Compute a 1-D, plane-parallel, LTE, Grey plane–parallel stellar atmosphere model over the optical depth range $10^{-3} \leq \tau \leq 10^2$. Select a star as defined by its surface gravity, $g$, and effective temperature, $T_{\text{eff}}$, and mass fractions $X$, $Y$, and $Z$. You should assume solar abundance pattern, but do not have to use solar mass fractions. Use the OPAL opacities from the on-line Opacity Project (you will need to download the proper table—see the class notes on opacity tables). If you choose a cool star, you may need to supplement the OPAL opacity table using the Cox & Tabor (1976) table I provided in the notes. You must include at least one metal species, but are encouraged to incorporate as many as you can (for extra credit points).

Present your results in a formal paper using Latex AAS macros for submitting to ApJ and AJ. I prefer the emulateapj style so that your paper looks like a real journal article. Present the following plots:

1. Main Physical Quantities (four panels): (a) Physical Depth, $x$, versus optical depth, $\tau$ (b) Temperature, $T$, versus $\tau$ (c) Opacity, $\chi' [\text{cm}^2/\text{g}]$, versus $\tau$ (d) Total Mass Density, $\rho_{\text{tot}}$, versus $\tau$

2. Pressures (on a single panel): $P_g$, $P_e$, and $P_e/P_g$ versus $\tau$ (you will need a different scale on the right hand vertical axis for $P_e$)

3. Ionization fractions: $f_{jk} = n_{jk}/n_k$ versus $\tau$ for all species

4. Electron density, $n_e$, versus $\tau$ and the partial electron density (not fraction) contributed from each $n_{jk}$ versus $\tau$.

5. Hydrogen excitation fractions: $n_{0,\text{hi}}/n_{\text{hi}}$, $n_{1,\text{hi}}/n_{\text{hi}}$, and $n_{3,\text{hi}}/n_{\text{hi}}$ versus $\tau$.

Properly encapsulate your figures and provide descriptive captions. I strongly prefer that your plots are presented throughout the paper at the appropriate pages where each is introduced, described, and discussed in the Results and Discussion sections (like a real paper)... please do not simply place all plots at the end of the paper. I also want you to present a single table with following columns in order (1) optical depth, $\tau$, (2) physical depth $x$ ($x = 0$ at the “surface”), (3) temperature $T$, (4) total gas pressure $P_g$, (5) total mass density, $\rho_{\text{tot}}$, (6) electron pressure $P_e$, (7) electron density $n_e$, and (8) opacity $\tau$, (9) the neutral hydrogen ionization fraction $f_{\text{hi}}$. Your paper should be structured as follows (including the points given below):

1. Abstract: Begin by describing of what you did; then include (a) the motivation for undertaking the study, (b) the methods for undertaking the study, (c) the results of the study, and (d) the interpretation and conclusion of the results of the study. (Astrophysics and Astronomy articles have good abstract structures for authors and you might check out a few articles to see how they force their authors to structure the abstract).
2. Introduction: This section describes the importance of stellar atmospheres in astrophysical studies, i.e., motivates the problem, then it quotes some previous works in the literature as to what work has come before and what the main results were (you should look up previous work on stellar atmosphere models, both Grey and non-Grey— these papers will provide a lot of motivating material as well as describe the usefulness of atmosphere modeling). Then the last paragraph states what project you are doing in this paper and provides an outline of the paper structure. Hopefully, your introduction has motivated why you are doing it.

3. Methods: Provide a detailed description of the mathematical model including physical assumptions (plane parallel, mean opacity and its definition, etc), equations used, and methods of computation, including the iterative scheme employed (to obtain pressure gradient), root-solving and interpolation schemes with references to pre-written codes used (if any). After describing all the components, describe your algorithm for solution.

4. Results: Present the results of the model calculations, especially introduction to and presentation of your table of results; in this section there is a description of behavior of the model output quantities, but a minimum of interpretative discussion. One by one, present and describe the plot. If there are any features that you believe should be pointed out in the plot, then point them out (but save deeper physical interpretation for the Discussion section).

5. Discussion: This section is where the interpretive discussion should be. Here, step through the detailed physical behavior of the model (for example, as a function of depth in the atmosphere). How does property X behave with optical depth? What physical insight into the behavior of the atmosphere is obtained by the feature (what causes it)? Etc. Focus your discussion around the physical interpretation as shown in your plots. Where appropriate, mention where the input assumptions (or possibly computational methods) are limiting the realism of the model (its shortcomings).

6. Conclusion: This is a brief section in which you restate what you did, restate what is the most important physical insights you gained from this study (normally the points chosen are relevant to improving, bettering, or changing the results of works quoted previously in the introduction to the paper). Sometimes, the paper is ended with a short blurb on what improvements could be followed in the future to increase the realism of them of your model.

7. References: In journal style, provide the references for all work cited.

GRADING: I will be looking for a clearly presented paper that follows the above structure closely (addressing each of the points stated above). As for the content, I will be looking for physical insight into the behavior of the model.
Do your best to describe the behavior as accurately and concisely as possible (avoid words such as “this”, “it” etc., since the meaning of these type of words can be confused depending on the sentence structure... just be specific even if it sound repetitive. I will be looking for each plot being presented as if it were being presented in a journal article, including labels, legends, and captions, and description in the text.

**PARTICLE/MASS DENSITY CONSERVATION**

\[ n = n_N + n_e, \quad \rho = \rho_N + \rho_e \]

\[ n_N = \sum_k n_k, \quad n_k = \alpha_k n_N = \sum_{j=1}^{k+1} n_{jk}, \quad n_{jk} = f_{jk} n_k = \sum_i n_{ijk} \]

\[ \rho_N = m_a \sum_k A_k n_k = m_a n_N \sum_k \alpha_k A_k, \quad \rho_e = m_e n_e \]

**DETAILED BALANCING**

\[ \frac{n_{ijk}}{n_{jk}} = \frac{g_{ijk}}{U_{jk}(T)} \exp \left\{ -\chi_{ijk} \frac{kT}{kT} \right\} \]

\[ Y_{jk} = \frac{n_{j+1,k}}{n_{jk}} = \frac{C_\Phi U_{j+1}(T) T^{3/2}}{U_{jk}(T)} \exp \left\{ -\chi_{ijk} \frac{kT}{kT} \right\} \]

\[ f_{jk} = \frac{n_{jk}}{n_k} = \frac{P_{jk}}{S_k} \quad \text{or} \quad f_{jk} = f_{j-1,k} Y_{j-1,k} \quad \text{with} \quad f_{1k} = \frac{1}{S_k} \]

\[ P_{jk} = P_{j-1,k} Y_{j-1,k} \quad \text{with} \quad P_{1k} = 1 \quad \text{and} \quad S_k = \sum_{j=1}^{k+1} P_{jk} \]

**CHARGE DENSITY CONSERVATION**

\[ n_e = \sum_k \sum_{j=1}^{k+1} (j-1)n_{jk} = \sum_k n_k \sum_{j=1}^{k+1} (j-1)f_{jk} \]

\[ n_e = (n - n_e) \sum_k \alpha_k \sum_{j=1}^{k+1} (j-1)f_{jk}, \]

\[ f_{jk} = \frac{n_{jk}}{n_k} \]
EQUATION OF STATE AND HYDROSTATIC PRESSURE

\[ P = P_g + P_r = (P_N + P_e) + P_r \]

\[ P_N = n_s kT = \frac{k}{\mu_s m_a} \rho T, \quad P_e = n_e kT = \frac{k}{\mu_e m_a} \rho T, \quad P_r = \frac{a T^4}{3} \]

\[ \mu_N = \left[ \sum_k \left( \frac{x_k}{A_k} \right) \right]^{-1}, \quad \mu_e = \left[ \sum_k \left( \frac{x_k}{A_k} \right) \sum_{j=1}^{k+1} (j-1) f_{jk} \right]^{-1} \]

\[ \frac{dP}{dx} = -g \rho, \quad g = \frac{GM_*}{R^2} \]

OPTICAL DEPTH RELATIONS

\[ \Delta \tau = \chi' (\rho, T) \rho \Delta x \]

\( \chi'(\rho, T) = \) Rossland Mean Mass Absorption Coefficient, from OPAL table

\[ \Delta P = g \frac{\Delta \tau}{\chi'(\rho, T)} \]

\[ \frac{T(\tau)}{T_{\text{eff}}} = \left[ \frac{3}{4} \{ \tau + q(\tau) \} \right]^{1/4} = \frac{1}{p(\tau)} \]

\( q(\tau) = \) Hopf Function, from Table (hand out)

FLUX

\[ \frac{F_\alpha(\tau)}{F} = \frac{4\pi k^4}{h^3 e^2 \sigma} \alpha^3 \beta(\tau) \]

\[ \alpha = \frac{h \nu}{k T_{\text{eff}}} = \frac{hc}{\lambda k T_{\text{eff}}} \]

\[ F = \sigma T_{\text{eff}}^4 \]

\[ \beta(\tau) = \int_{\tau}^{\infty} \frac{E_2(t-\tau) dt}{\exp \{ \alpha p(\tau) \} - 1} - \int_{0}^{\tau} \frac{E_2(\tau - t) dt}{\exp \{ \alpha p(\tau) \} - 1} \]

\[ E_n(y) = \int_{1}^{\infty} \frac{\exp \{-wy\}}{w^n} dw \]
ALGORITHMIC STEPS
(a little help)

1. Your model inputs are
   (i) effective temperature, $T_{\text{eff}}$
   (ii) surface gravity $g = GM_*/R_*$
   (iii) composition, mass fractions X, Y, Z
   (iv) Rosseland mean opacity table (for given X, Y, Z)
   (v) boundary condition: total pressure at the top layer, $P_1$

2. Obtain the boundary condition from the plots from the class notes; be sure to scale by $P_1 \propto g^{2/3}$ as appropriate for your choice of star.

3. Set up the $\tau_i$ and $T_i = T(\tau_i)$ arrays (the optical depth and temperature grid); where $\tau_1 = 10^{-3}$. Use a $\tau$ grid that samples $\tau$ ten times per unit decade.

4. Using $P_1$, solve the detailed balance of the $i = 1$ layer, obtaining $n_e(\tau_1)$ and $\rho_1 = \rho(\tau_1)$. Compute $\chi'(\rho_1, T_1)$ by interpolating on the the opacity table. You have solved the top layer.

5. ($\tau_i$ loop): increment $i$.

6. Make the initial estimate of the pressure, $P'_i$, in next layer $i$ based upon the opacity of the above layer, $i-1$

   $$P'_i = P_{i-1} + g \frac{(\tau_i - \tau_{i-1})}{\chi(\rho_{i-1}, T_{i-1})}.$$ 

7. (pressure convergence loop): using the guess $P'_i$, solve the detailed balance of layer $i$, obtaining your current estimate of $\rho'_i = \rho(\tau_i)$. Compute $\chi'(\rho'_i, T_i)$ by interpolating on the the opacity table.

8. Refine the pressure guess by averaging the pressure gradient. Do this using the average opacity of layer $i$ for your current estimate (i.e., $\chi'(\rho'_i, T_i)$ for $P'_i$ and $\rho'_i$) and the above (solved) layer $i-1$, 

   $$P''_i = P_{i-1} + 2g \frac{(\tau_i - \tau_{i-1})}{\chi(\rho_{i-1}, T_{i-1})} + \chi'(\rho'_i, T_i).$$

   This new pressure is an improved estimate of the hydrostatic gradient.

9. Check for convergence: is $|P''_i - P'_i|/P'_i \leq \epsilon$, where $\epsilon$ is your specified tolerance level?
   If “YES”, then you have achieved hydrostatic equilibrium. Set $P_i = P''_i$ and solve the detailed balance for the layer; store or print all quantities and go to step 5.
   If “NO”, then set $P'_i = P''_i$ and go to step 7; repeat until step 9 provides a “YES".