ASTR 545
Homework 2

1. On the class web page, I have placed seven (7) stellar spectra having resolution 5 Å. The files are simple text files with two columns, wavelength in nanometers and flux in units of erg $s^{-1}cm^{-2}$ Å^{-1} times an arbitrary constant. They can be downloaded at

   http://astronomy.nmsu.edu/cwc/Teaching/ASTR545/HW/spectra/

   (a) Plot each of the spectra from 340–680 nm. Identify as many lines and/or features on each spectrum as you can and label them on the plot (give the ID and wavelength of the feature; you do not need to go beyond the identifications that we covered in lecture). (b) To the accuracy afforded by the data, determine the spectral class of each star as accurately as you can (you can provide a range if you are uncertain). If you think you have the information needed to determine the luminosity class, also attempt to provide the luminosity class. Be sure to address any ambiguities you might have had in the classifications you provided.

2. Appearing in the Boltzmann and Saha Equations are the term

   $$e^{-\chi_{ij}/kT},$$

   where χ_{ij} is the excitation potential in ergs for an atom in ionization state j and with an electron in excited level i. It is often convenient to express χ_{ij} in electron Volts [eV] as a power of 10. Show

   $$e^{-\chi_{ij}/kT} = 10^{\theta\chi_{ij}},$$

   where $\theta = 5040/T$ and χ_{ij} is in eV on the right hand side.

3. Using the Boltzmann and Saha equations, (a) write out a single expression to calculate the fraction of hydrogen atoms available for absorbing the Balmer lines as a function of temperature, T, and electron density, n_e. If $n_{2,0}$ is the number of neutral hydrogen atoms in the $n = 2$ excitation state, N_0 is the total number of neutral hydrogen atoms, and N_1 is the total number of ionized hydrogen atoms, then what is needed is

   $$\frac{n_{2,0}}{N} = \frac{n_{2,0}}{N_0 + N_1}$$

   (b) Write a code (language of your choice) to calculate $n_{0,2}/N$ over the temperature range $3500 \leq T \leq 20,000$ K. Calculate $n_{2,0}/N$ for three cases assuming $P_e = n_e kT$ for electron pressures of $P_e = 1$, 10, and 100 dynes cm^{-2}. You may use interpolation of the tabulated partition functions, $\log U(T)$, tabulated in Appendix D of Gray). (c) On the same plot, graph $n_{2,0}/N$ versus temperature for the three values of P_e. Report the temperature where the Balmer strengths are a maximum for each P_e. (d) Finally, make a brief comparison of your solution with the observed strengths of the Balmer lines in the spectra of main sequence stars as a function of temperature. (Present your code with your write-up, be sure that all variable names are identified with clear comments).

4. Starting with particle and charge conservation

$$n_e = (N - n_e) \sum_k \alpha_k \sum_{j=1}^{J_k} j f_{jk}(n_e, T),$$

where N is the total particle density (neutrals + ions + electrons), (a) show that in a *pure hydrogen gas*

$$n_e = \hat{\Phi}_0^{-1} \left\{ (N\hat{\Phi}_0 + 1)^{1/2} - 1 \right\}.$$

(b) Does the value of $\hat{\Phi}_0$ increase or decrease as the degree of ionization decreases? (c) For $N \gg 1$ and fixed T, simplify the above expression to show that $n_e \sim N^{1/2}$ in the case of a low degree of ionization.

5. (a) Calculate (you will need a second code) the fraction of iron atoms in each of the first three stages of ionization (i.e., Fe I, Fe II, and Fe III) in the atmosphere of a solar type star ($n_e = 4 \times 10^{13}$ cm^{-2}, $T = 5800$ K). You can assume iron does not get further ionized. Compute the partition functions, $U_j(T)$, using the tabulations in Appendix D of Gray. (b) Compare the solar case with that of an A star ($n_e = 1 \times 10^{13}$ cm^{-2}, $T = 10,000$ K). (c) Compare the fractions in a giant star with a solar temperature ($n_e = 1 \times 10^{12}$ cm^{-1}). Hand in your code with all variables clearly commented.