N.M.S.U. Astronomy Department: Cumulative Exam #322
27^{th} January, 2007 – Nicole Vogt

This exam pays tribute to the work of Albert Einstein, who laid a strong ground-work for much of modern high energy astrophysics. (Somewhere, in a reference frame far, far away, it is his birthday today.)

Please start a new page for each problem, and when you are done staple the pages together in order. You may use your calculators only as calculating machines (do not access constants other than π and e, and do not access stored formulas).

I anticipate a passing score for papers marked above 70%.

1. Use your knowledge of the temperature and the shape of the solar spectrum to estimate Planck's constant, h. (20 pts)
2. The apparent thickness of the Galaxy disk as observed from the vantage point of a relativistic cosmic ray will not be equal to its rest-frame thickness, due to Lorentz contractions. If the disk is of order 100 pc wide, determine its perceived width to an observer traveling with a 10^{20} eV proton moving perpendicular to the disk. (10 pts)
3. The decay time of a 10^{20} eV neutron is roughly 900 seconds in the rest-frame of the particle. How far could it move before undergoing beta-decay? Could it escape the Galaxy? (10 pts)
4. What observational evidence do we have to support the existence of stellar mass black holes? (15 pts)
5. Consider a 1.4 $M_\odot$ neutron star. Estimate the maximum temperature/energy emitted upon impact of a proton which has been tidally stripped from a companion object. Given that strong x-ray fluxes are observed, what can we conclude about the accretion process? (15 pts)
6. What observational evidence do we have to support the existence of supermassive black holes? (15 pts)
7. Do quasars chew their food before swallowing? Let us assume that a quasar consists of a supermassive black hole at the center of a galaxy, fueled by an accretion disk. Estimate the minimum radius r_{tdl} at which the Sun could withstand tidal disruption in orbit around the black hole of mass M_{bh}, and compare it to the Schwarzschild radius r_{sch}. For what mass range of black holes will the stars be tidally disrupted before being accreted, in units of solar masses? (20 pts)

Selected Physical Constants

c	=	3.00×10^{10} cm s^{-1}	$M_\odot$	=	1.99×10^{33} gm
h	=	6.63×10^{-27} erg s	$L_\odot$	=	3.90×10^{33} erg s^{-1}
σ	=	5.67×10^{-5} erg cm^{-2} K^{-4} s^{-1}	$R_\odot$	=	6.96×10^{10} cm
k	=	1.38×10^{-16} erg K^{-1}	m_p	=	1.67×10^{-24} gm
G	=	6.67×10^{-8} cm^3 gm^{-1} s^{-2}	1 eV	=	1.60×10^{-12} erg

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1. **Use your knowledge of the temperature and the shape of the solar spectrum to estimate Planck's constant, h.**

We begin with the Planck function for blackbody radiation,

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(e^{\frac{h\nu}{kT}} - 1\right)^{-1}$$

and find the peak frequency of the blackbody curve by taking the first derivative with respect to frequency and setting it equal to zero.

$$\frac{\partial}{\partial\nu}B_\nu(T) = \frac{2h}{c^2}\left[3\nu^2\left(e^{\frac{h\nu}{kT}} - 1\right)^{-1} - \nu^2\left(e^{\frac{h\nu}{kT}} - 1\right)^{-2}\frac{h\nu}{kT}e^{\frac{h\nu}{kT}}\right] = 0.$$

This tells us that

$$\left(\frac{h\nu}{kT}\right)\frac{e^{\frac{h\nu}{kT}}}{e^{\frac{h\nu}{kT}} - 1} = 3$$

and through iteration, we quickly find that $\frac{h\nu}{kT} \approx 2.8$. Assuming a surface temperature of 5780 K and a peak frequency corresponding to a wavelength of 6000Å, we estimate that

$$h \approx \frac{2.8kT\lambda}{c} \approx 4.47 \times 10^{-27}\,\text{erg-s}$$

or roughly 70% of h.

2. **The apparent thickness of the Galaxy disk as observed from the vantage point of a relativistic cosmic ray will not be equal to its rest-frame thickness, due to Lorentz contractions. If the disk is of order 100 pc wide, determine its perceived width to an observer traveling with a 10^{20} eV proton moving perpendicular to the disk.**

We calculate $\gamma = \frac{1}{\sqrt{(1-(v/c)^2}}$ to be

$$\gamma = \frac{E}{m_oc^2} = \frac{10^{20} \times 1.60 \times 10^{-12}}{1.67 \times 10^{-24} \times (3 \times 10^{10})^2} = 1.07 \times 10^{11}.$$

The apparent width of the Galaxy disk scales as

$$W_{ap} = \frac{W_o}{\gamma} = \frac{100\,\text{pc}}{1.07 \times 10^{11}} = 9.34 \times 10^{-10}\,\text{pc} = 29,000\,\text{km}$$

or roughly the circumference of the Earth.

3. **The decay time of a 10^{20} eV neutron is roughly 900 seconds in the rest-frame of the particle. How far could it move before undergoing beta-decay? Could it escape the Galaxy?**

Utilizing the Lorentz transformation for time,

$$T_{ap} = \gamma T_o = 900\,\text{sec} \times 1.07 \times 10^{11} = 9.63 \times 10^{13}\,\text{sec} = 3.07\,\text{Myr}.$$

Assuming a velocity roughly equal to c, the neutron could travel 0.93 Mpc, far beyond the bounds of the Galaxy.

4. **What observational evidence do we have to support the existence of stellar mass black holes?**

The main challenge in finding stellar mass black holes is to distinguish them from neutron stars. They are detected within binary systems, where the orbital pattern of a stellar companion allows for mass estimation.

Much work has been done at x-ray wavelengths, starting with Cygnus X-1 over thirty years ago. This binary system consists of an OB supergiant and an unseen companion, demonstrating x-ray variability on timescales as small as one millisecond (indicating an extremely compact object). Radial velocity curves suggest a mass greater than or equal to ten $M_\odot$, indicative of a black hole rather than a neutron star.

Similar studies of low mass x-ray binaries, in particular x-ray transients such as V404 Cygni, have produced several tens of additional viable candidates. The transients, where the strong signal from the accretion disk is intermittent, thus allowing the low mass, low luminosity object to be studied at optical wavelengths in the quiescent phase, are the most convincing cases (with confirmed masses as high as $14\,M_\odot$). Advection Dominated Accretion Flow (ADAF) models also suggest that the infalling material is encountering a black hole rather than a neutron star, based upon the observed radiation during the quiescent phase.

Future efforts will doubtless be aided by gravitational wave detectors.

5. **Consider a 1.4 $M_\odot$ neutron star. Estimate the maximum temperature/energy emitted upon impact of a proton which has been tidally stripped from a companion object. Given that strong x-ray fluxes are observed, what can we conclude about the accretion process?**

The maximum energy of the proton can be modeled as

$$kT = \frac{M_{ns} m_p G}{r_{ns}} = \frac{(1.4 \times 1.99 \times 10^{33}) \times 1.67 \times 10^{-24} \times 6.67 \times 10^{-8}}{10^6} = 3.1 \times 10^{-4}\,\text{erg} = 194\,\text{MeV},$$

with an associated temperature $T = 2.25 \times 10^{12}$ K. Radiation is typically observed from neutron star accretion disks in the form of x-rays, four orders of magnitude lower in energy. This suggests that the energy of the infalling material is distributed over a range of radii before being emitted. It may carry too much angular momentum to be deposited directly on the neutron star, and instead lands upon the accretion disk, to be drawn in through dissipation.

6. **What observational evidence do we have to support the existence of supermassive black holes?**

There are several independent lines of evidence supporting the existence of supermasive black holes within galactic nuclei, though none can currently completely eliminate alternative explanations. A key component is spatial compactness, with $10^{12} L_\odot$ sources constrained to regions a light year in size. X-ray observations are important, suggesting variability timescales of less than an hour and gas velocities of thousands of km s^{-1}.

High resolution HST stellar studies (measuring positions and spectroscopic velocities for samples of stars in galactic cores of nearby galaxies) suggest dark masses between 10^6 and $10^9 M_\odot$. Radio and infrared measurements of gas motions have also been made, including studies of stars within the central hundredth of a parsec (less than 10^{12} km) of our own galaxy. Orbital motions areound Sgr A, being closest to us, are of the highest resolution and thus most tightly constrain the central mass.

Water masers have been used successfully to probe gas dynamics with the VLBA at 1.3 cm, producing angular resolution of less than a milliarcsecond (100 times better than that of HST), including the case of the edge-on spiral galaxy NGC 4258.

7. **Do quasars chew their food before swallowing? Let us assume that a quasar consists of a supermassive black hole at the center of a galaxy, fueled by an accretion disk. Estimate the minimum radius r_{tdl} at which the Sun could withstand tidal disruption in orbit around the black hole of mass M_{bh}, and compare it to the Schwarzschild radius r_{sch}. For what mass range of black holes will the stars be tidally disrupted before being accreted, in units of solar masses?**

We visualize a Hill sphere with radius equal to that of the Sun and calculate

$$r_{tdl} = r_\odot \left(\frac{3M_{bh}}{M_\odot}\right)^{1/3}$$

while the Schwarzschild radius is

$$r_{sch} = \frac{2GM_{bh}}{c^2}.$$

Infalling stars will be tidally disrupted for $R_{tdl} > R_{sch}$, where

$$r_\odot \left(\frac{3M_{bh}}{M_\odot}\right)^{1/3} > \frac{2GM_{bh}}{c^2}$$

and

$$M_{bh} < \left[\frac{(3\times10^{10})^2 \times 6.96\times10^{10}}{2\times6.67\times10^{-8}}\right]^{3/2} \sqrt{\frac{3}{1.99\times10^{33}}}, \text{ or}$$

$$M_{bh} < 1.98\times10^8\, M_\odot.$$