$, where $j_{ijk}(\lambda, T)$ is the emission coefficient per absorber.
 - (a) What are the units of $j_{ijk}(\lambda, T)$.

(b) Assuming an isothermal cloud, i.e., $T \neq T(s)$, and neglecting scattering, use the definition of the source function to show that the line source function of a single emission feature from a single absorber type is independent of location, s, in the gas, i.e., can be expressed $S_{\lambda} = j(\lambda, T)/\alpha(\lambda)$. Also, show that the units work out.

2. Consider the OII] λ 3727 transition (actually a doublet, but we will ignore that) for which

$$\alpha_l(\lambda) = \alpha_l^0 \exp\left\{-\left(\frac{(\lambda - \lambda_r)}{\Delta \lambda_{\rm D}}\right)^2\right\},\,$$

where $\alpha_l^0 = 3 \times 10^{-10}$, and

$$j_l(\lambda, T) = j_l^0(T) \exp\left\{-\left(\frac{(\lambda - \lambda_r)}{\Delta \lambda_{\rm D}}\right)^2\right\},$$

where $j_l^0(T) = 1 \times 10^{-8} (T/30,000)$. [We are ignoring the natural atomic broadening and only considering thermal broadening, thus α_l^0 and $j_l^0(T)$ behave as δ functions (to avoid convolution)].

(a) If λ is expressed in units of angströms, what is the value of $\Delta \lambda_{\rm D}$ for T = 10,000 K? For T = 30,000 K?

(b) On a single figure, plot $\alpha_l(\lambda)$ for T = 10,000 and T = 30,000 K and label the curves. Be sure to include units on your axes labels. Use a linear-linear plot.

(c) On a single figure, plot $j_l(\lambda, T)$ for T = 10,000 and T = 30,000 K and label the curves. Be sure to include units on your axes labels. Use a linear-linear plot.

3. You are viewing a plane parallel gas cloud. It is isothermal and has constant density with n = 10 atoms cm⁻³. The cloud is perpendicular to your line of sight (direction of physical depth). It is illuminated on its front side (s = 0, cloud face) by a hot star. Let's simplify and work with the normalized specific intensity [later, should we desire, we could multiply through by the stellar SED to recover the shape of the spectrum].

Assume the continuum absorption coefficient is $\sigma_c(\lambda) = 7 \times 10^{-11} \text{ cm}^2$ and that the cloud has a total physical depth of $L = 10^{9.5} \text{ cm}$.

(a) Write down the expression for the total column density through the cloud from the point of view of the observer. What is the column density as seen by the observer?

(b) Write down the expression for the total optical depth through the cloud from the point of view of the observer. What is the continuum optical depth as seen by the observer?

(c) What fraction of the incident specific intensity is observed?

(d) Write down the expression to determine at what physical depth, s, the optical depth achieves unity. Solve for and compute the physical depth, s at which the optical depth is unity. What percentage into the cloud does this correspond to?

(e) Write down the solution to the equation of transfer for this observing configuration (symbolically). Compute and plot the continuum specific intensity as a function of depth through the cloud from s = 0 to $L = 10^{9.5}$ cm. I suggest computing as a function of log s in steps of $\Delta \log s = 0.2$ or so (see my fortran program). Make your plot in linear–linear space, providing all labels and label where $\tau = 1$.

4. Now you are viewing a different part of the above structure– off to the side of the star (no incident continuum specific intensity on the cloud face along the line of sight). Here the density is $n_1 = 1000 \text{ cm}^{-3}$. In the continuum, the cloud has a constant source function throughout, which we will again normalize to $S_{\lambda}^c \equiv \eta_{\lambda}^c/\kappa_{\lambda}^c = 1$ for simplicity (perhaps it is the Planck function in reality?).

(a) Show that the solution to the radiative transfer equation can be written (symbolically)

$$I_{\lambda}^{c}(s) = \frac{\eta_{\lambda}^{c}}{\kappa_{\lambda}^{c}} (1 - \exp\{-n_{1}s\sigma_{c}(\lambda)\})$$

(b) Using $\sigma_c(\lambda) = 7 \times 10^{-11} \text{ cm}^2$, plot the specific intensity as a function of cloud depth from s = 0 cm to $L = 10^{9.5}$.

(c) Assuming the observer "sees" (i.e., the photons that reach the observer) about 1 mean free path into the cloud. Over what range of physical depth are the *observed* photons originating from within the cloud? [Give both the physical depth Δs seen into the cloud, and give the percentage into the cloud to which this corresponds]. Show how you computed Δs .

5. Assume the same cloud as Problem 4 with $T_1 = 30,000$ K, $n_1 = 1000$ cm⁻³, $S_{\lambda}^c \equiv \eta_{\lambda}^c / \kappa_{\lambda}^c = 1$, and $\sigma_c(\lambda) = 7 \times 10^{-11}$ cm² in the continuum. Now introduce the line source function from the OII] $\lambda 3727$ transition from Problem 2.

(a) Given that the total source function is the ratio of the *sum* of the emission coefficients to the *sum* of the absorption coefficients, show that the solution to the radiative transfer equation can be written (symbolically)

$$I_{\lambda}(s) = \left[\frac{(\eta_{\lambda}^{c}/\kappa_{\lambda}^{c})\sigma_{c}(\lambda) + j_{l}^{0}(\lambda, T_{1})}{\sigma_{c}(\lambda) + \alpha_{l}(\lambda)}\right] (1 - \exp\left\{-n_{1}s\left[\sigma_{c}(\lambda) + \alpha_{l}(\lambda)\right]\right\}).$$

(b) Do you expect the OII] feature to be an emission line or an absorption line? Explain.

(c) At specific physical depths into the cloud, compute and plot the observed transmitted spectrum centered on the absorption line (as a function of wavelength over a range of wavelength local to the the line center). Make your plot a single figure, and on this figure plot the observed transmitted spectrum for the cloud depths $6 \leq \log s \leq 9.5$ in steps of 0.5 dex. Label the curves by physical depth.

(d) Describe the behavior of the spectrum with increasing cloud depth. What is the behavior of the continuum? What is the behavior of the OII] line? For what cloud depths is the line strength proportional to column density? At what depth does the line show signs of saturating?

6. Now, consider that this cloud has an outer region that extends to $L = 10^{9.8}$ cm. Assume the extended portion of the cloud, from $s_2 = 10^{9.5}$ to $L = 10^{9.8}$ cm, is cooler with $T_2 = 10,000$ K, and less dense with $n_2 = 10$ cm⁻³. Also assume that, because of the cooler temperature in this s range, the continuum source function is $S_{\lambda}^{c'} \equiv \eta_{\lambda}^{c'}/\kappa_{\lambda}^{c'} = 0.3$. Assume that $\sigma_{c'}(\lambda) = \sigma_c(\lambda)$ and that $\alpha_l(\lambda)$ is the same as the inner cloud region.

(a) Show that the solution to the radiative transfer equation in the range $s > 10^{9.5}$ cm can be written (symbolically)

$$I_{\lambda}(s) = I_1 I_2 + I_3$$

where

$$I_{1} = \left[\frac{(\eta_{\lambda}^{c}/\kappa_{\lambda}^{c})\sigma_{c}(\lambda) + j_{l}^{0}(\lambda, T_{1})}{\sigma_{c}(\lambda) + \alpha_{l}(\lambda)}\right] \left(1 - \exp\left\{-n_{1}s_{2}\left[\sigma_{c}(\lambda) + \alpha_{l}(\lambda)\right]\right\}\right),$$

and

$$I_2 = \exp\left\{-n_2(s-s_2)\left[\sigma_c(\lambda) + \alpha_l(\lambda)\right]\right\},\,$$

and

$$I_{3} = \left[\frac{(\eta_{\lambda}^{c'}/\kappa_{\lambda}^{c'})\sigma_{c}(\lambda) + j_{l}^{0}(\lambda, T_{2})}{\sigma_{c}(\lambda) + \alpha_{l}(\lambda)}\right] (1 - \exp\left\{-n_{2}(s - s_{2})\left[\sigma_{c}(\lambda) + \alpha_{l}(\lambda)\right]\right\}).$$

Describe what each term, I_1 , I_2 , and I_3 , accounts for physically.

(b) At specific physical depths into the cloud, compute and plot the observed transmitted spectrum centered on the OII line (as a function of

wavelength over a range of wavelength local to the the line center). Make your plot a single figure, and on this figure plot the observed transmitted spectrum for the cloud depths $9.5 \le \log s \le 9.8$ in steps of 0.0334 dex (1/30). Label the curves by physical depth.

(c) Describe the behavior of the spectrum with increasing cloud depth. What is the behavior of the continuum? What is the behavior of the OII] line? Why does the transmitted specific intensity of the emission peaks decrease with physical depth? What about the strength of the peaks?

(d) The structure of the line changes from single peaked to double peaked and back to single peaked as physical depth increases. Explain why this is happening physically in terms of the opacity and source function of the radiative transfer. Is the line core of the observed emission line saturated? How do you know?