

ASTR 545
Homework 6 (120 points)
Due: Thursday November 17

for data files see

<http://astronomy.nmsu.edu/cwc/Teaching/ASTR545/AtomicData/>

This is the preliminary of your stellar model code. Basically, once you have completed this problem set, all that needs to be added is the hydrostatic equilibrium from the pressure and temperature gradient. I strongly suggest that you start this homework as soon as possible and contact me if you are having struggles. For the tables and plots, I want you to try to achieve publishable quality tables and plots like the ones you see in the journals.

1. **Excitation States** Consider a gas structure in thermal equilibrium. We will examine the excitation conditions for neutral hydrogen as a function of temperature and electron density.
 - (a) For arbitrary i , derive and write out the full expression for n_{i11}/n_1 in terms of n_e and $\Phi_{11}(T)$. Separately, write the full expression for $\Phi_{11}(T)$.
 - (b) Compute n_{i11}/n_1 for $i = 1, 2, \text{ and } 3$ as a function of temperature over the range $3000 \leq T \leq 20,000$ for three different electron pressures $P_e = 1, 10, \text{ and } 100$ dynes cm^{-2} . On a single graph, plot $\log(n_{i11}/n_1)$ versus T for each P_e (use different color curves for each P_e).
 - (c) For each electron pressure, P_e , at which temperature would you find that the Lyman lines are strongest? The Balmer lines are strongest? The Paschen lines are strongest?
 - (d) Consider the behavior of the “Balmer hydrogens”. Can you provide a physical explanation of what causes the peak of n_{211}/n_1 shift with changing P_e ?

2. **Charge Conservation** Consider a gas structure in thermal equilibrium with $T = 6000$ K and $P_{\text{gas}} = 100$ dynes cm^{-2} comprising hydrogen, helium, and one metal, calcium. Assume the gas mass fractions are $X = 0.71, Y = 0.27, \text{ and } Z = 0.02$. Assume all ionization states for H and He and that Ca can be ionized twice (Ca, $\text{Ca}^+, \text{Ca}^{++}$). Compute the equilibrium conditions of the gas using particle-charge conservation. Converge on the electron density to at least a tolerance of 10^{-5} .
 - (a) In a clear tabular form, report the total gas density, n , the density of nuclear matter, n_N , the electron density, n_e , the mean molecular weight of the nuclear matter, μ_N , the mean molecular weight of free electrons μ_e , the total mean molecular weight, μ , the nuclear particle mass density, ρ_N , the electron mass density, ρ_e , the total gas density, ρ , the electron pressure, P_e , and the ratio P_e/P_{gas} .
 - (b) In clear tabular form, report the number densities of each species, n_k , the number densities for each ionization stage of each species, n_{jk} , the ionization fractions f_{jk} , and the fractional contribution to n_e from each ionization species, which we denote $f(n_e)$. Which ion contributes the most to the free electron pool at this T ?

3. **Thermal Dependence** Expand your code to compute the equilibrium conditions of the gas structure you solved in Problem 2 over the range $1500 \leq T \leq 25,000$ K.
 - (a) Plot the following versus T . Use different colors or line types when necessary.
 - Plot 1: (a) $\log n_e, \log n_N, \log n$; (b) $\log n_k, \log n_{jk}$
 - Plot 2: $\log \rho_e, \log \rho_N, \text{ and } \log \rho$
 - Plot 3: P_e/P_{gas}
 - Plot 4: (a) $\log f_{jk}$; (b) f_{jk}
 - Plot 5: $\log f(n_e)$ for each ion

(b) [i] From plots 1 & 2, explain why n and ρ decrease with increasing T , and why they have the slopes they do. [ii] From plot 5, report which ionization species is the major electron donor over which temperature range. [iii] Going back to plot 3, describe the behavior of P_e/P_{gas} with temperature. Explain each feature in terms of the behavior of the particle ionizations.

YOUR CODES staple you codes to the end of your homework assignment. Clearly comment your codes.