

N.M.S.U. Astronomy Department: Cumulative Exam #356
23th October 2010 – Nicole Vogt

Please start a new page for each problem, and when you are done staple the pages together in order. Keep your work clear and organized, and state all assumptions. You may use your calculators only as simple calculating machines (do not access constants other than π and e , and do not access stored formulas, including any forms of sums or fits).

The exam comprises 60 points. I anticipate a passing score for papers marked above 75%.

This exam is inspired by the recent publication "The Lick-Carnegie Exoplanet Survey: A $3.1 M_{\oplus}$ Planet in the Habitable Zone of the Nearby M3V Star Gliese 581" concerning the planet Gliese 581g, by Steven Vogt (no relation) and collaborators, accepted for publication in ApJ. For convenience, we have extracted key physical parameters from the 44-page preprint into a data table, shown on the back page.

1. The existence of a habitable-zone planet a mere 20 light-years from Earth begs the question "If we are looking up, who is looking down, and what can they see?" (18 pts total)
 - (a) Which planet in the Sol system would be most likely to be discovered through exoplanetary searches conducted from other stellar systems? (2 pts)
 - (b) Assume that the two stellar systems (Sol and Gliese 581) share a common ecliptic plane. Describe the type of evidence for planets around Sol that might be obtained through solar spectroscopy from the Gliese 581 system, and explain the expected trend. Include a plot, with labels. (4 pts)
 - (c) Calculate the expected shift in the observed velocity of the Sun due to your projected trend. Does this shift fall within the sensitivity limits of our current exoplanetary surveys? (6 pts)
 - (d) At what orbital radius would this planet need to be located to produce a maximum shift of $0.01\text{\AA}$ in the observed wavelength of the $H\alpha$ line in the solar spectrum? (6 pts)
2. Let's now consider a parallel effect that might be observed through imaging. (14 pts total)
 - (a) Describe the event that might reveal the presence of the solar planet through imaging. Include a plot, with labels. (4 pts)
 - (b) Calculate the expected change in the apparent solar magnitude due to your projected trend, observed from Gliese 581. (6 pts)
 - (c) How far could the planet lie above the ecliptic plane for this event to occur? (4 pts)
3. The title of this paper suggests that the planet Gliese 581g lies within a habitable zone. (22 pts total)
 - (a) What is the definition of a habitable zone? (4 pts)
 - (b) Describe one feature of Gliese 581g that has journalists placing the word "habitable" in quotes. (2 pts)
 - (c) Test the authors' claim by calculating the effective temperature of Gliese 581g. (6 pts)
 - (d) How bright is the Sun viewed from Earth? How bright is Gliese 581 from Gliese 581g? What is the ratio of the observed fluxes? (6 pts)
 - (e) How far does Gliese 581 appear to shift across the sky in six months, due to parallax? Could the distance to Gliese 581 have been derived via parallax? (4 pts)
4. The rotational period of Gliese 581 is 94 days. (6 pts total)
 - (a) Derive a lower-limit for all stellar rotation periods, by equating gravity with centrifugal force at the stellar equator. How does this compare to the observed rotational period for Gliese 581? (4 pts)
 - (b) Compare the observed rotational velocity with that caused by Gliese 581g (a few meters per second). Could stellar rotation actually be responsible for the "detection" of this planet? (2 pts)

Selected Physical Constants

For Sol:

$$\begin{aligned} M_{\odot} &= 1.99 \times 10^{33} \text{ gm} \\ R_{\odot} &= 6.96 \times 10^{10} \text{ cm} \\ L_{\odot} &= 3.90 \times 10^{33} \text{ ergs s}^{-1} \\ T_{\odot} &= 5780 \text{ K} \\ M_{\odot} &= 4.8 \text{ magnitudes} \end{aligned}$$

$$\begin{aligned} M_J &= 1.90 \times 10^{30} \text{ gm} \\ R_J &= 7.14 \times 10^9 \text{ cm} \\ x_J &= 7.80 \times 10^{13} \text{ cm} \\ M_{\oplus} &= 5.98 \times 10^{27} \text{ gm} \\ R_{\oplus} &= 6.37 \times 10^8 \text{ cm} \\ A_{\oplus} &= 0.29 \end{aligned}$$

For Gliese 581:

$$\begin{aligned} M_{st} &= 0.31 M_{\odot} \\ R_{st} &= 0.29 R_{\odot} \\ T_{st} &= 3480 \text{ K} \\ P_{st} &= 94 \text{ days} \\ D_{st} &= 20.4 \text{ light-years} \end{aligned}$$

$$\begin{aligned} M_p &= 3.1 M_{\oplus} \\ R_p &= 1.6 R_{\oplus} \\ x_p &= 0.146 \text{ AU} \\ P_p &= 36.56 \text{ days} \end{aligned}$$

$$\begin{aligned} G &= 6.67 \times 10^{-8} \text{ cm}^3 \text{ gm}^{-1} \text{ s}^{-2} \\ \sigma &= 5.67 \times 10^{-5} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-4} \\ k &= 1.38 \times 10^{-16} \text{ erg K}^{-1} \end{aligned}$$

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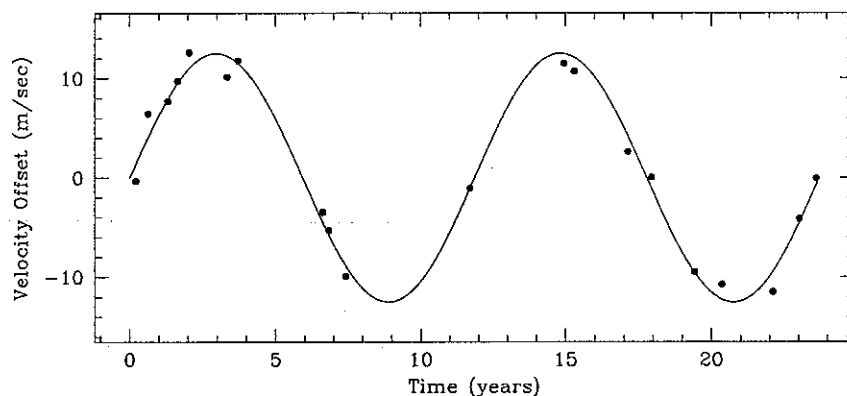
1. The existence of a habitable-zone planet a mere 20 light-years from Earth begs the question "If we are looking up, who is looking down, and what can they see?" (18 pts total)

- (a) Which planet in the Sol system would be most likely to be discovered through exoplanetary searches conducted from other stellar systems? (2 pts)

Jupiter is the most massive planet, and the innermost of the gas giants. It is thus the easiest to detect of the solar planets.

- (b) Assume that the two stellar systems (Sol and Gliese 581) share a common ecliptic plane. Describe the type of evidence for planets around Sol that might be obtained through solar spectroscopy from the Gliese 581 system, and explain the expected trend. Include a plot, with labels. (4 pts)

Solar spectroscopy could reveal a periodic variation in the velocity of Sol, due to its rotation with Jupiter about their common center of mass. One would observe a periodic offset from the systemic velocity, with a period equal to the orbital period of Jupiter and an amplitude defined by the relative masses of Jupiter and Sol and Jupiter's orbital radius.



One might also sketch a plot showing the shift from rest-frame wavelength of a particular absorption feature in the solar spectrum over time.

- (c) Calculate the expected shift in the observed velocity of the Sun due to your projected trend. Does this shift fall within the sensitivity limits of our current exoplanetary surveys? (6 pts)

We begin by calculating the position of the center of mass of the Sol-Jupiter system. As $x_{\odot}M_{\odot} = x_J M_J$,

$$x_{\odot} = x_J \frac{M_J}{M_{\odot}} = 7.80 \times 10^{13} \text{ cm} \left(\frac{1.90 \times 10^{30} \text{ g}}{1.99 \times 10^{33} \text{ g}} \right) = 7.44 \times 10^{10} \text{ cm}$$

which is (slightly) greater than the solar radius.

We next calculate the orbital period of Jupiter, using Kepler's third law.

$$P_J^2 = \frac{4\pi^2}{GM_\odot} x_J^3$$

and so

$$P_J = \sqrt{\frac{4\pi^2}{GM_\odot} x_J^3} = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-8} \times 1.99 \times 10^{33}} (7.80 \times 10^{13})^{\frac{3}{2}} \text{ sec}} = 3.76 \times 10^8 \text{ sec}.$$

The added velocity component will thus have an amplitude V of

$$V = \frac{2\pi x_\odot}{P_J} = \frac{2\pi \times 7.44 \times 10^{10} \text{ cm}}{3.76 \times 10^8 \text{ sec}} = 1.24 \times 10^3 \frac{\text{cm}}{\text{sec}} = 12.4 \frac{\text{m}}{\text{sec}}.$$

Current exoplanetary surveys can detect velocity shifts of a meter per second, so finding a Jupiter at a Jovian radius would not be difficult.

- (d) At what orbital radius would this planet need to be located to produce a maximum shift of $0.01\text{\AA}$ in the observed wavelength of the $\text{H}\alpha$ line in the solar spectrum? (6 pts)

We can connect this shift in velocity with a shift in the observed wavelength of $\text{H}\alpha$ as follows.

$$\frac{V}{c} = \frac{\lambda_o - \lambda_r}{\lambda_r}, \quad \text{and so}$$

$$\lambda_o = \left(\frac{V}{c} + 1\right) \lambda_r = \left(\frac{1.24 \times 10^3}{3 \times 10^{10}} + 1\right) 6563\text{\AA} = 6563.00027\text{\AA}.$$

The shift in wavelength is clearly much less than $0.01\text{\AA}$ for a Jupiter placed at 5.2AU , so we will need to move it inwards towards the Sun. For a shift $\Delta\lambda$ of $0.01\text{\AA}$, we observe that

$$\Delta\lambda = 0.01\text{\AA} = \frac{V\lambda_r}{c} = \frac{\lambda_r}{c} \times \frac{2\pi x_\odot}{P_J} = \frac{\lambda_r}{c} \times \frac{2\pi x_J}{P_J} \times \frac{M_J}{M_\odot}$$

where

$$P_J = \sqrt{\frac{4\pi^2}{GM} x_J^3}$$

and thus

$$\Delta\lambda = \frac{\lambda_r}{c} \times 2\pi x_J \times \frac{M_J}{M_\odot} \times \sqrt{\frac{GM_\odot}{4\pi^2} x_J^{-\frac{3}{2}}} = \frac{\lambda_r}{c} \sqrt{\frac{GM_J^2}{x_J M_\odot}}.$$

Solving for x_J , we see that

$$x_J = \left(\frac{\lambda_r}{\Delta\lambda c}\right)^2 \frac{GM_J^2}{M_\odot} = \left(\frac{6563}{0.01 \times 3 \times 10^{10}}\right)^2 \frac{6.67 \times 10^{-8} \times (1.90 \times 10^{30})^2}{1.99 \times 10^{33}} \text{ cm, and so}$$

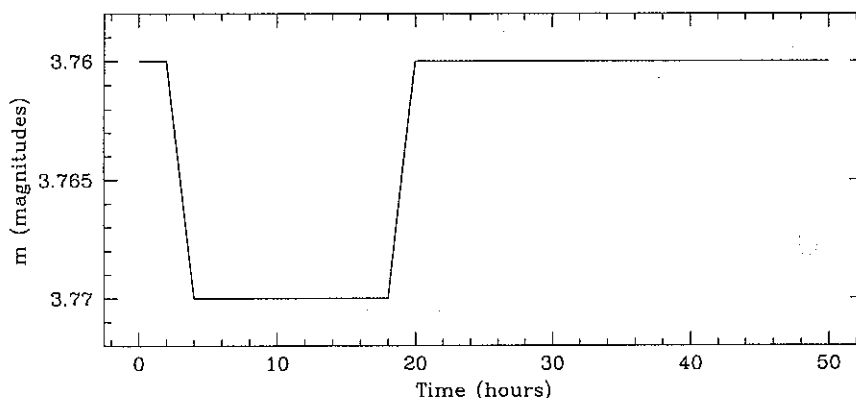
$$x_J = 5.79 \times 10^{10} \text{ cm} < R_\odot.$$

As the proposed position lies well within the radius of the Sun, we would have some difficulty in implementation.

2. Let's now consider a parallel effect that might be observed through imaging. (14 pts total)

- (a) Describe the event that might reveal the presence of the solar planet through imaging. Include a plot, with labels. (4 pts)

During a Jovian transit, Jupiter would pass in front of the Sun and block some of its light from reaching the Gliese 581 system. The apparent magnitude of Sol would thus dip down for almost 15 hours, by 0.05 magnitudes. With an orbital period of 12 years, this would be a rare event indeed.



- (b) Calculate the expected change in the apparent solar magnitude due to your projected trend, observed from Gliese 581. (6 pts)

While transiting, Jupiter would obscure a small portion of the Sun's surface from view. The fractional area hidden A would be

$$A = \frac{\pi R_J^2}{\pi R_\odot^2} = \left(\frac{7.14 \times 10^9}{6.96 \times 10^{10}} \right)^2 = 0.0105, \quad \text{or } 1.05\%.$$

The absolute magnitude of the Sun M is 4.8 magnitudes, so the apparent magnitude m is

$$m = M + 5 \log_{10} \left(\frac{d}{10 \text{ parsecs}} \right), \quad \text{and at a distance of 20.4 light-years,}$$

$$m = 4.8 + 5 \log_{10} \left(\frac{20.4/3.26}{10} \right) = 3.78 \text{ magnitudes.}$$

The change in magnitude due to the change in flux will be

$$\Delta m = -2.5 \log_{10} \left(\frac{0.9895}{1} \right) = 0.011 \text{ magnitudes,}$$

requiring truly precision photometry to be detected.

- (c) How far could the planet lie above the ecliptic plane for this event to occur? (4 pts)

In order for Jupiter to obscure the Sun, it would have to lie no more than a solar radius above or below the plane of the ecliptic. At a distance of 7.8×10^{13} cm,

$$\Theta = 2 \times \arctan \left(\frac{6.96 \times 10^{10}}{7.80 \times 10^{13}} \right) = 6'.$$

3. The title of this paper suggests that the planet Gliese 581g lies within a habitable zone. (22 pts total)

- (a) What is the definition of a habitable zone? (4 pts)

A habitable zone has classically been defined as the radial range around a star within which water could exist in liquid form on a Terrestrial planet. At smaller orbital radii, the stellar flux will boil away any liquid water, while at larger radii it will freeze. This presupposes a carbon-based form for life.

Various attempts have been made to extend the concept of the habitable zone to include the orbits of satellites of gas giants such as Europa and Callista, those which are heated sufficiently by tidal heating to maintain liquid water, and to limit it to the galactic radii at which high-energy radiation would not destroy any life, while remaining at small enough galactic radii that sufficient metals exist for Terrestrial planetary formation.

Venus and Mars are at times included in the habitable zone definition for the solar system.

- (b) Describe one feature of Gliese 581g that has journalists placing the word “habitable” in quotes. (2 pts)
 Gliese 581g is tidally locked to its primary, meaning that one face always sees the star and the other face is always cast in darkness.
- (c) Test the authors’ claim by calculating the effective temperature of Gliese 581g. (6 pts)
 The effective temperature is calculated by equating the stellar flux incident upon the planet with that radiated outwards.

$$F_{in} = (1 - A_b) \frac{L_{st}}{4\pi r_p^2} \pi R_p^2$$

where A_b is the bond Albedo (0.29 for Earth), L_{st} the luminosity of the star, and r_p and R_p the orbital and planetary radii of the planet, and

$$F_{out} = \frac{4\pi R_p^2}{2} \sigma \epsilon T_{eff}^4$$

where σ the Stefan-Boltzmann constant, ϵ is the emissivity, and T_{eff} the effective temperature of the planet. Note the factor of two reduction in the assumed radiative area, as the planet is tidally locked. Equating the input and output fluxes, we see that

$$T_{eff}^4 = \frac{1 - A_b}{8\pi r_p^2} \frac{L_{st}}{\epsilon \sigma} = \frac{1 - 0.30}{8\pi(0.146 \times 1.5 \times 10^{13})^2} \times \frac{0.011 \times 3.9 \times 10^{33}}{5.67 \times 10^{-5}} = 6.28 \times 10^9 \text{ K}^4.$$

assuming $A_b = 0.3$, $\epsilon = 1$, and deriving a stellar luminosity from the radius ($R_{st} = 0.29R_{\odot}$) and the temperature (3480 K) of Gliese 581. The effective temperature is thus $T_{eff} = 257 \text{ K}$, or 216 K if the entire $4\pi R_p^2$ surface is taken to radiate, as Vogt et al. assume.

- (d) How bright is the Sun viewed from Earth, and how bright is Gliese 581 from Gliese 581g, in apparent magnitude? What is the ratio of the observed fluxes? (6 pts)
 We begin by calculating the apparent magnitude of the Sun viewed from Earth.

$$m_{\odot} = M + 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right) = 4.8 + 5 \log_{10} \left(\frac{1.5 \times 10^{13}}{3.09 \times 10^{19}} \right) = -26.77 \text{ magnitudes.}$$

and for Gliese 581g,

$$M = M_{\odot} - 2.5 \log_{10} \left(\frac{0.011 L_{\odot}}{L_{\odot}} \right) = 4.8 + 4.90 = 9.7.$$

$$m_{st} = 9.7 + 5 \log_{10} \left(\frac{0.146 \times 1.5 \times 10^{13}}{3.09 \times 10^{19}} \right) = -26.05 \text{ magnitudes.}$$

The ratio of the observed fluxes is merely

$$\frac{F_{\odot}}{F_{st}} = 10^{-\Delta M/2.5} = 10^{0.72/2.5} = 1.94$$

indicating that the Sun shines almost twice as brightly here on Earth as Gliese 581 does on its sixth planet.

- (e) How far does Gliese 581 appear to shift across the sky in six months, due to parallax? Could the distance to Gliese 581 have been derived via parallax? (4 pts)
 Gliese 581 lies at a distance of 20.4 light-years, or 6.26 parsecs. Its apparent angular shift is thus

$$\alpha = \arctan \left(\frac{2 \times 1.5 \times 10^{13}}{6.26 \times 3.09 \times 10^{18}} \right) = 8.89 \times 10^{-5} \text{ degrees} = 0.320''.$$

putting it easily within the reach of parallax surveys (which extend out more than an order of magnitude further in distance).

4. The rotational period of Gliese 581 is 94 days. (6 pts total)

- (a) Derive a lower-limit for all stellar rotation periods, by equating gravity with centrifugal force at the stellar equator. How does this compare to the observed rotational period for Gliese 581? (4 pts)
Equating gravity and centrifugal force, we see that for a test particle of mass m ,

$$m\omega^2 R = \frac{GMm}{R^2}, \text{ where } \omega^2 R = \frac{4\pi^2 R}{P^2}, \text{ and so}$$

$$P = \sqrt{\frac{4\pi^2 R^3}{GM}} = \sqrt{\frac{3\pi}{G\rho}} = \frac{3.3}{\sqrt{\rho}} \text{ hours}$$

where ρ is measured in units of grams per cubic centimeter, and is of order unity for stars.

The 94-day period of Gliese 581 lies far above this lower limit.

- (b) Compare the observed rotational velocity with that caused by Gliese 581g (a few meters per second). Could stellar rotation actually be responsible for the "detection" of this planet? (2 pts)

$$V_{st} = \frac{2\pi R_{st}}{P_{st}} = \frac{2\pi \times 0.29 \times 6.96 \times 10^{10} \text{ cm}}{94 \times 24 \times 3600 \text{ sec}} = 1.56 \times 10^4 \frac{\text{cm}}{\text{sec}} = 156 \frac{\text{m}}{\text{sec}}$$

which is much larger than the shift in velocity due to Gliese 581g. Stellar rotation is not producing the shift in velocity attributed to this planet.

Selected Physical Constants

For Sol:

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