ASTR 545
Homework 3 (130 points)
DUE: October 4, 2012
see "http://astronomy.nmsu.edu/cwc/Teaching/ASTR545/HW3/"

Read the file "readme.txt" (in the on-line homework directory) for additional information required for solving these problems. REPORT ALL ANSWERS IN CGS UNITS.

Problem 1 (20 points) Consider a pure hydrogen gas structure in thermal equilibrium. For neutral hydrogen, compute the number density ratios, n_{i11}/n_1, for $i = 1$, 2, 3 as a function of temperature over the range $3000 \leq T \leq 20,000$ for three different electron pressures $P_e = 1$, 10, and 100 dynes cm^{-2}. Use $g_n = 2n^2$ for the statistical weight, and $\chi_n = R(1 - 1/n^2)$ for the excitation energy, where n is the principle quantum number of the excited level i, and $R = 13.598434$ eV is the Rydberg constant.

(a) Derive and write out the full expression for n_{i11}/n_1 in terms of n_e and $\Phi_{11}(T)$. Separately, write the full expression for $\Phi_{11}(T)$. Identify/define each term in your equations.

(b) Plot $\log(n_{i11}/n_1)$ versus T. Create a separate panel for each excitation level, i and plot the three electron pressures, P_e on each panel; use a unique line type or color for each pressure. Limit your "y" axis to five decades in the log.

(c) For each electron pressure, P_e, at which temperature would you find that the Lyman lines are strongest? The Balmer lines are strongest? The Paschen lines are strongest?

(d) Consider the behavior of the "Balmer hydrogens" ($i = 2$). Explain the physical reasons why the temperature peaks in n_{211}/n_1 shift the way they do with increasing P_e. Include in your answer both the excitation behavior and the ionization behavior with increasing P_e and how the balance in these change with P_e and T such that the curves behave the way they do. [HINT: behavioral insight to the physics is provided directly by the formula, but then the next step is to describe that behavior in terms of the physical processes!]

Problem 2 (60 points) Consider a gas structure in thermal equilibrium with $T = 6000$ K and $P_{\rm gas} = 100$ dynes cm^{-2} comprising hydrogen, helium, and one metal, calcium, with mass fractions $X = 0.71$, $Y = 0.27$, and $Z = 0.02$. Assume all ionization states for H and He and that Ca can be ionized twice (Ca, Ca$^+$, Ca^{++}). Compute the equilibrium conditions of the gas using particle-charge conservation,

$$n_e = (n_{tot} - n_e) \sum_k \alpha_k \sum_{j=1}^{J_k} (j-1) f_{jk}(n_e, T) \tag{1}$$

In solving Eq. 1, apply a tolerance of at least $n_e^+ - n_e^- = 10^{-5}$, where n_e^+ and n_e^- are the values that bracket the root, n_e. I have provided the solutions ("Prob2-solutions.txt") to this problem. Please present your solutions in the idendical format.

(a) Table 1. For the given T and $P_{\rm gas}$, compute the quantities that constrain Eq. 1, the total density, n_{tot}, and the abundance fractions, α_k. Report these quantities.

(b) Table 2. Report the equilibrium values of the density of nuclear matter, $n_{\rm N}$, the electron density, n_e, the mean molecular weight of the nuclear matter, $\mu_{\rm N}$, the mean molecular weight of free electrons μ_e', the total mean molecular weight, μ_{tot}, the nuclear particle mass density, $\rho_{\rm N}$, the electron mass density, ρ_e, the total gas density, ρ_{tot}, the electron pressure, P_e, and the ratio $P_e/P_{\rm gas}$.

(c) Table 3. Report the number densities of each species, n_k, the number densities for each ionization stage of each species, n_{jk}, the ionization fractions $f_{jk}(n_e, T)$, the fractional contribution to n_e from each ionization species, $f(n_e) = (j-1)n_{jk}/n_e$, the Saha term $\log \Phi_{jk}(T)$, and the partition function $\log U_{jk}(T)$.

(d) [i] Identify which of these quantities in Tables 2 and 3 are independent of the ionization conditions of the gas? [ii] From Table 3, rank, and report which ionization stages of each species are dominanating. [iii] From Table 3, identify and rank which ionization stages of which species are the top three donors of free electrons to the gas. [iv] What do these data tell you about the role of metals in gas where hydrogen and helium are not highly ionized?

Problem 3 (50 points) Expand your code to compute the equilibrium conditions of this gas structure over the range $1500 \leq T \leq 25,000$ K.

(a) Produce fully labeled plots of the following versus T. Plot 1: $\log n_e$, $\log n_N$, and $\log n_{\rm tot}$; Plot 2: $\log \rho_e$, $\log \rho_N$, and $\log \rho_{\rm tot}$; Plot 3: $P_e/P_{\rm gas}$. Use different colors or line types when necessary. For Plots 1 and 2, limit your "y" axis to five decades in the log.

(b) Plot 4: f_{jk} versus T (linear-linear: the f_{jk} range between zero and unity).

(c) Plot 5: $\log n_k$, and $\log n_{jk}$ versus T. Make a separate panel for each element and use different colors or line types for each ionization stage. Limit your "y" axis to five decades in the log.

(d) Plot 6: $\log f(n_e)$ versus T, where $f(n_e) = (j-1)n_{jk}/n_e$, for all jk on a single panel. Make the "y" axis of your plot over the range $-6 \leq \log f(n_e) \leq +0.5$.

(e) [i] Examining Plot 1, explain why $n_{\rm tot}$ and $\rho_{\rm tot}$ decrease with increasing T, and why they have the slopes they do. [ii] From Plot 6, report which ionization species is the major electron donor and over which temperature range. [iii] Going back to your Plot 3, note that $P_e/P_{\rm gas}$ rises to 0.5, levels off, and then rises slightly above 0.5. Incorporating the data in Plot 6, explain physically why this is happening (why $\simeq 0.5$?) and explain the physics of why these values and changes happen at the temperatures they do.

YOUR CODES staple you codes to the end of your homework assignment. Clearly comment your codes.