CUME 488 Exam: Potential Answers and Rubric

Total Points: 49
Guaranteed Passing: 36.75

Answer 1)
Start with Faraday's Law: $\frac{\partial B}{\partial t} = -\nabla \times E.$

Substitute the generalized Ohm's Law ($E = -v \times B + \eta J + \frac{1}{ne}(J \times B)$) into Faraday's Law:

$$\frac{\partial B}{\partial t} = -\nabla \times (-v \times B + \eta J + \frac{1}{ne}(J \times B))$$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \nabla \times (\eta J) - \nabla \times (\frac{1}{ne}(J \times B))$$

$\nabla \times B = \mu_0 J$
$J = \frac{1}{\mu_0}\nabla \times B.$ substtude for J

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \nabla \times (\eta \frac{1}{\mu_0}\nabla \times B) - \nabla \times (\frac{1}{ne}(\frac{1}{\mu_0}\nabla \times B) \times B)$$

$\eta_{diff} = \eta_{resistivity}/\mu_0\$$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) - \eta_{diff}\nabla \times (\nabla \times B) - \nabla \times (\frac{1}{n_e e \mu_0}(\nabla \times B) \times B)$$

$\nabla \cdot B = 0$ and $\nabla \times (\nabla \times B) = \nabla(\nabla \cdot B) - \nabla^2 B$
so; $\nabla \times (\nabla \times B) = -\nabla^2 B$

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \eta_{diff}\nabla^2 B - \nabla \times (\frac{1}{n_e e \mu_0}(\nabla \times B) \times B)$$

Rubric:
- Substituting Ohm's law into Faraday's law (2 pts)
- Substituting Ampere's law for J (2 pts)
- Correctly applying vector identity and $\nabla \cdot B = 0$ (2 pts)
- Reaching the final correct equation as given in the question (2 pt)

(b) (1 point): $-\nabla \times (\frac{1}{n_e e \mu_0}(\nabla \times B) \times B)$
Rubric: Correct identification (1 pt).

2.
(a)

Answer: $\nabla \times B = \mu_0 J + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$.

Rubric:
"A relationship between circulation of the magnetic field around a closed loop is proportional to the current flowing through that loop": 0.5 pt

$\mu_0 J$: 0.5 point
$\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$: 2 points

(b) The displacement current term,

$$\mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

is typically negligible compared to the conduction current term ($\mu_0 J$) for the low frequencies and slow characteristic velocities ($v \ll c$) typical of MHD phenomena in the solar atmosphere.

In large-scale magnetic events on the Sun (MHD), things generally happen much slower than the speed of light, meaning electric fields don't change extremely rapidly. Because of these slow changes and speeds, the magnetic effect created by changing electric fields is insignificant compared to the magnetic effect created by the actual flow of electric charges (the conduction current). Therefore, for simplicity in studying solar MHD, the displacement current term is usually ignored.

"low frequency variations of the plasma currents and the dominance of free currents over displacement currents.'

Rubric: Statement and regonising electric fields don't change rapidly $\frac{\partial E}{\partial t} <<< 1$; 2 points

3. Reynold's number
(a) Convection (or advection) term: $\nabla \times (v \times B)$. Diffusion term: $\eta \nabla^2 B$.
Correct identification of convection term (1 pt).
Correct identification of diffusion term (1 pt).

(b) R_m compares the magnitude of the convection term to the diffusion term. Let L be a characteristic length scale and V be a characteristic velocity.

Magnitude of convection term $\sim |\nabla \times (v \times B)| \sim \frac{VB}{L}$

Magnitude of diffusion term $\sim |\eta \nabla^2 B| \sim \frac{\eta B}{L^2}$

$$R_m = \frac{\text{Convection}}{\text{Diffusion}} \sim \frac{VB/L}{\eta B/L^2} = \frac{VL}{\eta}$$

Rubric;
R_m is the ratio of convection to diffusion (1 pt).
Stes to divide:1 pt
Shows correct derivation leading to $R_m = VL/\eta$ (1 pt).

(c)
RM >>1: Occurs in regions with high conductivity (low η), large length scales (L), and/or high velocities (V). This applies to most of the solar corona and large portions of the photosphere and chromosphere. Physical meaning: Convection dominates diffusion. The magnetic field is effectively "frozen-in" to the plasma.

R_m << 1: Occurs where resistivity η is high, or length/velocity scales are small. Expected in regions with strong current sheets, sites of magnetic reconnection (like EBs), or weakly ionized regions. Physical meaning: Diffusion dominates convection. The magnetic field can slip through the plasma, allowing field lines to break and reconnect.

Rubric
η low : 1 pt, where it occurs 1 pt.

η high : 1 pt, where it occurs 1 pt.

Part 2: Polarisation and atomic processes (18 points)

4. Zeeman and Stark effects**

(a)
Zeeman splitting: The splitting of atomic energy levels (and spectral lines) into multiple components when an atom is in an external magnetic field. The splitting magnitude is proportional to the B-field strength, and components show specific polarization.
Stark effect: The splitting or shifting of atomic energy levels (and spectral lines) when an atom is in an external electric field. For hydrogen, this is mainly the linear Stark effect (proportional to E).
Rubric: Correct description of Zeeman effect (level splitting by B-field, polarization) (2 pts).
Correct description of Stark effect (level splitting/shifting by E-field) (2 pts).

(b)
Sunspot: Dominated by strong magnetic fields (kG). Zeeman splitting is significant and the primary diagnostic. Stark effect is generally negligible.
Ellerman Bomb: Associated with weaker, complex magnetic fields undergoing reconnection. Zeeman effect is present, but significant E-fields (~100 V/m) due to reconnection can make Stark effects or related phenomena (E-field induced A-O conversion) important for sensitive lines, causing unique polarization signatures (e.g., symmetric Stokes V).

Rubric:
Sunspot: Strong B, dominant Zeeman, negligible Stark (2 pts).
EB: Weaker/complex B, Zeeman present but E-field/Stark effects can become significant/detectable for specific diagnostics (e.g., H lines) due to reconnection (2 pts).

5.

(a)

The Stokes V/I profile for Hϵ (Fig 3d/4c) is largely *symmetric* around the line center, showing a negative peak. This is unusual because a typical Zeeman-induced Stokes V profile is antisymmetric (opposite signs in wings, crossing zero at line center). While field/velocity gradients can cause some asymmetry/NCP, they generally don't produce such a symmetric profile.

Rubric:
Describes observed profile as symmetric (1 pt).
Describes typical Zeeman V profile as antisymmetric (1 pt).
Highlights the symmetry as the key unusual feature (1 pt).
Acknowledges gradient effects but notes they are insufficient (1 pt).

(b)
The authors propose the presence of an ambient electric field at the EB, likely at a magnetic null point. This E-field induces circular polarization via the Alignment-to-Orientation (A-O) conversion mechanism in hydrogen.

Rubric:
Mentions presence of an electric field (1 pt). Mentions Alignment-to-Orientation (A-O) conversion / Stark effect influence (1 pt).

6. Electric fields from VISP
(i) Linear Stark broadening/shifting (2 points):
Signature: Broadening of the Stokes I profile, or shifts in line center position.
Reason: The electric field directly splits/shifts atomic energy levels (linear Stark effect), broadening the range of observed transition energies.
Rubric: Correct signature (broadening/shift) (1 pt) + correct reason (level splitting/shifting) (1 pt).

(ii) Alignment-to-Orientation (A-O) conversion (2 points)
Signature: A symmetric Stokes V/I profile, often with significant Net Circular Polarization (NCP).
Reason: Anisotropic radiation creates atomic alignment. An E-field converts this alignment to orientation, directly producing circular polarization (Stokes V). This is efficient for hydrogen even with weak B-fields if an E-field exists.
Rubric: Correct signature (symmetric V profile/NCP) (1 pt) + correct reason (E-field converts alignment to orientation) (1 pt).

7

(a)

Answer:
Preventing Factors: Expected E-fields were weak and/or confined to very small spatial scales, below the resolution/sensitivity of previous instruments. Detecting the predicted subtle polarization signatures was challenging.

DKIST Advances: DKIST's large 4-meter aperture provides unprecedented spatial resolution. Advanced adaptive optics correct atmospheric blurring. Highly sensitive spectro-polarimeters like ViSP measure the full Stokes vector with high precision across relevant lines, enabling resolution of small structures (EBs) and detection of faint polarization signals indicative of E-fields.

Rubric:
Identifies prior limitations (resolution, sensitivity, weak/small signals) (2 pts).
Identifies DKIST advantages (large aperture/high resolution, high polarimetric sensitivity/ViSP) (2 pts).

(b)
Answer: The distinct symmetric Stokes V/I signal is localized to the EB (~< 1 arcsec).

With 5x poorer resolution, the signal from the small E-field region would be averaged (diluted) with the signal from the surrounding area. The measured amplitude of the symmetric Stokes V/I profile would significantly decrease.

NCP is the integrated V/I. Since the amplitude decreases due to spatial averaging, the total integrated signal (NCP) from the E-field region would also substantially decrease, possibly becoming indistinguishable from noise or averaged Zeeman signals.

Rubric:
Explains effect of poorer resolution (averaging/dilution) (1 pt).
Predicts Amplitude Decrease + Justification (1 pt).
Predicts NCP Decrease + Justification (based on amplitude decrease and averaging) (2 pts).