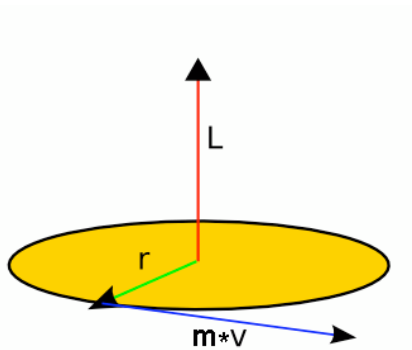


ASTR 308 "Into the Final Frontier"

Rocket Science

1. Angular Momentum

- (a) Draw a clear picture of a spinning wheel, like a bicycle tire. Include and label its spin axis and rotation direction.



(credit: rpmedia.ask.com)

L represents the angular momentum vector. L always points along the rotation axis of the spinning wheel. Using the “right hand rule” we see that the counter clockwise spinning wheel in the diagram has an angular momentum L pointing parallel to the upward direction of the spin axis.

- (b) Now, on your drawing, sketch an arrow in the direction of the momentum that results from this spin (the angular momentum). Explain the affects of this angular momentum on the orientation of the spinning wheel. (Perhaps you can use the example of a spinning top in comparison to a non-spinning top; or of riding your moving bike with no hands!?)

The direction of the angular momentum is provided and described above. If you draw the L vector, it must follow the right hand rule for the sense that you have your wheel spinning. **The angular momentum has the affect that it stabilizes the 3D spatial orientation of the wheel.** Much like a billiard ball moving in a straight line has linear momentum in the direction of motion; a force is required to overcome the momentum to change its motion ($F=ma$), the same applies to angular momentum... it requires a force to change the orientation of the wheel (meaning to change the direction of its spin axis in 3D space). Consider the spinning top... why does it not tip over when it is spinning but will naturally tip over when it is not spinning? Gravity is the force that will cause the top to tip over (because most of the mass is contained in the upper portion of the top). When the top is spinning, it has angular momentum that points upward (if it rotates counter clockwise like in the diagram- or downward if rotating clockwise). This stabilizes the spinning top from gravity tipping it over to its lowest energy state (massive part down). When the spin dies down (due to friction), then the angular momentum decreases, the top is not being stabilized any longer and the gravity topples it to the lowest energy state. Over it goes.

- (c) Explain in principle how such rapidly spinning massive wheels, called gyroscopes are used to change the orientation of a rocket (i.e.; change its roll, pitch, and yaw). Roll is rotation around the long axis of the rocket. Pitch is the angle of the long axis of the rocket “up” or “down” from the astronauts’ point of view. Yaw is the angle of the long axis of the rocket “left” or ”right” from the astronauts’ point of view.

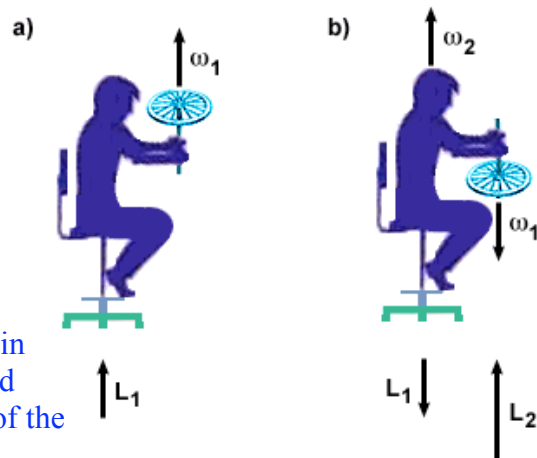
The TOTAL angular momentum of the system (rocket + gyroscope) must be conserved through the principle of conservation of momentum. Thus, if the angular momentum of the gyroscope is change (by a mechanical motor), the angular momentum of the rocket will change by an amount and direction that will conserve the angular momentum that the system had before the motor rotated the gyroscope. (see explanation below)

- (d) Say you want to use this gyroscope to change the roll of the rocket (its orientation with respect to the long axis of the rocket). Can you figure out how you would want to orientate **the axis** your wheel in the rocket so that when you used a mechanical motor to change the direction of the wheel’s axis it will cause the rocket to roll around its long axis. You also have to move the axis of the wheel in a certain direction. Can you figure that you too? [If you recall our experiment in class with the bicycle wheel in class you should be able to figure this out.]

(credit: www.sparknoress.com)

Pretend that the person in the chair is the motor in a rocket that changes the orientation of the spinning wheel (gyroscope) and the chair is the rocket.

- (a) The gyroscope spins at a rate ω_1 such that its angular momentum is L_1 pointing in the upward direction (see the arrow labeled L_1). L_1 is the initial angular momentum of the total system.



- (b) The person (motor) rotates the wheel so that its angular momentum, L_1 , is now pointing downward. The system must respond so that the final angular momentum after the rotation of the gyroscope is equal to the initial angular momentum, L_1 , before the rotation of the gyroscope (L_1 was pointing upward!). So, the person (motor) and chair (rocket) now rotate (“rolls”) with a rate ω_2 having an angular momentum L_2 such that L_2 upward plus L_1 (now downward) equals the original L_1 [as drawn in panel (a)]. So, the orientation of the gyroscope to get rocket to roll about its long axis is as given here in the above diagram.

2. Newton's Laws and Defining Motion

(a) Briefly state, in your own words, each of Newton's three Laws of Motion.

LAW 1. Law of Inertia: A body in a state of constant motion will remain in constant motion unless a net force acts on the body.

LAW 2. Law of Acceleration: When a net force, F , acts on a body of mass m , the body will accelerate with magnitude $a=F/m$ in the direction of the net force.

LAW 3. Law of Reaction: For every action there is an opposite and equal reaction. OR... if body 1 exerts a force on body 2, simultaneously, body 2 exerts an equal magnitude force on body 1, but in the opposite direction.

(b) When we concern ourselves with motion; we describe it in terms of "location", "velocity", and "acceleration"; the latter two are vectors. What is a vector? Now define the terms "velocity" and "acceleration", being sure to consider the vector nature/properties of their definitions.

A **vector** is a quantity that describes both magnitude and direction. For example, velocity measures both the speed of an object (the magnitude) and the direction in which it is moving.

Velocity is defined as $v=\Delta x/\Delta t$ (change in position/change in time). Since a change in position occurs in a given direction, velocity describes the direction of the change too. An object may move 10 feet to the right in 2 second. It has a speed of 5 feet per second. It has a velocity of 5 feet per second to the right.

Acceleration is defined as $a=\Delta v/\Delta t$ (change in velocity/change in time). Since velocity has both magnitude and direction, acceleration has both magnitude and direction.

2. Newton's 1st Law of Motion.

(a) The property we call "mass" really is a measure of an object's "inertia". What is inertia?

Inertia is a quantity that measures an object's resistance to change in motion. [So... how do we measure mass? A change in motion implies non-constant motion, which further implies that a net force, F , is acting on the object. The net force impels the object to accelerate, a . If we apply a force of known magnitude and direction, F , we then witness an acceleration, a , which we can measure. The mass is then "measured" by solving the equation $F=ma$, giving $m=F/a$. That is how we measure mass!]

- (b) When we say “constant motion”, we have a precise meaning and definition. What is constant motion? Give an example of an object in constant motion. If there are any forces acting on this object, what special condition has to hold?

Constant motion is motion in which the acceleration is zero, $a=0$. Under this condition the velocity is constant, whether it is zero (rest) or not zero (constant speed in a constant direction, i.e., a “straight line”). An example of constant motion is a car that is at rest. Similarly, constant motion of a car can be one that is moving down the freeway at a constant speed in a straight line. In the case of the car moving down the freeway, forces are acting on it. The engine propels the car forward and air resistance and friction in the car’s axel and friction of the tires on the road are all acting in the opposite direction. But for constant motion to occur, the forces must cancel (be of equal magnitude and opposite in direction) such that **the net force is zero**.

- (c) If I put my coffee on a smooth flat surface in my car, which is speeding along at a constant velocity on the freeway, and I slam on my breaks, what will the motion of the cup of coffee be in then next instant of time? Be specific (recall motion is a very well defined quantity!)

The coffee cup will continue in its state of constant motion (until it hits the dashboard and makes a horrible mess and pisses you off). Specifically, this means the coffee cup will continue at the **same speed and in the same direction** it was heading in just before you applied the breaks! The coffee cup is obeying Newton’s first law of motion.

3. Newton’s 2nd Law of Motion and Gravity.

- (a) You have an object with mass m which in a large open space with no air resistance and no other objects exist in this space. The object has two forces constantly acting on it. The forces are 100 lbs upward, 150 lbs downward. What is the magnitude and direction of the unbalanced force on the object? Will the object accelerate? If so, in what direction?

The unbalanced force has a magnitude of **50 lbs in the downward direction**. The object will accelerate. It will accelerate in the direction of the unbalanced force, i.e., downward.

- (b) Does the acceleration at any given time depend upon the object’s velocity at that time? Or is the acceleration constant regardless of where the object move’s to or how fast it is going (its velocity)?

Acceleration is totally and completely independent of velocity and never depends upon the velocity (or speed) of the object at any given moment during its motion. Recall, $F=ma$, in this little gem of an equation do you see any reference to the velocity? No. Only that acceleration depends upon the force and the mass.

- (c) What is the incredible truth about the acceleration due to the force of gravity on objects of different masses (neglect air resistance). If you are a given planet... what does properties of the planet does the acceleration of ALL objects on the planet depend on? [For 5 points extra credit; can you show how this is determined mathematically?]

In a gravity field of a planet, the acceleration of an object is independent of the object's mass! That is, no matter what the object is made of or our how massive it is, it will accelerate at the same rate as every other object no matter what it is made of or how massive it is! The acceleration depends only upon the mass of the planet and the radius of the planet! How can this be?

Consider an object of mass m being accelerated by the force of gravity on a planet of mass M and radius R .

$$F(\text{gravity}) = ma \quad (\text{general expression for Newton's Second Law of Motion})$$

$$F(\text{gravity}) = GMm/R^2 \quad (\text{Newton's Law of Gravity})$$

Equating the two expressions because $F(\text{gravity})$ is the same in both...

$$ma = GMm/R^2$$

The mass of the object cancels since it appears on both sides of the equation, yielding $a=GM/R^2$. Thus, we see that the acceleration due to gravity has no dependence on the mass m of the object! Q.E.D.

4. Netwon's 3rd Law of Motion and Rocket Propulsion

- (a) We know that rocket propulsion is based upon Newton's 3rd Law of Motion. Briefly explain the principle of rocket propulsion, be sure to account for the impulse of individual atoms on the rocket in your description (i.e., first describe what one individual atom is propelled and how it subsequently propels the rocket, and then describe how the billions of atoms affect enough reaction force to propel the single massive rocket.

When on atom is forced out the back of a rocket by the rocket engine, the atom imparts an equal and opposite force on the rocket. The atom has a miniscule mass and is highly accelerated. The rocket has a huge mass and it experiences a minuscule acceleration in the opposite direction that the atom is accelerated. Now, force billions and billions of atoms out the back of the rocket and add up all the reaction (equal an opposite) force on the rocket and you begin to have a large enough force on the rocket that it can be highly accelerated even though it has a huge mass

- (b) Explain why rocket propulsion is most efficient in the vacuum of space as opposed to while the rocket is still in the Earth's atmosphere?

There are two reasons. The first is that there is **no air resistance in space**. Air resistance is a force acting on the rocket in the opposite direction as the thrust force. So, the net force on the rocket (thrust – air resistance) is smaller than if there is no air resistance. The second comes from the rocket equation. In the rocket equation the thrust force has two components. The first term is the product of the velocity at which the propellant is ejected out of the engine nozzle and the rate at which the mass of the propellant is ejected out of the nozzle. That is

$(\Delta m/\Delta t) * V_e$ where V_e is the exhaust velocity

This term is independent of whether the rocket is in space or in an atmosphere; it just is a fundamental relationship. The second term in the rocket equation is the one that depends upon the atmosphere... in particular it depends upon the **pressure of the atmosphere**. The second term of the rocket equation is the product of exit area of the engine nozzle and the **difference** between the pressure of the exhaust propellant coming out of the nozzle and the atmospheric pressure. That is

$A * (P_e - P_a)$ where P_e is the exhaust pressure and P_a is the atmospheric pressure

In an atmosphere, the term $P_e - P_a$ is smaller than in space, when $P_a = 0$. Thus, the total thrust of a rocket in space is

Thrust = $(\Delta m/\Delta t) * V_e + A * P_e$

Where as the thrust in an atmosphere is

Thrust = $(\Delta m/\Delta t) * V_e + A * (P_e - P_a)$

Note that the second term is smaller when $P_a > 0$. Thus, for both reasons (air resistance and the principle of rocket thrust pressure), rockets work more efficiently in space.

- (c) Why are rockets built in stages? What benefits is there to making the mass of the rocket get smaller by jettisoning parts of the rocket as it is achieving higher velocities?

$F = ma!$ For a given force (thrust) smaller mass objects accelerate more rapidly! This is the first reason. The second reason is that fuel is massive. To accelerate a rocket, one must also accelerate the massive fuel. It requires more fuel to do this and so a single stage rocket is very massive. By dropping off the heavy engines and empty gas tanks of part of the rocket, the remaining rocket can be accelerated more rapidly at a much lower expense in fuel.