ASTR 545 Fall 2018
Homework 5B (XXX points)
ALWAYS SHOW YOUR WORK

Throughout this assignment, we adopt the notation n_{ijk}, n_{jk}, and n_k etc., where i is the excitation level, j is the ionization stage, and k is the species index. I have included two data files for your convenience. The first is the ionization potentials ("IonPotentials.txt") and the second is the partition functions ("PartFuncs.txt") of the relevant atoms and ions.

1. (XX) Ionization Fractions of Calcium. We will assume that there are three ionization stages for caclium, with number densities denoted $n_{1,20}$ for neutral, $n_{2,20}$ for singly ionized, and $n_{3,20}$ for doubly ionized, where the total calcium number density is $n_{20} = n_{1,20} + n_{2,20} + n_{3,20}$.

   (a) [X pts] Starting with the definition of ionization faction for stage j, $f_{j,20} = n_{j,20}/n_{20}$, where $j = 1, 2, 3$, show that

$$f_{1,20} = \frac{1}{1 + Y_{1,20} + Y_{1,20}Y_{2,20}}\,, \quad f_{2,20} = f_{1,20}Y_{1,20}\,, \quad f_{3,20} = f_{2,20}Y_{2,20}\,, \tag{1}$$

   where $Y_{j,20} = n_{j+1,20}/n_{j,20}$.

   (b) [X pts] Invoking $Y_{j,20} = n_{j+1,20}/n_{j,20} = n_e^{-1}\Phi_{j,20}(T)$, compute the calcium ionization fractions in the atmosphere of a G3 Ia star with $T = 6000$ K and $n_e = 8 \times 10^{11}$ cm^{-3}.

   (c) [X pts] Recompute the calcium ionization fractions in the atmosphere of a G3 V star with $T = 6000$ K and $n_e = 2 \times 10^{13}$ cm^{-3}.

   (d) [X pts] Assuming these stars have identical metallicities and abundance patterns, which star, the G2 V or the G2 Ia, will have the stronger Ca II H&K lines in its spectrum? Based on physical principles, explain why this occurs.

2. (XX) Excitation and Ionization– the "Balmer Hydrogens".

   (a) [X pts] Starting with the Boltzmann excitation equation for n_{211}/n_{11}, and the Saha ionization equation for n_{21}/n_{11}, combine them to derive the expression for the quantity n_{211}/n_1, i.e., the number density of neutral hydrogen atoms in the $n = 2$ excitation state relative to the number density of all hydrogens.

   (b) [X pts] Using your expression, for the gas conditions you computed for Problem 2 in Homework 5A, compute n_{211}/n_1. Now compute n_{211}. [I recommend adding this calculation to your existing code]

3. (XX pts) Equlibrium Gas Physics. Consider a gas structure in thermal equilibrium with $T = 6000$ K and $P_g = 100$ dynes cm^{-2} comprising hydrogen, helium, and one metal, calcium, with mass fractions $X = 0.71$, $Y = 0.27$, and $Z = 0.02$. Assume all ionization states for H and He and that Ca can be ionized twice (Ca^0, Ca^+, Ca^{+2}). Compute the equilibrium conditions of the gas using particle-charge conservation,

$$n_e = (n - n_e)\sum_k \alpha_k \sum_{j=1}^{J_k} (j-1) f_{jk}(n_e, T) \tag{2}$$

   In solving Eq. 2, apply a tolerance of at least $n_e^+ - n_e^- = 10^{-5}$, where n_e^+ and n_e^- are the values that bracket the root, n_e. I have provided the solutions to this problem so you can ensure you are obtaining the correct equilibrium solution for all ionization stages of all species. Please present your solutions in the identical format.

(a) [XX pts] Table 1. For the given T and P_g, report the quantities n and the abundance fractions, α_k.

(b) [XX pts] Table 2. Report the equilibrium values $n_{\rm N}$, n_e, $\mu_{\rm N}$, μ_e, μ, $\rho_{\rm N}$, ρ_e, ρ, P_e, and P_e/P_g.

(c) [XX pts] Table 3. Report the equilibrium values n_k, n_{jk}, f_{jk}, $\Phi_{jk}(T)$, $\log U_{jk}(T)$ and fraction of electrons donated for ion j, k,, i.e., $f(n_e, j, k) = (j-1)\, n_{jk}/n_e$.

(d) [XX pts] From Table 3, identify and rank which ionization stages of which species are the top three donors of free electrons to the gas. What do these data tell you about the role of metals in gas where hydrogen and helium are not highly ionized?

4. (XX pts) Equilibrium Gas Physics as a Function of Temperature. Expand your code to compute the equilibrium conditions of this gas structure over the range $1500 \leq T \leq 25,000$ K. I provided solutions to this problem in the form of several plots. Reproduce the plots and hand in your versions. Answer the below questions.

(a) [XX pts] Plot 1: Densities, Molecular Weights, and Pressure
Panel 1a: $\log n_e$, $\log n_N$, and $\log n$
Panel 1b: $\log \rho_e$,$\log \rho_N$, and $\log \rho$
Panel 1c: $\log \mu_e$,$\log \mu_N$, and $\log \mu$
Panel 1d: $P_e/P_{\rm g}$.

(b) [XX pts] Plot 2: Ionization Fractions and Atom/Ion Densities
Panel 2a: f_{jk} for hydrogen; Panel 2b: $n_k j$ and n_{jk} for hydrogen
Panel 2c: f_{jk} for helium; Panel 2d: $n_k j$ and n_{jk} for helium
Panel 2e: f_{jk} for calcium; Panel 2f: $n_k j$ and n_{jk} for calcium.

(c) [XX pts] Plot 3: Fraction Contributions to Free Electrons
Plot 3: $\log f(n_e)$, where $f(n_e) = (j-1)\, n_{jk}/n_e$, for all jk.

(d) [XX pts] Examining Plot 1, explain why n and ρ decrease with increasing T, and why they have the slopes they do.

(e) [XX pts] On Plot 1, what is the physical reason for the rise, turnover, and rise again of n_e and ρ_e across the temperature range 2000-7000 K? (HINT: examine the behavior of Plot 2).

(f) From Plot 3 report which ionization species is the major electron donor as a function of temperature.

(g) Going back to Plot 1, note that $P_e/P_{\rm g}$ rises to 0.5, levels off, and then rises slightly above 0.5. Explain physically why this is happening (i.e., why $\simeq 0.5$, and they slighlty above 0.5?).

YOUR CODES Hand in hard copies of your codes in class. Upload your results to Canvas.