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N.M.S.U. Astronomy Department: Cumulative Exam #368
18^{th} February 2012 – Nicole Vogt

Please start a new page for each problem, and when you are done staple the pages together in order. Keep your work clear and organized, state all assumptions, and show your work. You may use your calculators only as simple calculating machines (do not access constants other than π and e, and do not access stored formulas).

The exam comprises 100 points. I anticipate a passing score for papers marked above 80%.

1. The Canaries Great Telescope is located on the island of La Palma, with a longitude of 18°W and a latitude of 28°N.
   (a) If while observing you notice the third quarter Moon transiting at the zenith, what is the month, approximate day of the month, and time of day? (10 pts)
   (b) Could you observe the Galactic Center during your observing run? If so, at what time, and where would it lie in the sky? Would moonlight be an issue? (10 pts)

2. Which gets more hours of daylight per year, the North Pole or the South Pole? (5 pts)

3. What is the angular resolution of the human eye, in arcseconds? Could an astronaut on Mars separate the Moon from the Earth? (10 pts)

4. How large (in grams) would an asteroid impacting at 100 kilometers per second need to be to destroy the Earth? (Define "destroy" – and keep it simple.) (10 pts)

5. If ice has an albedo of 0.35, at what minimum orbital radius (in A.U.) would a tumbling ice cube stay frozen? (15 pts)

6. What is the maximum stable orbital radius (in A.U.) for solar comets, given the tidal force from Alpha Centauri ($M_{total} = 2\ M_{\odot}$)? How does this compare to the radius of the Oort cloud? (15 pts)

7. In an eclipsing binary, is the deeper eclipse associated with the eclipse of the hotter, the smaller, or the more luminous star? State your argument clearly. (15 pts)

8. Where would a star with a 10% distance error appear on the Hertzsprung-Russell diagram, in magnitudes, relative to its correct location? (10 pts)

Selected Astronomical Measurements and Physical Constants

$M_\odot$	=	1.99×10^{33}	gm	$M_\oplus$	=	5.98×10^{27}	gm
$R_\odot$	=	6.96×10^{10}	cm	$R_\oplus$	=	6.37×10^{8}	cm
$L_\odot$	=	3.90×10^{33}	ergs s^{-1}				
$T_\odot$	=	5780 K					
$M_\odot$	=	4.8 magnitudes					

G	=	6.67×10^{-8}	$cm^3\ gm^{-1}\ s^{-2}$
σ	=	5.67×10^{-5}	erg $cm^{-2}\ s^{-1}\ K^{-4}$
k	=	1.38×10^{-16}	erg K^{-1}

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1. The Canaries Great Telescope is located on the island of La Palma, with a longitude of 18°W and a latitude of 28°N.

   (a) If while observing you notice the third quarter Moon transiting at the zenith, what is the month, approximate day of the month, and time of day?

   The third quarter Moon forms a right triangle with the Sun and Earth, lying in front of the Earth. If the Moon transits at the zenith at 28° then the Earth must be tipped over toward the moon a full 23.5° (the Moon dances up and down by an additional 5°, bringing the maximum declination to $\approx 28°$).

   The Earth's North Pole tips away from the Sun during northern winter, toward the Sun during northern summer, and neither during autumn and spring. It thus points towards the third quarter Moon in September, around the autumnal equinox (September 22-23). The Moon transits around 6 am in this configuration (third quarter).

   (b) Could you observe the Galactic Center during your observing run? If so, at what time, and where would it lie in the sky? Would moonlight be an issue?

   The coordinates of the Galactic Center are (17^h45^m, -29°0′). On the autumnal equinox, an object with a right ascension of 24 hours transits around midnight, and so one at 18 hours transits around sunset. The Galactic Center thus transits at dusk, and lies 28° + 29° = 57° to the south (33° above the southern horizon). The Moon doesn't rise until midnight, so there is no moonlight issue.

2. Which gets more hours of daylight per year, the North Pole or the South Pole?

   Perihelion occurs during January, so northern winter is shorter than southern winter, northern summer is longer than southern summer, and the North Pole should receive more hours of sunlight each year.

3. What is the angular resolution of the human eye, in arcseconds? Could an astronaut on Mars separate the Moon from the Earth?

   Assuming the human eye is diffraction limited,

   $\theta = 1.22 \times \frac{\lambda}{d_{eye}} = 1.22 \times \frac{5{,}000 \times 10^{-8}\,\mathrm{cm}}{0.75\,\mathrm{cm}} = 8.13 \times 10^{-5}\,\mathrm{radians} = 17''.$

   With Mars at opposition,

   $\alpha = \tan^{-1}\left(\frac{r_{E-M}}{d_{M-M}}\right) = \tan^{-1}\left(\frac{3.84 \times 10^5\,\mathrm{km}}{7.80 \times 10^7\,\mathrm{km}}\right) = 4.92 \times 10^{-3}\,\mathrm{radians} = 1{,}015'' = 17'.$

   At conjugation, $d_{M-M} = 3.78 \times 10^8$ km, and $\alpha = 1.02 \times 10^{-3}$ radians, or 3.5′. The separation of the Earth and Moon is thus 12 to 60 times larger than the diffraction limit of the human eye, so keen-eyed astronauts should be able to discern it.

4. How large (in grams) would an asteroid impacting at 100 kilometers per second need to be to destroy the Earth? (Define "destroy" – and keep it simple.)

   Let's assume the Earth is destroyed when it becomes gravitationally unbound. Comparing the binding energy of the planet to the kinetic energy of the impact,

$G\frac{M_\oplus^2}{R_\oplus} = \frac{1}{2}mv^2$

and so

$m = \frac{2GM_\oplus^2}{R_\oplus v^2} = \frac{2\times6.67\times10^{-8}\times(5.98\times10^{27})^2}{6.37\times10^8\times(10^7)^2}\,\frac{\mathrm{cm^3/gm/s^2-gm^2/cm}}{\mathrm{cm^2/s^2}} = 7.49\times10^{25}\,\mathrm{gm}.$

Note that this is equal to the mass of the Moon.

5. If ice has an albedo of 0.35, at what minimum orbital radius (in A.U.) would a tumbling ice cube stay frozen?

   Let's assume the ice cube is spherical. Equating the input and output flux,

   $(1-A_b)\,\frac{L}{4\pi r^2}\times\pi R^2 = 4\pi R^2\epsilon\sigma T^4.$

   Solving for the orbital radius r,

   $r = \sqrt{\frac{(1-A_b)L}{16\pi\epsilon\sigma T^4}}.$

   We might first assume a temperature of 273 K (0° C), so that

   $r = \sqrt{\frac{(1-0.35)\,3.9\times10^{33}}{16\pi\times5.67\times10^{-5}\times273^4}}\sqrt{\frac{\mathrm{erg/s}}{\mathrm{erg/cm^2/s/K^4-K^4}}} = 1.26\times10^{13}\,\mathrm{cm} = 0.84\,\mathrm{A.U.}$

   This seems small! Recalling that ice sublimates at around 175 K in a vacuum,

   $r = \sqrt{\frac{(1-0.35)\,3.9\times10^{33}}{16\pi\times5.67\times10^{-5}\times175^4}}\,\mathrm{cm} = 3.08\times10^{13}\,\mathrm{cm} = 2.05\,\mathrm{A.U.}$

6. What is the maximum stable orbital radius (in A.U.) for solar comets, given the tidal force from Alpha Centauri ($M_{\mathrm{total}} = 2\,M_\odot$)? How does this compare to the radius of the Oort cloud?

   We can calculate the size of the Hill sphere around the Sun as follows. If Alpha Centauri lies at 1.3 parsecs, then

   $R_H = a\sqrt[3]{\frac{M_2}{3(M_1+M_2)}} = 1.3\times206265\sqrt[3]{\frac{1}{3(2+1)}}\,\mathrm{A.U.} = 130,000\,\mathrm{A.U.}$

   The Oort cloud has a radius of 100,00 to 200,000 A.U., in reasonable agreement.

7. In an eclipsing binary, is the deeper eclipse associated with the eclipse of the hotter, the smaller, or the more luminous star? State your argument clearly.

   Let's assume that the first star (S_1) is smaller than the second star (S_2). Then S_1 partly occults S_2, blocking $\Delta F_2 \propto R_1^2 T_2^4$. S_1 is later completely blocked, losing $\Delta F_1 \propto R_1^2 T_1^4$. The stellar size cancels out, leaving a dependency on temperature. There will be a larger decrease in flux when the hotter star is eclipsed.

8. Where would a star with a 10% distance error appear on the Hertzsprung-Russell diagram, in magnitudes, relative to its correct location?

   The luminosity of the star will be

   $L = 4\pi r^2 f$

   and so

   $dL = 8\pi r f\,dr = 2L\,dr = 0.20\,L$ if $dr = 10\%$.

   The change in stellar magnitude will be

   $\Delta m = -2.5\log(1.20) = \pm0.20\,\mathrm{magnitudes},$

   moving the star up or down on the H-R diagram by 0.20 magnitudes.