

**ASTR 545**  
**Homework 5**  
**DUE: October 25, 2010**

1. Consider an LTE gas structure with  $T = 6000$  K and  $P_{tot} = 100$  dynes  $\text{cm}^{-2}$  comprising hydrogen, helium, and calcium with mass fractions  $X = 0.71$ ,  $Y = 0.27$ , and  $Z = 0.02$ . Assume all ionization states for H and He and that Ca can be ionized twice (Ca,  $\text{Ca}^+$ ,  $\text{Ca}^{++}$ ). Write a code to compute the equilibrium conditions of the particles in this gas by root-solving the equation,

$$n_e = (n_{tot} - n_e) \sum_k \alpha_k \sum_{j=1}^{J_k} j f_{jk}(n_e, T), \quad (1)$$

for  $n_e$ . The result also yields the equilibrium ionization fractions,  $f_{jk} = n_{jk}/n_k$ . Solve for  $n_e$  to a tolerance  $0.5(n_e^+ - n_e^-) = 10^{-5}$ , where  $n_e^+$  and  $n_e^-$  are the values that bracket the root,  $n_e$ .

- (a) At the top of your code, before proceeding to root solve Eq. 1, calculate the abundance fractions,  $\alpha_k$ , and the mean molecular weight of the nuclear matter,  $\mu_N$ . Have your code report these values.
  - (b) Having solved Eq. 1, have your code report the equilibrium values of  $n_e$ ,  $\mu_e$ ,  $P_e$ , and  $\beta = P_e/P_{tot}$ . In tabular form, have your code report the values of  $f_{jk}$ , where the rows are the elements ( $k$ ) and the columns are the  $f_{jk}$ .
  - (c) Have your code apply particle conservation to determine  $n_N$ , the  $n_k$  and the  $n_{jk}$  and report the values. For the  $n_{jk}$ , have your code report the values in tabular form (same format as for the  $f_{jk}$ .)
  - (d) Have your code compute the quantities  $j n_{jk}/n_e$ , the fractional contribution to  $n_e$  from each ionization species, and report the values for the top three electron donors.
  - (e) Assuming massless electrons, have your code compute and report the values of  $\mu_{tot}$  and  $\rho_{tot}$  as computed from  $\mu_{tot}$ . Also compute  $\rho_{tot} = \rho_N + \rho_e = \sum (n_k A_k m_{\text{amu}}) + m_e n_e$ . Compare the two  $\rho_{tot}$  and  $\rho_N$  and  $\rho_e$ .
2. Expand your code to compute the equilibrium conditions of this gas structure as a function of temperature over the range  $3500 \leq T \leq 25,000$  K in  $T$  intervals of  $\Delta T = 250$  K.
    - (a) Produce fully labeled plots of the following versus  $T$ : (i)  $n_e(T)$ , (ii)  $f_{jk}(n_e, T)$ , (iii)  $\beta(T) = P_e(T)/P_{tot}$ , and (iv)  $j n_{jk}(T)/n_e(T)$ . Comment on the physical reasons governing the  $T$  behavior of  $n_e(T)$ .
    - (b) Examining plot (iv), report which ionization species is the major electron donor and over which temperature range. Comment on the temperature behavior of  $\beta(T)$  given the insights gained from plot (iv).