ASTR 545
Stellar Atmopshere Model
DUE: Friday, December 1, 2006; by 3:00pm

Compute a 1–D, plane–parallel, LTE, Grey stellar atmosphere model over the optical depth range $10^{-3} \leq \tau \leq 10^{2.5}$ for a star with surface gravity, g, and effective temperature, $T_{\rm eff}$. Assume mass fractions $X = 0.7$, $Y = 0.28$, and $Z = 0.02$ and use the opacities from Table 8 of Cox & Tabor (1976, ApJS, 31, 271). You must include at least one metal, but are encouraged to incorporate a few. Assume the atmosphere thickness is negligible compared to R.

Present your results in a formal paper using Latex AAS macros for submitting to ApJ and AJ. Properly encapsulate your figures and provide descriptive captions. Your write up should be structured to include (1) a description of the model including equations used, physical assumptions, and methods of convergence, including interpolation schemes with references to data tables used; (2) a results section that discusses the *physical behavior* of the model as a function of depth in the atmosphere. If applicable, discuss how certain behavior departs from the "real world" based upon applied assumptions. Focus your discussion around the following figures (curves labeled, not hand written). Present your well documented code and data tables in an Appendix.

Fig 1a. Physical Depth x versus optical depth, τ

Fig 1b. Temperature, T, versus τ

Fig 1c. Opacity, χ, versus τ

Fig 1d. Density, ρ, versus τ

Fig 2. P_g, P_e, and P_e/P_g versus optical depth, τ (requires two scales)

Fig 3. N_{jk}/N_k versus τ for all species

Fig 4. Electron density, n_e, contributed from each N_{jk} versus τ

Fig 5. $n_{0,0}/N_0$, $n_{1,0}/N_0$, and $n_{3,0}/N_0$ versus τ for hydrogen

Fig 6. Flux, $F_\lambda(\tau)/F$ versus λ for several representative τ layers

Below are some useful relationships...

PARTICLE/MASS CONSERVATION

$$N_{\rm tot} = N_n + n_e$$

$$N_n = \sum_k N_k$$

$$N_k = \alpha_k N_n = \alpha_k (N_{\rm tot} - n_e) = \sum_{j=0}^{J_k} N_{jk}$$

$$\rho_{\rm tot} = \rho_n + \rho_e$$

$$\rho_n = m_{\rm H} \sum_k A_k N_k = m_{\rm H} N_n \sum_k \alpha_k A_k$$

$$\rho_e = m_e n_e$$

PRESSURE BALANCE

$$P_g = P_n + P_e$$

$$P_n = N_n kT$$

$$P_e = n_e kT$$

$$\frac{dP_g}{dx} = -g\rho_{\rm tot}$$

$$g = \frac{GM}{R^2}$$

CHARGE CONSERVATION

$$n_e = \sum_k \sum_{j=1}^{J_k} j N_{jk} = \sum_k N_k \sum_{j=1}^{J_k} j f_{jk}(n_e, T)$$

$$f_{jk}(n_e, T) = \frac{N_{jk}}{N_k}$$

$$n_e = (N_{\rm tot} - n_e) \sum_k \alpha_k \sum_{j=1}^{J_k} j f_{jk}(n_e, T),$$

DETAILED BALANCING

$$\frac{n_{ijk}}{N_{jk}} = \frac{g_{ijk}}{U_j(T)} \exp\left\{-\frac{\chi_{ijk}}{kT}\right\}$$

$$\frac{N_{jk}}{N_{j+1,k}} = n_e \frac{U_j(T)}{U_{j+1}(T)} C_I T^{-3/2} \exp\left\{\frac{\chi_{Ijk}}{kT}\right\}$$

$$f_{jk}(n_e, T) = \frac{N_{jk}}{N_k} \equiv \frac{P_{jk}}{S_k} = \frac{(N_{jk}/N_{0k})}{\sum_{j=0}^{J_k}(N_{jk}/N_{0k})}$$

$$P_{jk} = \prod_{l=1}^{j}\left(\frac{N_{lk}}{N_{l-1,k}}\right) \qquad S_k = 1 + \sum_{m=1}^{J_k} P_{mk}$$

OPTICAL DEPTH RELATIONS

$$d\tau = \chi(\rho, T)\rho_{\rm tot}dx$$

$\chi(\rho, T)$ = Rossland Mean Opacity, from table (hand out)

$$dP_g = -g\frac{d\tau}{\chi(\rho, T)}$$

$$\frac{T(\tau)}{T_{\rm eff}} = \left[\frac{3}{4}\{\tau + q(\tau)\}\right]^{1/4} \equiv \frac{1}{p(\tau)},$$

$q(\tau)$ = Hopf Function, from Table (hand out)

FLUX

$$\frac{F_\alpha(\tau)}{F} = \frac{4\pi k^4}{h^3c^2\sigma}\alpha^3\beta(\tau)$$

$$\alpha = \frac{h\nu}{kT_{\rm eff}} = \frac{hc}{\lambda kT_{\rm eff}}$$

$$F = \sigma T_{\rm eff}^4$$

$$\beta(\tau) = \int_\tau^\infty \frac{E_2(t-\tau)dt}{\exp\{\alpha p(t)\} - 1} - \int_0^\tau \frac{E_2(\tau - t)dt}{\exp\{\alpha p(t)\} - 1}$$

$$E_2(y) = \int_1^\infty \frac{\exp\{-wy\}}{w^2}dw$$

ALGORITHMIC STEPS

1. Start at the top layer by settng the index $i = 1$ and assuming $\tau_1 = 1 \times 10^{-3}$.
2. Make a guess at the pressure, $P(\tau_i)$
3. Compute $T(\tau_i)$.
4. Compute $N_{\rm tot}$ from $P(\tau_i)$.
5. Compute n_e, and $f_{jk}(n_e, T)$ from charge conservation.
6. Compute $\rho_{\rm tot}$ from particle conservation.
7. Obtain the opacity $\chi(\rho, T)$.
8. Compute the hydrostatic pressure, P_g, for τ_i and $\chi(\rho, T)$.
9. Check if $P(\tau_i)$ agrees with P_g ...

    If YES, compute $F_\lambda(\tau_i)/F$, step to τ_{i+1}, and repeat from Step 2.

    If NO, set $P(\tau_i) = P_g$ and repeat from Step 4.