

ASTR 545
Grey Stellar Atmosphere Model
DUE: Thursday, Dec 8, 2016

Compute a 1-D, plane-parallel, LTE, Grey plane-parallel stellar atmosphere model over the optical depth range $10^{-3} \leq \tau \leq 10^2$. Select a star as defined by its surface gravity, g , and effective temperature, T_{eff} , and mass fractions X , Y , and Z (you are not required to use solar mass fractions). Use the OPAL opacities from the on-line Opacity Project (I will provide on the web page). If you choose a cool star, you may need to supplement the OPAL opacity table using the Cox & Tabor (1976) table (I will also provide on the web page). You must include at least one metal species, but are encouraged to incorporate as many as you can (for extra credit points).

Present your results in a formal paper using Latex AAS macros for submitting to ApJ and AJ. *I prefer the emulateapj style* so that your paper looks like a real journal article. Include the following plots:

1. Main Physical Quantities (four panels): (a) Physical Depth, x , versus optical depth, τ (b) Temperature, T , versus τ (c) Opacity, χ' [cm^2/g], versus τ (d) Total Mass Density, ρ_{tot} , versus τ
2. Pressures: (left panel) P , P_g , P_N , P_e , and P_r versus τ . (right panel) P_e/P_g versus τ .
3. Ionization fractions: $f_{jk} = n_{jk}/n_k$ versus τ for all species
4. Electron density, n_e , versus τ and the partial electron density (not fraction) contributed from each n_{jk} versus τ .
5. Hydrogen excitation fractions: n_{111}/n_1 , n_{211}/n_1 , and n_{311}/n_1 versus τ .

Provide proper descriptive figure captions. Please do not simply place all plots at the end of the paper but place them throughout as done in a journal article. Also, include a table with the following columns: (1) optical depth, τ . (2) physical depth x ($x = 0$ at the “surface”), (3) temperature T , (4) total gas pressure P_g , (5) total mass density, ρ_{tot} , (6) electron pressure P_e , (7) electron density n_e , and (8) opacity τ , (9) the neutral hydrogen ionization fraction f_{11} .

Your paper should be structured as follows:

1. Abstract: Begin by describing of what you did; then include (a) the motivation for undertaking the study, (b) the methods for undertaking the study, (c) the results of the study, and (d) the interpretation and conclusion of the results of the study.
2. Introduction: This section describes the importance of stellar atmospheres in astrophysical studies, i.e., motivates the problem, then it quotes some previous works in the literature as to what work has come before and

what the main results were and what work still needs to be done. Then the last paragraph states what project you are doing in this paper. The last paragraph provides an outline of the paper structure.

3. **Methods:** Provide a detailed description of the mathematical model including physical assumptions (plane parallel, mean opacity and its definition, etc), equations used, and methods of computation, including the iterative scheme employed (to obtain pressure gradient), root-solving and interpolation schemes with references to pre-written codes used (if any). After describing all the components, describe your algorithm for solution.
4. **Results:** Present the results of the model calculations and your table. One by one, present and describe the plots. If there are any stand-out features that you believe should be pointed out in the plot, then point them out (but save deeper physical interpretation for the Discussion section)
5. **Discussion:** This section is where the interpretive discussion should be. Here, step through the *detailed physical behavior* of the model as a function of depth in the atmosphere. Discuss the *physical interpretation* of features seen in your plots. Where appropriate, mention where the input assumptions (or possibly computational methods) are limiting the realism of the model (its short comings).
6. **Conclusion:** This is a brief section in which you restate what you did and then state what is the most important physical insights you gained. Papers dare often ended with a short blurb on what improvements could be made and implemented in the future to increase the realism of them of your model.
7. **References:** In journal style, provide the references for all work cited.

GRADING: I will be looking for a clearly presented paper that follows the above structure closely. I will be looking for each plot being presented as if it were being presented in a journal article, including labels, legends, and captions, and description in the text. As for the content, I will be looking for physical insight into the behavior of the model.

PARTICLE/MASS DENSITY CONSERVATION

$$n = n_N + n_e, \quad \rho = \rho_N + \rho_e$$

$$n_N = \sum_k n_k, \quad n_k = \alpha_k n_N = \sum_{j=1}^{k+1} n_{jk}, \quad n_{jk} = f_{jk} n_k = \sum_i n_{ijk}$$

$$\rho_N = m_a \sum_k A_k n_k = m_a n_N \sum_k \alpha_k A_k, \quad \rho_e = m_e n_e$$

DETAILED BALANCING

$$\frac{n_{ijk}}{n_{jk}} = \frac{g_{ijk}}{U_{jk}(T)} \exp \left\{ -\frac{\chi_{ijk}}{kT} \right\}$$

$$Y_{jk} = \frac{n_{j+1,k}}{n_{jk}} = \frac{C_{\Phi} U_{j+1}(T)}{n_e U_{jk}(T)} T^{3/2} \exp \left\{ -\frac{\chi_{Tjk}}{kT} \right\}$$

$$f_{jk} = \frac{n_{jk}}{n_k} = \frac{P_{jk}}{S_k} \quad \text{or} \quad f_{jk} = f_{j-1,k} Y_{j-1,k} \quad \text{with} \quad f_{1k} = \frac{1}{S_k}$$

$$P_{jk} = P_{j-1,k} Y_{j-1,k} \quad \text{with} \quad P_{1k} = 1 \quad \text{and} \quad S_k = \sum_{j=1}^{k+1} P_{jk}$$

CHARGE DENSITY CONSERVATION

$$n_{e,jk} = (j-1)n_{jk} = (j-1)f_{jk}n_k$$

$$n_e = (n - n_e) \sum_k \alpha_k \sum_{j=1}^{k+1} (j-1)f_{jk},$$

$$f_{jk} = \frac{n_{jk}}{n_k}$$

EQUATION OF STATE AND HYDROSTATIC PRESSURE

$$P = P_g + P_r = (P_N + P_e) + P_r$$

$$P_N = n_N kT = \frac{k}{\mu_N m_a} \rho T, \quad P_e = n_e kT = \frac{k}{\mu_e m_a} \rho T, \quad P_r = \frac{a}{3} T^4$$

$$\mu_N = \left[\sum_k \left(\frac{x_k}{A_k} \right) \right]^{-1}, \quad \mu_e = \left[\sum_k \left(\frac{x_k}{A_k} \right) \sum_{j=1}^{k+1} (j-1)f_{jk} \right]^{-1}$$

$$\frac{dP}{dx} = -g\rho, \quad g = \frac{GM_*}{R_*^2}$$

OPTICAL DEPTH RELATIONS

$$\Delta\tau = \chi'(\rho, T)\rho\Delta x$$

$\chi'(\rho, T)$ = Rossland Mean Mass Absorption Coefficient, from OPAL table

$$\Delta P = g \frac{\Delta\tau}{\chi'(\rho, T)}$$

$$\frac{T(\tau)}{T_{\text{eff}}} = \left[\frac{3}{4} \{ \tau + q(\tau) \} \right]^{1/4} \equiv \frac{1}{p(\tau)},$$

$q(\tau)$ = Hopf Function, from Table (hand out)

FLUX

$$\frac{F_{\alpha}(\tau)}{F} = \frac{4\pi k^4}{h^3 c^2 \sigma} \alpha^3 \beta(\tau)$$

$$\alpha = \frac{h\nu}{kT_{\text{eff}}} = \frac{hc}{\lambda kT_{\text{eff}}}$$

$$F = \sigma T_{\text{eff}}^4$$

$$\beta(\tau) = \int_{\tau}^{\infty} \frac{E_2(t - \tau) dt}{\exp \{ \alpha p(\tau) \} - 1} - \int_0^{\tau} \frac{E_2(\tau - t) dt}{\exp \{ \alpha p(\tau) \} - 1}$$

$$E_n(y) = \int_1^{\infty} \frac{\exp \{ -wy \}}{w^n} dw$$

ALGORITHMIC STEPS

(a little help)

1. Your model inputs are
 - (i) effective temperature, T_{eff}
 - (ii) surface gravity $g = GM_*/R_*$
 - (iii) composition, mass fractions X, Y, Z
 - (iv) Rosseland mean opacity table (for given X, Y, Z)
 - (v) boundary condition: total pressure at the top layer, P_1
2. Obtain the boundary condition from the plots from the class notes; be sure to scale by $P_1 \propto g^{2/3}$ as appropriate for your choice of star.
3. Set up the τ_i and $T_i = T(\tau_i)$ arrays (the optical depth and temperature grid); where $\tau_1 = 10^{-3}$. Use a τ grid that samples τ ten times per unit decade.
4. Using P_1 , solve the detailed balance of the $i = 1$ layer, obtaining $n_e(\tau_1)$ and $\rho_1 = \rho(\tau_1)$. Compute $\chi'(\rho_1, T_1)$ by interpolating on the the opacity table. You have solved the top layer.
5. (τ_i loop): increment i .
6. Make the initial *estimate* of the pressure, P'_i , in next layer i based upon the opacity of the above layer, $i-1$

$$P'_i = P_{i-1} + g \frac{(\tau_i - \tau_{i-1})}{\chi(\rho_{i-1}, T_{i-1})}.$$

7. (pressure convergence loop): using the guess P'_i , solve the detailed balance of layer i , obtaining your current estimate of $\rho'_i = \rho(\tau_i)$. Compute $\chi'(\rho'_i, T_i)$ by interpolating on the the opacity table.
8. Refine the pressure guess by averaging the pressure gradient. Do this using the average opacity of layer i for your current estimate (i.e., $\chi'(\rho'_i, T_i)$ for P'_i and ρ'_i) and the above (solved) layer $i-1$,

$$P''_i = P_{i-1} + 2g \frac{(\tau_i - \tau_{i-1})}{\chi'(\rho_{i-1}, T_{i-1}) + \chi'(\rho'_i, T_i)}.$$

This new pressure is an improved estimate of the hydrostatic gradient.

9. Check for convergence: is $|P''_i - P'_i|/P'_i \leq \epsilon$, where ϵ is your specified tolerance level?

If “YES”, then you have achieved hydrostatic equilibrium. Set $P_i = P''_i$ and solve the detailed balance for the layer; store or print all quantities and go to step 5.

If “NO”, then set $P'_i = P''_i$ and go to step 7; repeat until step 9 provides a “YES”.