ASTR 545
FALL 2018
Mid-term Exam (170 Points)

1. **True/False Questions. (60 points; 4 points each)** Write a "T" following the statement if *all* the statement is true or an "F" if *any part* of the statement is false. If the statement is false, provide a brief explanation (in complete sentence form) of how/why the statement is false. Stay in the space allotted (be brief and to the point).

(a) The optical portion of a stellar continuum is known as the Balmer continuum.

> FALSE. It is known as the Paschen continuum.

(b) A K0 V star has a redder $B-V$ than a K0 Ia star by about $+0.5$.

> FALSE. A K0 V star has a bluer $B-V$ than a K0 Ia star by about -0.5.

(c) The AB magnitude system is based upon a theoretical source with a constant flux density, f_ν, the value of which is designed to give $V_{\rm vega} = V_{\rm AB}$ in the Johnson-Cousins photometric system.

> TRUE.

(d) On the Strömgren system, the index c_1 is an indicator of a stars metallicity, with a value of $c_1 = 0$ for solar metallicity stars.

> FALSE. The index c_1 measures the strength of the Balmer decrement, and the value $c_1 = 0$ is the maximum value, corresponding to an A0 star.

(e) In a given stellar atmosphere, the thermal Doppler width, $\Delta\lambda_{\rm D}$, of the Hα line will have the same value as that of the Hβ line.

> FALSE. The Doppler width is defined as $\Delta\lambda_{\rm D} = (\lambda/c)b = (\lambda/c)\sqrt{2kT/m}$. Since the temperature T is the same and the mass of the absorbing atom is the same for the two transitions, we see that the Doppler b parameters are the same. However, the Doppler widths in wavelength are not because they are proportional to the transition wavelength. Thus, the Doppler width of Hα is slightly larger than the Doppler width of Hβ.

(f) A bound-bound absorption process is one in which a photon changes direction after collision with a free electron, and to an excellent approximation does not change energy.

> FALSE. A bound-bound transition is one in which an atom changes from one energy state to another via an electron transitioning from one bound energy level to another bound energy level. The energy of the emitted or absorbed photon equals that energy difference of the two energy levels.

(g) The reason the continuum of an O star spectrum has the underlying shape of a blackbody spectrum is that a dominant opacity in the atmosphere is H^- ion scattering, which has a cross section that is flat with wavelength.

FALSE. The reason the continuum of an O star spectrum has the underlying shape of a blackbody spectrum is that a dominant opacity in the atmosphere is Thompson electron scattering, which has a cross section that is flat with wavelength, i.e., $\sigma = (8\pi/3)r_e^2$, where r_e is the classical electron radius.

(h) It is true that $g/g_\odot = (M/M_\odot)^\alpha (L/L_\odot)^{-\beta} (T/T_\odot)^\gamma$ for stars of any spectral class, where g is surface gravity, M is mass, L is luminosity, and where $\alpha = 1$, $\beta = 2$ and $\gamma = 4$.

FALSE. $\beta = 1$. We have $g \propto M/R^2$, and $L \propto R^2 T^4$. This $R^2 \propto L/T^4$, yielding $g \propto MT^4/L$. So, we have $\alpha = 1$, $\beta = 1$ and $\gamma = 4$.

(i) The ionization energy of a bound electron in a hydrogen atom is $\chi = R(1 - 1/n^2)$, where R is the Rydberg constant and n is the principle quantum number of the energy level in which the bound electron resides.

FALSE. This is the excitation energy, not the ionization energy.

(j) For a fixed temperature, as the electron pressure increases, all ionization fractions of a given ion will decrease.

FALSE. What happens is that all ionization get pushed to a higher temperature. So, for a fixed temperature, for upper ionization states the ionization fractions will be relatively smaller, whereas for lower ionization states will become larger. I may need to show this in a class discussion.

(k) The only condition for both the Boltzmann equation and the Saha equation to hold is that the gas must be in radiative equilibrium.

FALSE. The gas must be in thermal equilibrium. This means the kinetic temperatures of the atomic and electron particle fields is equal to the temperature of the radiation field.

(l) The total Damping constant of an absorption line due to a transition from lower level "l" to upper level "u" is the sum of the Einstein A coefficients of spontaneous decay from the upper level plus the sum of the Einstein A coefficients of spontaneous decay from the lower level.

TRUE.

(m) One would expect that rotational broadening of absorption lines in the spectra of G, K, and M stars to be quite common, where as it would be very rare in B, A, and F stars.

FALSE. The case is the opposite. One would expect that rotational broadening of absorption lines in the spectra of B, A, and F stars to be quite common, where as it would be very rare in G, K, and M stars.

(n) The bound-bound Gaunt factor, $g_{\rm II}(n)$, provides the number of possible states that an electron can occupy energy level E_n in a hydrogen atom.

> FALSE. The bound-bound Gaunt factor is denoted $g_{\rm I}(n)$. At any rate, $g_{\rm II}(n)$ is a quantum mechanical correction factor ranging from 0.8 to about 1.3 that provides a correction to Kramer's bound-free cross section for hydrogen.

(o) The total mean molecular weight in a gas depends on $\sum(x_k/A_k)$, and is independent of the ionization conditions in the gas.

> FALSE. The total mean molecular weight depends on both the nuclear particle and the free electron particle mean molecular weight. The nuclear mean molecular weight is independent of the ionization conditions. However, the electron mean molecular weight depends on the number density of free electrons, which is dependent on the ionization conditions of the gas.

2. **Multiple Choice. (50 points; 5 points each)** Mark an "X" in the best response.

(a) The Balmer decrement is due to
[ ] the ionization edge of ground state ($n = 1$) neutral hydrogen
[ ] line blending of higher order Lyman lines
[**X**] the ionization edge of first excited state ($n = 2$) neutral hydrogen
[ ] line blending of higher order Balmer lines
[ ] the Ca II H-K break of cool stars

(b) As surface gravity increases, absorption line shapes change such that
[**X**] the higher $\log g$, the broader the lines
[ ] the lower $\log g$, the broader the lines
[ ] the higher $\log g$, the narrower the lines
[ ] the effects of $\log g$ on line shapes is random
[ ] the line shapes aren't effected by $\log g$

(c) A given stellar spectrum has strong H Balmer lines and G-band absorption that is roughly as strong as that of the H_γ line. The spectral class of the star is roughly
[ ] A0
[ ] A5
[ ] F0
[**X**] F5 (I realize this was an ambiguous condition,
[**X**] G0 therefore, I will accept either answer.)

(d) The *natural* width of an absorption line is on the order of
[ ] 10^{-8} Å
[ ] 10^{-6} Å
[**X**] 10^{-4} Å
[ ] 10^{-2} Å
[ ] 1 Å

(e) Consider an absorption line from, for example, Hα. On the flat part of the curve of growth, for a fixed equivalent width, the larger the Doppler b parameter

[**X**] the smaller the column density will be
[ ] the larger the column density will be
[ ] the column density does not depend on b
[ ] the column density could be larger or smaller (need more information)
[ ] the narrower the absorption profile be

(f) For Vega, which is true on the Vega magnitude system

[ ] $m_U = 0$
[ ] $m_V = 0$
[ ] $m_B = 0$
[ ] $B - V = 0$
[**X**] all of the above are true

(g) Consider a gas in ionization equilibrium at fixed T, P, and mass fractions. Identify the response for which it is true that *all* the listed quantities depend on the ionization conditions.

[ ] $n_{\rm N}$, $\mu_{\rm N}$, n_e, μ_e
[ ] n_k, n_{jk}, f_{jk}, n_e
[ ] $\rho_{\rm N}$, Φ_{jk}, ρ_e, μ_e
[**X**] f_{jk}, n_e, μ_e, n_{jk}
[ ] ρ_e, α_k, f_{jk}, μ_e

(h) Indicate which of the following stellar property or measured value is used for the Morgan-Keenan (MK) classification system.

[ ] mass
[ ] metallicity
[**X**] relative strength/broadening of absorption lines
[ ] color index
[ ] effective temperature

(i) The type of pressure broadening the most strongly influences the shape of an Hα absorption line is

[ ] $n = 2$ linear Stark broadening
[**X**] $n = 3$ resonance broadening
[ ] $n = 4$ quadratic Stark broadening
[ ] $n = 5$ damped broadening
[ ] $n = 6$ van der Waals broadening

(j) If I observe an absorption line with an unsaturated Gaussian core and Lorentzian wings, I am probably seeing _____________ on the _____________ of the cruve of growth.

[ ] a thermally broadened line; linear part
[ ] a thermally broadened line; flat part
[ ] a thermally broadened line; square root part
[**X**] a thermally and pressure broadened line; linear part
[ ] a thermally and pressure broadened line; square root part

3. **Knowledge Spot Check. (60 points; 6 points each)** Provide your best answer to the following short questions. In some cases, the answer can be a direct recall; it's OK to just to give the answer (in a sentence and with units). In other cases, a short statement or calculation may be in order. You can work on scratch paper for this part, but you must provide your complete answer in the space provided, so be precise in your presentation. Do not present derivations of formulae, but do present the formulae you use, if any. Don't be overly detailed, but for full credit **be specific**.

(a) The mean free path of a photon is 2000 meters and the gas has a cross section for absorption of $\sigma = 1 \times 10^{-20}$ cm^2. What is the number density of the absorbing material?

> The relationship between opacity, χ, the mean free path, ℓ, and the product $n\sigma$, is
>
> $$\chi = \frac{1}{\ell} = n\sigma \,,$$
>
> where n is the number density and σ is the cross section for absorption. We obtain
>
> $$n = \frac{1}{\ell\sigma} = \frac{1}{(2\times 10^5 \text{ cm})(1\times 10^{-20} \text{ cm}^2)} = 5\times 10^{14} \text{ cm}^{-3}$$

(b) If the optical depth at wavelength λ in an absorption line is $\tau_\lambda = 2/3$ at the line center, the flux *decrement*, $1 - I_\lambda/I^0_\lambda$, at the line center is?

> For pure absorption, we have $I_\lambda/I^0_\lambda = \exp\{-\tau_\lambda\} = \exp\{-2/3\} = 0.513$. Thus the flux decrement is $1 - 0.513 = 0.487$.

(c) Within an order of magnitude, the the radius, $R/R_\odot$, of an O main sequence star is? Within an order of magnitude, the the radius, $R/R_\odot$, of an M main sequence star is?

> The answer can be read directly off of the HR diagram (in the notes) that provides the lines of constant radii. One can immediately see that the radii of OV stars are roughly $R = 10R_\odot$ and the radii of MV stars are roughly $R = 0.1R_\odot$. Of course, one can apply the relationship $L \propto R^2T^4$ and plug in appropriate values for L and T from the HR diagram. The results should be the same within the precision of an order of magnitude.

(d) The hydrogen mass fraction of a star is $X = 0.7032$ and the mass fraction of the metals is $Z = 0.0219$. What is the mass fraction of helium in this star, Y? If all the metals are iron, what are the abundance ratios, $\alpha_{\rm H}$, $\alpha_{\rm He}$, and $\alpha_{\rm Fe}$?

> From mass density conservation, we invoke $X+Y+Z = 1$ to obtain $Y = 1-X-Z = 0.2749$. Similarly, from number density conservation, we invoke $\alpha_{\rm H} + \alpha_{\rm He} + \alpha_{\rm Fe} = 1$. After some algebra, we obtain
>
> $$\alpha_{\rm H} = \left[1 + \frac{\alpha_{\rm He}}{\alpha_{\rm H}} + \frac{\alpha_{\rm Fe}}{\alpha_{\rm H}}\right]^{-1} ,$$
>
> and from the relationship $x_k = A_k\alpha_k / \sum A_k\alpha_k$, we have
>
> $$\frac{\alpha_{\rm He}}{\alpha_{\rm H}} = \frac{A_{\rm H}}{A_{\rm He}}\frac{Y}{X} = \frac{1}{4}\cdot\frac{0.2749}{0.7032} = 0.0977 \qquad \frac{\alpha_{\rm Fe}}{\alpha_{\rm H}} = \frac{A_{\rm H}}{A_{\rm Fe}}\frac{Z}{X} = \frac{1}{56}\cdot\frac{0.0219}{0.7032} = 0.0006 \,,$$
>
> where the sums have divided out (which works only when there is a single metal species). We thus have $\alpha_{\rm H} = 0.9105$, $\alpha_{\rm He} = 0.0977\alpha_{\rm H} = 0.0890$, and $\alpha_{\rm Fe} = 0.0006\alpha_{\rm H} = 0.0005$, which recovers number density conservation (a good double check!).

(e) Explain the utility of (reason for applying) the Unsöld Approximation when computing the bound-free hydrogen absorption cross section.

The bound-free absorption cross section for a hydrogen atom in excitation state n scales as $(\lambda/n)^3$. Thus, the cross section has a long tail toward shorter wavelengths and the higher order n terms must be included where the lower n terms have highly peaked edges and dominate the cross section at shorter wavelengths. As such, one cannot neglect the low wavelength tails of the higher n contributions. This becomes especially true as temperature increases and the term $\exp\{-\chi_n/kT\} \rightarrow 1$. The Unsöld approximation allows one to account for these tails by providing a simple closed expression for the summation to $n = \infty$.

(f) The difference (or a condition that can be relaxed) between strict thermodynamic equilibrium and local thermodynamic equilibrium is?

In strict thermodynamic equilibrium, the temperatures describing the kinetic energy of the particle field (nuclear and free electrons), the relative excitation and ionization of the nuclear particles, and the radiation field, must all be equal. In the context we have been discussing, the relaxed condition is that the temperature that would describe the radiation field, i.e., the Planck function, can be different than the kinetic, excitation, and ionization temperature of the particle field. (This occurs in stellar atmospheres, when the mean free path of photons is less than the pressure scale height of the atmosphere; in this case the radiation at a given layer comes from lower hotter layer).

(g) Explain what a Voigt profile is (what functions and what physical parameters are incorporated and how)?

The Voigt profile is a function that is the convolution of a Lorentzian and a Gaussian. For the optical depth of absorption lines, the Lorentzian is derived from the natural broadening cross section of bound-bound atomic transitions and is parameterized by the central wavelength, λ, the oscillator strength, f, and the damping constant, Γ. The Gaussian is derived from the line of sight thermal (and possible turbulent) Doppler motions of the absorbing atoms in the gas. A key parameter is the gas temperature, T, which is folded into the thermal component of the Doppler b parameter, $b = \sqrt{2kT/m}$. If turbulence is included then the total b parameter is the quadrature sum of the thermal and turbulent b parameters.

(h) If [X/H] $= -0.3$ is determined for a stellar atmosphere of a distant star, and the solar logarithmic abundance of X is $\hat{A}_x = 8$ on a scale where $\hat{A}_{\rm H} = 12$, what is $n_x/n_{\rm H}$ in the distant star?

From the definition the abundances

$$[\mathrm{X/H}] = -0.3 = \log(n_x/n_{\rm H}) - \log(n_x/n_{\rm H})_\odot = \log(n_x/n_{\rm H}) - (8-12) = \log(n_x/n_{\rm H}) - 4\,,$$

which yields $\log n_x/n_{\rm H} = -4.3$, or $n_x/n_{\rm H} = 5 \times 10^{-5}$.

(i) Someone measures the Balmer decrement in two A0 stars, α Sith and β Jedi. They tell you that that the decrement is stronger in β Jedi than in α Sith. Apply all you know as to what the decrement is physically dependent on, and describe the possible physical differences between α Sith and β Jedi (don't just say "X could be different", but compare the two stars and explain your reasoning).

Option 1:. If both stars are of the same luminosity class, then α Sith has a higher metallicity than β Jedi, giving it a higher n_e, an elevated H^- b-f opacity, and a smaller decrement. Option 2:. If both stars are of the same metallicity, then α Sith has a higher surface gravity than β Jedi giving it a higher n_e, an elevated H^- b-f opacity, and a smaller decrement. Option 3. The differences in the combination of metallicity and surface gravity conspire to give α Sith a higher n_e, an elevated H^- b-f opacity, and a smaller decrement.

(j) Explain/describe what free-free absorption is (draw a picture if need be). In optical and infrared spectra of stars, what is the dominant atom/ion contribution to the free-free absorption cross section? Is this most important in cool stars or hot stars? Explain your answer to the last question.

> Free-free absorption is the process in which a free electron absorbs a photon. A free electron cannot absorb a photon unless a third body, such as an atom or an ion, can conserve angular momentum in the process. Thus, free-free absorption must occur in the presence of an atom or an ion, the most abundant of which are H, H^+, H^-, He, He^+, and He^{+2}. In star, the most prominent free-free absorption comes from the H^- ion in the infrared, as the cross section for the other forms of hydrogen and helium are far less important. Since H^- abundance and therefore opacity, decreases as $n_{H^-}/n_H \propto \exp\{-0.755/kT\}$, we see that it is much more important in cooler stars.