ASTR 545
Homework 6
DUE: November 19, 2010

1. Accounting for limb darkening, assume that the specific intensity at the surface of a star goes as

$$I_\lambda(0) = I^a_\lambda + I^b_\lambda \cos\theta$$

where I^a_λ is the isotropic component and I^b_λ is the amplitude of the non-isotropic component of the specific intensity at each wavelength.

(a) Derive the mean intensity, J_λ, in terms of I^a_λ and I^b_λ.

(b) Derive the astrophysical flux, $\mathcal{F}_\lambda$, in terms of I^a_λ and I^b_λ.

(c) Derive the radiation pressure density, P_λ, in terms of I^a_λ and I^b_λ.

2. Consider four stars with solar abundances, which have the following properties at $\tau = 2/3$ in their atmospheres:

K5V: $T = 4470$ K ; $n_e = 2.6 \times 10^{14}$ cm^{-2}

G5V: $T = 5930$ K ; $n_e = 1.2 \times 10^{14}$ cm^{-2}

A5V: $T = 8430$ K ; $n_e = 3.9 \times 10^{14}$ cm^{-2}

B5V: $T = 13,980$ K ; $n_e = 4.4 \times 10^{13}$ cm^{-2}

(a) Compute the neutral hydrogen bound-free cross section [cm^2 per neutral hydrogen atom] at $\tau = 2/3$ for these four stars over the wavelength range 1000–16,000 Å. Use the Unsöld approximation for $m = 6$. You may assume a unity Gaunt factor, $g_{\rm II}(n) = 1$.

(b) Compute the H$^-$ bound–free cross section [cm^2 per neutral hydrogen atom] at $\tau = 2/3$ for these four stars over the wavelength range 1000–16,000 Å.

(c) For each star, plot the individual cross sections and the total cross section in units 10^{-26} cm^2 per neutral hydrogen atom. Please present your plot of each star as a single panel and place them all on a single page. Label the curves in at least one of the panels. Hand in your code that computed the cross sections (not the plotting code).

(d) Based upon your plots, describe the anticipated behavior of the Balmer Decrement as one goes from K5V stars to B5V stars. Provide as much detail and insight as you can into the physics (excitation, ionization, abundances, surface gravity) that governs this behavior.

3. Consider the flux density integral for a grey atmosphere

$$F_\nu(\tau) = 2\pi \left[\int_\tau^\infty S_\nu(t) E_2(t-\tau)\, dt \; - \int_0^\tau S_\nu(t) E_2(\tau - t)\, dt \right] . \qquad (1)$$

Assume that the source function, $S_\nu(\tau)$, is given by the Planck function, $B_\nu[T(\tau)]$. In class we derived,

$$T(\tau) = T_{\rm eff} \left\{ \frac{3}{4} [\tau + q(\tau)] \right\}^{1/4} \equiv \frac{T_{\rm eff}}{p(\tau)} ,$$

where we have now defined the function $p(\tau)$. For computational ease, it is often convenient to use "code variables" that are dimensionless. This allows one to formulate a calculation so that it is independent of the specific parameters of the model, being a function of the dimensionless variable only [this is common practice]. The flux integral is an excellent example of how this can be applied in practice (see Mihalas, §3-5, pp73-74; "Emergent Flux from a Grey Atmosphere". Then, once the computation is made (a single time!), the results can be conveniently scaled to the specific parameters of *any* object under consideration.

We write the ratio of the radiation energy, $h\nu$, to the kinetic energy, $kT_{\rm eff}$, at $\tau \simeq 2/3$ as the dimensionless parameter α, i.e.,

$$\alpha = \frac{h\nu}{kT_{\rm eff}} ,$$

and account for energy conservation between the parameterized flux density and the actual flux density,

$$F_\alpha(\tau)\, d\alpha = F_\nu(\tau)\, d\nu .$$

Starting with Eq. 1, derive the parameterized flux density,

$$F_\alpha(\tau) = \frac{4\pi k^4 T_{\rm eff}^4}{h^3 c^2} \, \alpha^3 \beta_\alpha(\tau) ,$$

where

$$\beta_\alpha(\tau) = \left[\int_\tau^\infty \frac{E_2(t-\tau)\, dt}{\exp\{\alpha p(\tau)\} - 1} - \int_0^\tau \frac{E_2(\tau - t)\, dt}{\exp\{\alpha p(\tau)\} - 1} \right] .$$

Also show that, using $F = \sigma T_{\rm eff}^4$, we obtain

$$\frac{F_\alpha(\tau)}{F} = \frac{4\pi k^4}{h^3 c^2 \sigma} \alpha^3 \beta_\alpha(\tau) .$$

4. For 15 points extra credit: plot $F_\alpha(\tau)/F$ for $\tau = 0, 0.4, 1.0,$ and 2.0 over the range $0 \le \alpha \le 12$. (See Figure 3-1 of Mihalas for the solution). For $\tau = 0$, and $T_{\rm eff} = 6000$ K, convert your numeric values for $F_\alpha(0)/F$ to $F_\nu(0)/F$ and plot as a function of wavelength in angströms for $0 < \alpha \le 12$. Show your conversion for at least one value of α (or you can present code). [Partial credit given for partial steps taken on this problem]