ASTR 545
SPRING 2010
Homework 2
DUE: September 10, 2010

1. Appearing in the Boltzmann and Saha Equations is the term

$$e^{-\chi_{ij}/kT},$$

where χ_{ij} is the excitation potential in ergs for an atom in ionization state j and with an electron in excited level i. It is often convenient to express χ'_{ij} in electron Volts [eV] and rewrite the exponential as a power of 10. Derive the transformation

$$e^{-\chi_{ij}/kT} = 10^{-\theta\chi'_{ij}},$$

where $\theta = C/T$. In your transformation, explicitly show your derivation of the value of the constant C and evaluate it numerically. What are the units of C?

2. From Table D.2 in Gray, we see that $U(T) \simeq 2$ for neutral hydrogen and that it is fairly independent of temperature!

(a) Using the definition of $U(T)$ at $T = 10,000$ K, demonstrate numerically that the neutral hydrogen partition function converges to $2 + \epsilon$, where $\epsilon \ll 1$. (Carry out your sum to at least 4 terms in your demonstration).

(b) For $T = 10,000$ K, what is the value of ϵ (within a factor of a few)?

(c) Does $U(T)$ converge more rapidly for higher T or for lower T compared to $T = 10,000$ K? Explain why.

[HINT]: For part (a), if you want to derive this algebraically, which may be easier actually, then you would benefit by considering the unitless variable $x = \chi_{ij}/kT$, Taylor expanding the exponentials, $e^{-x_{ij}}$, and examining the behavior for small and/or large x_{ij}.

3. Consider the fraction of hydrogen atoms available for absorbing the Balmer lines as a function of temperature, T, and electron density, n_e. If n_{20} is the number of neutral hydrogen atoms in the $n = 2$ excitation state, n_0 is the total number of neutral hydrogen atoms (H^0), and n_1 is the total number of ionized hydrogen atoms (H^+), then

$$\frac{n_{20}}{n} = \frac{n_{20}}{n_0 + n_1}.$$

(a) Write out a single expression in full (all constants and variables) to calculate n_{02}/n.

(b) Write a code (language of your choice) to calculate n_{02}/n as a function of temperature over the range $3500 \leq T \leq 20,000$ K for three different electron pressures $P_e = 1$, 10, and 100 dynes cm^{-2}, where $P_e = n_e kT$ can be assumed.

(c) Generate three plots. (i) On Plot 1 graph n_{20}/n_0 versus temperature for the three values of P_e and label your curves. (ii) On Plot 2 graph n_1/n_0 versus temperature for the three values of P_e and label your curves. (ii) On Plot 3 graph n_{20}/n versus temperature for the three values of P_e and label your curves.

(d) Report the temperature where the Balmer strengths are a maximum for each P_e. Explain why *physically*, the peaks are different for the different P_e and why they follow the trend with P_e that you obtained?

(e) Present your code with your write-up, be sure that all variable names are identified with clear comments.

4. Consider the fraction of the densities of iron atoms in each of the first three stages of ionization. We will assume iron does not get further ionized, so that $n = n_0 + n_1 + n_2$, where $n_0 = \mathrm{Fe}^0$, $n_1 = \mathrm{Fe}^{+1}$, and $n_2 = \mathrm{Fe}^{+2}$, respectively.

   (a) In terms of the ratios of the densities of adjacent ionization stages, n_1/n_2, etc, write out the expressions for n_0/n, n_1/n, and n_2/n.

   (b) Compute and tabulate all n_j/n for a G2 V atmosphere with $n_e = 4 \times 10^{13}$ cm^{-2} and $T = 5800$ K.

   (c) Compute and tabulate all n_j/n in a G2 III atmosphere with a solar temperature and $n_e = 1 \times 10^{12}$ cm^{-1}. Compare with the solar type atmosphere: (i) in which is iron in higher/lower ionization stages? (ii) explain *physically* why the two stars are different and how this leads to the computed differences? (you are encouraged to also refer to the functional form of the equations in your answer).

   (d) Compute and tabulate n_j/n for an A0 V atmosphere with $n_e = 4 \times 10^{13}$ cm^{-2} and $T = 10,000$ K. Compare with the solar type atmosphere: (i) in which is iron in higher/lower ionization stages? (ii) explain *physically* why the two stars are different and how this leads to the computed differences? (you are encouraged to also refer to the functional form of the equations in your answer).

   (e) Present your code with your write-up, be sure that all variable names are identified with clear comments.

Type	T [K]	n_e [cm^{-3}]	n_0/n	n_1/n	n_2/n
G2 V	5800	4×10^{13}			
G2 III	5800	1×10^{12}			
A0 V	10,000	4×10^{13}			