ASTR 545 Fall 2018
Homework 2 (90 points)
ALWAYS SHOW YOUR WORK

1. (10 pts) HR Diagram. Mass, Radius, Luminosity, and Gravity:

(a) [2 pts] A star on the HR diagram has $T = 3000$ K, $L/L_\odot = 10^4$, and $M/M_\odot = 3$. Compute $\log(g/g_\odot)$ for this star.

Assuming $T_\odot = 5780$ K and invoking

$$\log\left(\frac{g}{g_\odot}\right) = \log\left(\frac{M}{M_\odot}\right) - \log\left(\frac{L}{L_\odot}\right) + 4\log\left(\frac{T}{T_\odot}\right)$$

We have

$$\log\left(\frac{g}{g_\odot}\right) = \log(3) - \log\left(10^4\right) + 4\log\left(\frac{3000}{5780}\right) = (0.477) - (4) + 4(-0.284) = -4.662$$

Meaning $g/g_\odot = 10^{-4.662} = 2.18 \times 10^{-5}$. Fun Fact: The sun has $g_\odot/g_\oplus = 27.94$, so this cool, luminous (clearly non-main sequence) star has $g/g_\oplus = 6.1 \times 10^{-4}$! Giant stars have some really low gravity!

(b) [2 pts] If the mass of this star were increased to $M/M_\odot = 30$, but the surface gravity was the same as in part (a), determine the luminosity $L/L_\odot$.

Again, assume $T_\odot = 5780$ K and rearranging

$$\log\left(\frac{L}{L_\odot}\right) = \log\left(\frac{M}{M_\odot}\right) - \log\left(\frac{g}{g_\odot}\right) + 4\log\left(\frac{T}{T_\odot}\right)$$

We have

$$\log\left(\frac{L}{L_\odot}\right) = (1.477) - (-4.662) + 4(-0.284) = 5.001$$

Meaning $L/L_\odot = 10^5$.

(c) [6 pts] Qualitatively describe the "Luminosity Effects at A0" by describing the behavior of the Balmer hydrogen absorption lines in the optical spectra of A0V stars through A0Ia stars. Estimate $\log(g/g_\odot)$ for an A0V star. If the typical A0Ia star has a mass of $M/M_\odot = 10$, estimate $\log(g/g_\odot)$ for an typical A0Ia star. What is the ratio of the two surface gravities. Relate your estimated $\log(g/g_\odot)$ to the observed behavior of the Balmer hydrogen absorption lines.

Assuming $T_\odot = 5780$ K and, from the HR diagram, I estimate $T = 10^4$ K and $\log L/L_\odot = 1.9$ for an A0V star, we have

$$\log\left(\frac{g}{g_\odot}\right) = (0.477) - (1.9) + 4(0.238) = -0.471$$

Adopting $M/M_\odot = 10$, and from the HR diagram, I estimate $T = 10^4$ K and $\log L/L_\odot = 4.5$ for an A0Ia star. We have

$$\log\left(\frac{g}{g_\odot}\right) = (1.0) - (4.5) + 4(0.238) = -2.548$$

The gravity ratios (A0V/AIa) are $10^{-0.471}/10^{-2.548} = 10^{2.548-0.471} = 10^{2.077} \simeq 100$. An A0V star as roughly 100 times greater surface gravity than an A0Ia star. As such, the pressure is greater in an A0V star so that pressure broadening is more prevelent than in the A0Ia star. This results in broader Balmer hydrogen lines in A0V stars.

2. (30 pts) Stellar Spectral Types: As part of this assignment, I have provided six stellar spectra (downloadable files named "starX-spec.txt", where X=1,2,3,4,5,6). The files have two columns: (1) wavelength in nanometers, (2) flux density, f_λ, in units of erg s^{-1}cm^{-2} Å^{-1} times an inconsequential arbitrary constant.

(a) [10 pts] A plot of the six stellar spectra is also provided ("stars1-6.pdf"). On this plot 12 vertical dashed lines are drawn. Identify the twelve lines/features indicated by the vertical dashed lines. Plot "star3" and clearly label your line identifications on the plot.

From blue to red:
1. Balmer break $\lambda3646$;
2. H10 $\lambda3799$;
3. H9 $\lambda3835$;
4. H8 $\lambda3890$;
5. Ca IIK $\lambda3834$;
6. blend of Ca IIH $\lambda3969$ + Hϵ $\lambda3970$;
7. Hδ $\lambda4102$;
8. CH (G) Band $\lambda4300$;
9. Hγ;
10. Hβ;
11. Na ID $\lambda\lambda$ 5891,5898;
12. Hα.

(b) [15 pts] Using the "Arm Chair" spectral classification method, and any additional information you may wish to apply, use your 12 identified lines to estimate the spectral class of each of the six stars. For each star, write out a succinct yet clear explanation of your decision tree of how you arrived at your answers.

The stars were obtain from the ESO on-line standard star archive. Each star has a known spectral and luminosity class. These spectra are low resolution, and as such, the "arm-chair" approach provides insight only into spectral class (as luminosity class effects mostly the Balmer hydrogen lines strengths and the ratios of other weak lines that are not resolved in these spectra). I believe it is also fair to account for the strength of the Balmer decrement for this problem for extra insight. The stars are:
Star1: A0I (strong H lines; strong Balmer decrement; no G-band; K$\ll$H)
Star2: B5I (weak H lines; no K line; weak Balmer decrement)
Star3: F0I (strong H lines; strong H&K; G-band $\ll$ Hγ)
Star4: G2V (weak H lines; G-band $>$ Hγ)
Star5: K4V (no H lines; G-band   Hγ; no TiO bands)
Star6: O5V (weak H; no K lines; no Balmer decrement)
Credit given for being within $\pm$ a 1/2 spectral class if sound logic is provided.

(c) [5 pts] For "star3", convert the flux density from f_λ to f_ν. Normalize the converted spectrum so that $f_\nu = f_\lambda$ at $\lambda = 550$ nm and plot both the f_ν and f_λ versions of the spectrum on a single plot. Explain why the spectral energy distribution shapes differ between the two spectra.

Because of the relationship $f_\nu d\nu = f_\lambda d\lambda$, the flux density in conserved per unit frequency and per unit wavelength. Since the unit frequency interval $d\nu$ scales as $d\lambda/\lambda^2$, the interval $d\nu$ decreases as wavelength increases, so that the energy per second per cm^2 per Hz increases in equal proportion. This creates a shape change in the flux density such that that f_ν is "flatter" than f_λ.

3. (50 pts) Magnitudes and Colors: Compute the Johnson-Cousins B and V magnitudes on the Vega system and then the $B - V$ colors of of the fours star, CD 34d241, HR 1544, HR 3454, and LTT 1020. For this problem you need the filter responses of the B and V Johnson-Cousins filters, the spectra of your target stars, and the spectrum of Vega. All are provided in down-loadable ".txt" files. The filter files have two columns: (1) wavelength in Å, (2) relative response, $R(\lambda)$, normalized to unity at the peak. The Vega spectrum has two columns (1) wavelength in Å, (2) flux density in units erg s^{-1} cm^{-2} Å^{-1}. The target stars have multiple columns but only the first two are needed: (1) wavelength in Å, (2) flux density in units $f_\lambda \times 10^{16}$ erg s^{-1} cm^{-2} Å^{-1}.

(a) [36 pts] Given that the filter response curves are not area normalized, write the equations for how you would compute the Vega V and the Vega B magnitudes of a flux calibrated spectrum of a star. Apply your equations to compute these magnitudes of the four target stars. Report your values. Hand in your well documented code.

CD 34d241: B = 11.69 ; V = 11.21
HR 1544: B = 4.34 ; V = 4.34
HR 3454: B = 4.08 ; V = 4.28
LTT 1020: B = 12.05 ; V = 11.49

(b) [10 pts] Compute $B - V$ for each star. Comparing to Figure 1.5 from Gray (in class notes) and applying your knowledge of the HR diagram, estimate the approximate spectral class and temperature of each of your target stars?

CD 34d241: B - V = 0.48 ; F5 to G0 star; T = 6000 - 7000 K
HR 1544: B - V = 0.00 ; A star; T = 10,000 K
HR 3454: B - V = -0.20 B star; T = 20,000 K
LTT 1020: B - V = 0.56 G star; T= 6000 K

(c) [4 pts] If the color excess is $E(B - V) = 0.3$, what is $(B - V)_0$ for your four target stars? How does this change your conclusion about the spectral classes and temperatures of your target stars?

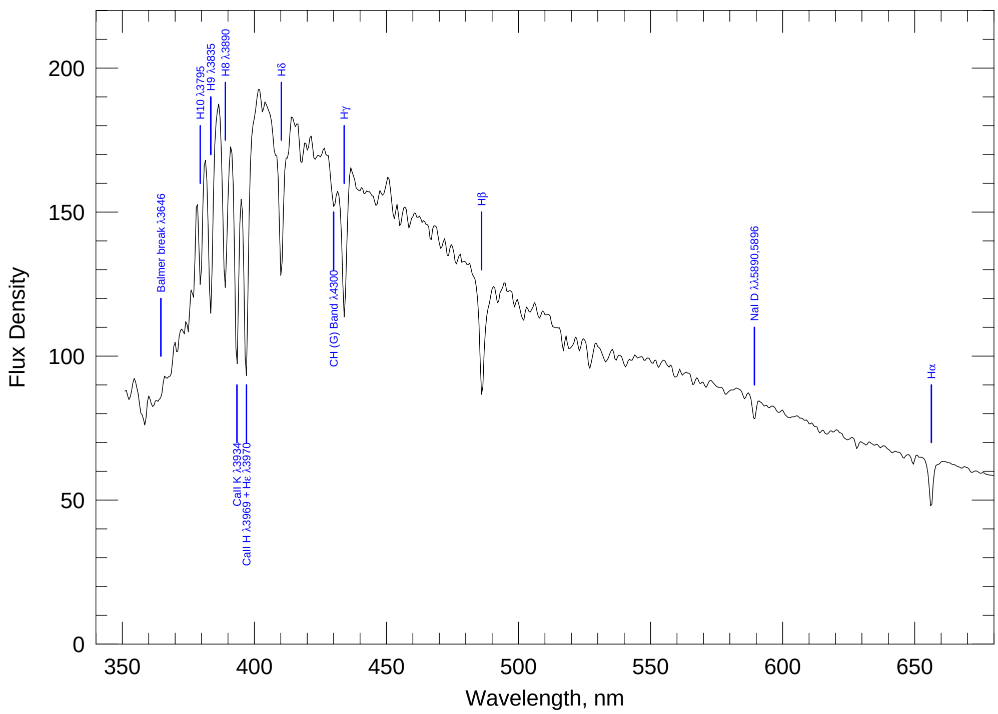

Figure 1: (red-dashed spectrum) Black body curve at $T = 10,000$ K from Eq. 2. (blue solid spectrum) Balmer break at 3646 Å and Balmer hydrogen lines through H40 from Eqs. 3 and 4.