

ASTR 545
Homework 5A (110 points)
Due: Tuesday November 1

All units are in cgs. Provide all answers in cgs. All problems should be calculated using computer code, which is to be extensively commented and handed in with your homework solutions (stapled to the very end of your problem set and in problem order). Please clearly mark which codes correspond to which problem(s) and to which parts of problems. For each response, you are expected to write out and/or derive the equations you are using in your codes and report the requested values clearly (box your answers or put in tabular form). Do not just write code and print out results! Also, be mindful of significant digits in your answers; I will grade you on that too.

1. **Gas Quantities: (50 pts)** Consider a gas of temperature $T = 10,000$ K and pressure of the nuclear particles $P_N = 100$ dynes cm^{-2} . The gas comprises two elements, H and He, with relative mass fractions $X = 0.7$, and $Y = 0.3$. The ionization fractions of H are $f_{11} = 0.15$ and $f_{21} = 0.85$. The ionization fractions of He are $f_{12} = 0.01$, $f_{22} = 0.40$, and $f_{32} = 0.59$.
 - (a) Compute the total density of the nuclear particles, n_N .
 - (b) Compute the H and He abundance fractions, α_k .
 - (c) Compute the H and He number densities, n_1 and n_2 .
 - (d) Compute the number densities of all ionization stages for H and He, n_{11} , n_{21} , n_{12} , n_{22} , and n_{32} .
 - (e) Compute the electron density of the gas, n_e .
 - (f) Compute the total nuclear particle density n .
 - (g) Compute the total gas (particle) pressure, P_g .
 - (h) Assuming thermal equilibrium, compute the pressure due to radiation, P_r . Compute the ratio $\beta = P_g/P$, where P is the total pressure.
 - (i) At this temperature, which ion contributes the greatest number density of free electrons to the gas? What percentage of the total electron density does this ion contribute to the gas.
 - (j) Compute the mean molecular weight of the nuclear particles, μ_N . Compute the mean molecular weight of the electrons μ_e . Compute the total mean molecular weight, μ .
 - (k) Compute the mass density for the nuclear particles, ρ_N . Compute the mass density for the electrons, ρ_e . Demonstrate that total mass density ρ obeys $\rho \simeq \rho_N$ to high accuracy.
2. **Excitation Balance of Hydrogen: (20 pts)** Consider the same gas as presented in Problem 1. Consider neutral hydrogen only, so $k = 1$ and $j = 1$.
 - (a) Compute the ratios n_{ijk}/n_{1jk} for excitation states $i = 2, 3, 4$, and 5. Describe the meaning of this ratio in words. What is the trend in this ratio with increasing excitation state, i ?
 - (b) Compute the ratios $n_{i'jk}/n_{ijk}$, where $i' = i + 1$, for the $i = 1, 2, 3$ and 4. Describe the meaning of this ratio in words. What is the trend in this ratio with increasing excitation state, i ?
 - (c) Compute the neutral H partition function, $U_{11}(T)$ accounting for the first five excitation levels. For this gas temperature, what is the value of $U_{11}(T)$ to a very good approximation?
 - (d) Compute n_{ijk}/n_{jk} for neutral H for excitation states $i = 1, 2, 3, 4$, and 5. Describe the meaning of this ratio in words. What is the trend in this ratio with increasing excitation state, i ?

3. **Ionization Balance: (40 pts)** Calculate the ionization fractions $f_{1,26}$, $f_{2,26}$, and $f_{3,26}$ of Fe I, Fe II, and Fe III for three different stellar atmospheres. To simplify the calculations, we will assume that iron does not get further ionized.
- (a) Star 1: A G2 V star having $n_e = 10^{14} \text{ cm}^{-3}$ at $T = 5800 \text{ K}$. The applicable partition functions are $\log U_{1,26}(T) = 0.763$ and $\log U_{2,26}(T) = 0.938$ and $\log U_{3,26}(T) = 0.000$ for $T = 5800 \text{ K}$.
- (b) Star 2: A G2 Ia star having $n_e = 10^{12} \text{ cm}^{-3}$ at $T = 5800 \text{ K}$.
- (c) Star 3: An A0 V star having $n_e = 10^{14} \text{ cm}^{-3}$ at $T = 10,000 \text{ K}$, for which the applicable partition functions are $\log U_{1,26}(T) = 0.770$ and $\log U_{2,26}(T) = 0.962$ and $\log U_{3,26}(T) = 0.000$ for $T = 10,000 \text{ K}$.
- (d) Compare the ionization fractions for the three stars. (i) State how they changed by computing the percent difference relative to Star 1. Present these in a table. (ii) Explain why you think the fractions changed in the manner you computed (I am asking for your insight into the physics itself).