N.M.S.U. Astronomy Department: Cumulative Exam #327
15^{th} September, 2007 – Nicole Vogt

Please start a new page for each problem, and when you are done staple the pages together in order. Be clear, state all assumptions, and do not hesitate to draw figures. You may use your calculators only as calculating machines (do not access constants other than π and e, and do not access stored formulas).

I anticipate a passing score for papers marked above 80%.

1. (a) Define and relate the terms *linear size* and *angular size.* (10 pts)
   (b) Calculate the angular resolution of the human eye, of the Hubble Space Telescope, and of a large radio telescope. (10 pts)

2. (a) Define the terms *parallax* and *parsec.* (10 pts)
   (b) To how many stars can parallax be successfully applied? What are the two limiting factors? (10 pts)

3. (a) Define the term *Eddington luminosity.* (10 pts)
   (b) A compact accreting object of mass M is radiating at its Eddington luminosity,

$$L_E = \frac{4\pi c G M m_p}{\sigma_T}.$$

   A white-suited astronaut is placed at rest an arbitrary distance from the object. If her cross-sectional area A is 1.5 m^2, what is her maximum mass m, in kg, such that she will not fall onto the object? (15 pts)

4. (a) Define the term *globular cluster.* (10 pts) ✓

   Consider a newly formed globular cluster with a total mass of 10^6 $M_\odot$ and an initial mass function of

$$f = \frac{dN}{dm} = a\,m^{-2.35} \qquad dN = a m^{-2.35} dm$$

   over the mass range 0.1 – 20 $M_\odot$, where m is in units of $M_\odot$. $\int dN =$ total ✱

   (b) Find the value of a. (10 pts)
   (c) Find the mean mass of a star in the cluster. (5 pts)
   (d) Find the total luminosity of the cluster, assuming a mass-luminosity relation $L \propto M^4$. What fraction is contributed by stars above $5M_\odot$? (10 pts)

Selected Physical Constants

c	=	3.00×10^{10}	cm s^{-1}	$M_\odot$	=	1.99×10^{33}	gm
h	=	6.63×10^{-27}	erg s	$L_\odot$	=	3.90×10^{33}	erg s^{-1}
σ	=	5.67×10^{-5}	erg cm^{-2} K^{-4} s^{-1}	$R_\odot$	=	6.96×10^{10}	cm
k	=	1.38×10^{-16}	erg K^{-1}	m_p	=	1.67×10^{-24}	gm
G	=	6.67×10^{-8}	cm^3 gm^{-1} s^{-2}	σ_T	=	6.7×10^{-25}	cm^2

Thompson cross section

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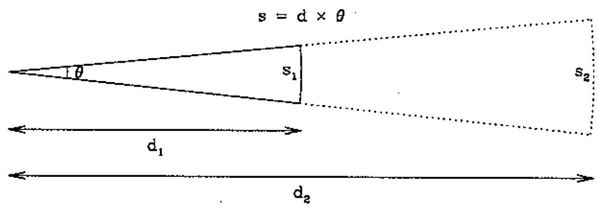
1. (a) **Define and relate the terms *linear size* and *angular size*.**

   (b) **Calculate the angular resolution of the human eye, of the Hubble Space Telescope, and of a large radio telescope.**

The angular size of an object is equivalent to the angle which it subtends on the sky. If the distance to an object is known, an angular size can be translated into a linear size, the length of the object in physical units. For objects at non-cosmological distances,

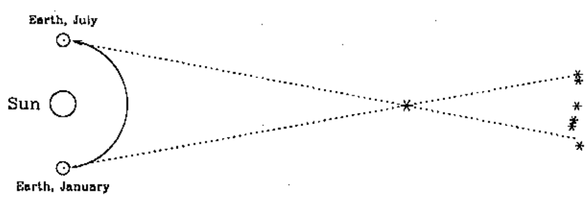


The best possible angular resolution of a lens goes as $\theta = 1.22\frac{\lambda}{d}$ (the diffraction limit), where λ is the wavelength of light under study and d is the diameter of the lens. For mid-range optical light ($\lambda = 5500\AA$), a half-centimeter human eye has a resolution of $28''$ and HST (with a 2.4-meter mirror) has a resolution of $0.06''$. For the Arecibo 300-meter radio telescope, HI (21-cm radiation) is resolved at $2.9'$.

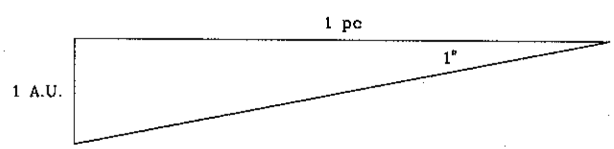
2. (a) **Define the terms *parallax* and *parsec*.**

   (b) **To how many stars can parallax be successfully applied? What are the two limiting factors?**

Parallax is the technique by which the distance to nearby stars can be measured by determining the shift in the apparent stellar positions in the sky relative to distance objects, as the Earth rotates around the Sun. Successive observations made every six months as shown below will find an object at the extrema of this pattern.



A parsec, or parallactic arcsecond, is in fact defined as the distance at which an angle of one arcsecond subtends one A.U., and so an object located a parsec from Earth has a parallax angle of $1''$.



To find out how many distances to stars can be found via parallax, we need to estimate two quantities: the minimum parallactic angle which can be measured ($0.01''$), and the local density of stars (0.1 pc^{-3}). The minimum angle corresponds to a distance of 100 parsecs, giving us a volume which contains 52,000 stars. As a simple check on luminosity limits, we verify that we have no difficulty in detecting individual bright stars out to a this distance (this limit lies well within the Milky Way, and we know that individual stars have been resolved as far away as our neighbor M31). Note that the Hipparcos satellite has indeed measured parallax angles for over 100,000 stars.

3. (a) **Define the term *Eddington luminosity*.**

   (b) **A compact accreting object of mass M is radiating at its Eddington luminosity,**

$$L_E = \frac{4\pi c G M m_p}{\sigma_T}.$$

   A white-suited astronaut is placed at rest an arbitrary distance from the object. If her cross-sectional area A is 1.5 m^2, what is her maximum mass m, in kg, such that she will not fall onto the object?

The Eddington luminosity of an object is the luminosity sufficient to balance its gravitational attraction, which defines a maximum stellar luminosity for stability of the outer stellar shells of a star. A proton at rest above such an object would hang in place, balanced by the equal and opposite forces of gravity and radiation pressure.

We can find the maximum mass of an astronaut in a similar position by comparing her cross-sectional area and mass to those of a proton.

$$R = \frac{m_p}{\sigma_T} \times 2\,\frac{A}{m} = \frac{1.67\times 10^{-24}\ \text{gm}}{6.7\times 10^{-25}\ \text{cm}^2} \times 2\,\frac{1.5\times 10^{4}\ \text{cm}^2}{m\times 10^{3}\ \text{gm}} = 1, \text{ and so}$$

$$m = 75\,\text{kg},$$

where the factor of two is due to the reflective nature of her white space-suit.

4. (a) **Define the term *globular cluster*.**

   Consider a young globular cluster with a total mass of 10^6 M$_\odot$ and an initial mass function of

$$\frac{dN}{dm} = a\,m^{-2.35}$$

   over the mass range 0.1 – 20 $M_\odot$, where m is in units of $M_\odot$.

   (b) **Find the value of a.**

   (c) **Find the mean mass of a star in the cluster.**

   (d) **Find the total luminosity of the cluster, assuming a mass-luminosity relation $L \propto M^4$. What fraction is contributed by stars above 5M$_\odot$?**

A globular cluster is a collection of 10,000 to 500,000 Population II (metal-poor, early forming) stars in a single, self-gravitating spherical cluster less than 100 parsecs in size. Because the stars all formed from the same cloud, they have essentially identical chemical compositions and formed over a relatively short period of time. Variations in properties are thus due primarily to variations in initial mass. Globular clusters hold a hallowed place in astronomical history, having been used to estimate the size of the Galaxy in the 1920 Shapley-Curtis debate.

The 150 globular clusters in the Milky Way are distributed spherically about the center, and can be divided into two populations on the basis of spatial position. (There is also a younger, metal-rich population, which may be associated with the thick disk.) Most are inner clusters, which are old, vary little in age, and orbit in a preferential direction, while the few outer clusters at $\sim$ 100 kpc cover a larger range of ages and have randomly oriented orbits. This suggests that the outer clusters could be dwarf spheroidals, or may have formed elsewhere and been captured by the Milky Way (e.g. tidal stripping of the LMC). Note that individual globular clusters have been successfully observed in other galaxies as well, such as giant ellipticals in the nearby Virgo cluster.

We can solve for the normalization constant a by using the initial mass function to constrain the total cluster mass.

$$M_{tot} = \int_{0.1}^{20} m \times a m^{-2.35}\, dm = a \int_{0.1}^{20} m^{-1.35}\, dm = 5.395\, a = 10^6, \text{ and so}$$

$$a = \frac{10^6}{5.395} = 185,357.$$

The total number of stars is

$$N = \int_{0.1}^{20} am^{-2.35}\, dm = 16.57\, a, \text{ and so the mean mass is}$$

$$\bar{m} = \frac{10^6}{16.57 \times 185{,}357} M_\odot = 0.33\, M_\odot.$$

The total luminosity is found similarly by assuming that all cluster members lie on the Main Sequence and setting $L \propto M^4$, so that $dl = 4m^3 dm$, and

$$L_{tot} = \int_{5^4}^{20^4} l \times a \frac{l^{-\frac{2.35}{4}}}{4\, l^{\frac{3}{4}}}\, dl = \frac{a}{4} \int_{0.1^4}^{20^4} l^{-0.3375}\, dl = 1.299 \times 10^8\, L_\odot, \text{ and for high mass stars}$$

$$L' = \frac{a}{4} \int_{5^4}^{20^4} l^{-0.3375}\, dl = 1.266 \times 10^8\, L_\odot \; (97\%).$$