

ASTR 545
Homework 4 (65 points)
Due: Tuesday October 11

1. **Multi-Electron Atoms I:** In class, we examined the MgII $\lambda\lambda 2796, 2803$ zero-volt resonance fine-structure doublet, which results from the ${}^2S_{1/2}-{}^2P_{3/2}^{\circ}$ and ${}^2S_{1/2}-{}^2P_{1/2}^{\circ}$ transitions.
 - (a) Write the electron sequence for the ground state of MgII? Which neutral atom's electron sequence (i.e., which iso-electron sequence) does this correspond to? For the six elements with 3p valence electrons on the period table, list the ionization state each would need to be in such that it would have this same iso-electronic. Will each of these ions have ${}^2S_{1/2}-{}^2P_{3/2}^{\circ}$ and ${}^2S_{1/2}-{}^2P_{1/2}^{\circ}$ doublet transitions? Explain why or why not.
 - (b) What is the excitation energy of the "active" 3s ground state electron? What are the possible combinations of n , l , and j of the excited electron in this transition. What is the excitation energy of higher energy excited state electron? What is the excitation energy of the lower energy excited electron?
 - (c) From the Russel-Saunders terms, determine ΔL , ΔJ , ΔS , and $\Delta\pi$ for both transitions. From the mathematical definition of π , show that it is the excited state that has odd parity and that it is the ground state that has even parity.
 - (d) From inspection of the Grotrian diagram, note that there is also a $\Delta n = 1$, 4p electron configuration with an excitation energy of roughly 9.99 eV. Write down the spectroscopic notations, in the form ${}^{2S+1}L_J^{\pi}$, for these excited states of the MgII ion. Write down the spectroscopic notation for the transitions between the excited states and the ground state. Are both these transition allowed by the dipole selection rule? State why or why not.
2. **Multi-Electron Atoms II:** Consider the NII ion.
 - (a) Write down the ground state electron configuration for NII. Write down the Russel-Saunders state symbol for NII. Write down the quantum numbers (n, l, m_l, m_s) for the ground-state electron. What is the value of the excitation energy for this ground state electron (eV)? Explain why it has this value.
 - (b) Now consider the scenario in which the ground state electron is excited into the $n = 3$ shell, but you don't know its l , m_l , and m_s . Write down the electronic sequence for this excited NII ion. Write down all the possible Russel-Saunders state symbols of the NII excited states that are theoretically allowed to spontaneously transition back down to the ground state according to the dipole selection rules for multi-electron atoms. Write down the possible transitions using spectroscopic notation of Russel-Saunders symbols. How many possible transitions did you find?
 - (c) We now wish to determine which of these "theoretically allowed" dipole transitions are actually physical, meaning that the Russel-Saunders symbols you wrote in part (b) correspond to real physical electron configurations in the NII ion. To do this, begin by applying the dipole selection rules for multi-electron atoms to determine the allowed l , m_l , and m_s quantum states that the $n = 3$ excited electron must have to yield an allowed transition to the ground state of NII. From your results, which of the "theoretically possible" transitions that you wrote in part (b) exist for NII? Report the l , m_l , and m_s quantum states for these existing excited states, clearly labeled as to which corresponds to which transition.
3. **The Natural Line Width:** The natural width of emission and absorption lines depends upon the damping constant. The Einstein coefficient for a spontaneous transition from upper state $n'l'$ to lower state nl (i.e., $n'l' \rightarrow nl$) is $A_{nl}^{n'l'}$ and the damping constant is $\Gamma_{nl}^{n'l'}$. We are assuming the Schrödinger model, that these are

the averaged quantities over the m' and m states, and that you know the values of $A_{nl}^{n'l'}$ for all nl states. We will also assume we are working with hydrogenic atom or ion.

(a) Draw an accurate Grotrian diagram with all nl states up to $n = 6$. You may want to use a full page for this diagram. Consider the 6d–4p transition. Draw this transition on your diagram using a thick solid line. Label it using Russel-Saunders spectroscopic notation.

(b) Write out the fully *explicit* equation to compute the damping constant Γ_{41}^{62} in terms of the $A_{nl}^{n'l'}$. By “explicit” I mean write out the numeric values for the subscripts and superscripts for all terms in your expression; do not leave your answer in symbolic nl notation nor in terms of symbolic operations, like summation or multiplication operators.

(c) Now, add to your Grotrian diagram the transitions that correspond to the the decay channels that appear in your answer for Γ_{41}^{62} in terms of the $A_{nl}^{n'l'}$. Use dotted and dashed lines and label each transition using Russel-Saunders spectroscopic notation.

(d) Explain why all allowed decay channels must be included in the calculation of the damping constant, Γ . What are the units of Γ ? What is the physical interpretation of Γ ?