ASTR 545
Final Exam Take Home Part
DUE: December 9, 2010
(NO COLLABORATIONS)

You have observed the Hα line for four stars. The data are given in the files "star1.data", "star2.data", "star3.data" and "star4.data". The profiles are plotted in the attached Figure 1. The columns in the data files that you will need are: (col 1, λ_i) wavelength [angstroms], (col 3, I_i) relative flux, (col 4, σ_i) uncertainty in relative flux.

Using χ^2 minimization, you are to determine the total Doppler b parameter and the column density, N, for the Hα absorption profile for each star. To do this, you will need to generate Hα absorption profiles for a range of b and N, searching for the pair that provides the best reduced χ^2_ν (near unity), where the reduced χ^2 is

$$\chi^2_\nu = \frac{1}{\nu} \sum_{i=1}^{M} \left(\frac{I_i - f_i}{\sigma_i} \right)^2$$

where $\nu = M - n$ is the degrees of freedom, M is the number of pixels and $n = 2$ is the number of free parameters to be estimated (N and b), and where I_i is the measured relative flux in pixel i, σ_i is the uncertainty in I_i, and f_i is the absorption profile model at pixel i

$$f_i = \exp\left\{-N\alpha(\lambda_i)\right\},$$

where $\alpha(\lambda_i)$ is

$$\alpha(\lambda_i) = \frac{\pi e^2}{m_e c^2} \lambda_r^2 f \cdot U(x_i, y),$$

where $U(x_i, y)$ is the Voigt function with

$$x_i = \frac{\lambda_i - \lambda_r}{\Delta\lambda_{\rm D}} \qquad y = \frac{1}{\Delta\lambda_{\rm D}} \frac{\Gamma \lambda_r^2}{4\pi c},$$

where $\Delta\lambda_{\rm D}$ is the Doppler width, the atomic constants have their usual meaning, and λ_i is the wavelength of pixel i.

I am supplying you with the fortran subroutine "voigt.f" (which you can convert to your favorite language if you want). The call to this subroutine is as follows: the subroutine call and argument list is `voigt(`x_i`,`y`,`u`,`v`)`; at a given λ_i, the user passes x_i and y, the routine returns u and v, where u is the Voigt function $U(x_i, y)$, and v is a complex part to the complex probability function $w(z) = \exp(-z^2) \cdot \mathrm{erfc}(-iz) = u(x, y) + iv(x, y)$, where $z = x + iy$. You do not need v and may discard it.

Use a grid of b from $10 \leq b \leq 70$ km/s in steps of $\Delta b = 5$ km/s and a grid of N from $12.0 \leq \log N \leq 22.0$ in steps of $\Delta \log N = 0.1$ and compute the χ^2_ν for each N, b pair. For each b, plot the χ^2_ν curve as a function of $\log N$. Identify the N, b pair that provides the minimum χ^2_ν.

QUESTIONS (35 points total):

1. (5 points) For what absorber (element, ionization stage, excitation level) does this column density correspond to?

2. (5 points) For each star, present your χ^2_ν curves on a single plot and provide useful labels for the curves. Please create a four panel plot on a single page with one panel for each star.

3. (10 points) For each star, report your best estimate fitted parameters N and b for the observed profile. For each of these best fitted parameters report your value of χ^2_ν.

4. (5 points) For each star, superimpose your best fit profile on the data and plot (please provide each star's profile in a separate panel on a single page, i.e., four panels).

5. (10 points) Submit your *well commented* code for my review.

EXTRA CREDIT (20 points total):

The important part of any fitting method is to determine the uncertainties in the best fitter parameters. In many cases, the uncertainties are not normally distributed, i.e., are not symmetric about the best fitted parameter. If **a** represents the vector of the best fitter parameters, and da_i is the uncertainty in a_i, then $da_i = 1\ \sigma$ uncertainty in a_i when the condition

$$\Delta\chi^2_\nu = \chi^2_\nu(\mathbf{a} + da_i^+) - \chi^2_\nu(\text{best}) = +1$$

or

$$\Delta\chi^2_\nu = \chi^2_\nu(\mathbf{a} - da_i^-) - \chi^2_\nu(\text{best}) = +1$$

is met, where da_i^+ is the upward uncertainty in a_i and da_i^- is the downward uncertainty in a_i.

1. (5 points) for each star, estimate the uncertainties in your best $\log N$, and report them as $\log N^{+\sigma(\log N^+)}_{-\sigma(\log N^-)}$. To determine the upper and lower uncertainties individually, explore $\Delta\chi^2_\nu$ centered on your best N and b by varying N to slightly larger values [to obtain $\sigma(\log N^+)$] and then to slightly smaller values [to obtain $\sigma(\log N^-)$]. Hold b constant. For each exploration, when $\Delta\chi^2_\nu = +1$, you have determined the uncertainties.

2. (5 points) for each star, estimate the uncertainties in your best b, and report them as $b^{+\sigma(b^+)}_{-\sigma(b^-)}$. Follow the method described above for the column densities, but this time slightly vary the Doppler b parameters.

3. (5 points) Superimpose you best fit profile and your $\pm 1\ \sigma$ profiles over the data (in four panels as before) and label them. I suggest you use color: green for the N ±uncertainties and red for the b ±uncertainties.

4. (5 points) For which stars did you find that one or both of the fitted parameters are not so well constrained? Which parameters for which star(s)? Based upon the behavior of absorption profile shapes, thoroughly explain your results. The curve of growth behavior can be helpful here, but recall that the curve of growth applies to the equivalent widths, not to the profile shapes.

5. Submit your well commented code for the full extra credit.