CUME #380 (SOLUTION)
Saturday, April 27th, 2013
Jim Murphy

The following questions are motivated by the accompanying paper, "Gliese 581d is the first discovered terrestrial-mass exoplanet in the habitable zone", by Wordsworth et al, 2011.

The exam consists of 9 questions totaling 35 points. A percentage score of 70% (24.5 points) will be considered a 'PASS'.

Calculators are to be used ONLY for calculations, not for retrieving stored constants.

You are welcome to include drawings in any of your answers if they aid you in conveying your response or thinking about your response.

Table 1 on page 8 of the paper includes some values relevant to GJ581d. Additionally, a few possibly useful measurements are provided below. I expect that you will know the few other constant values necessary for the calculations. If you do not know a constant value, work the problem to the end with a variable in place of the constant so I can consider some partial credit for you.

Earth's mass = 6×10^{24} kg   1 AU = 1.5×10^{11} meters   Earth's radius = 6378 km
Solar Luminosity = 3.83×10^{26} W   Solar radius = 6.96×10^{8} m   Sun's mass=2×10^{30} kg
GJ581's emission temperature is provided near the end of paragraph 9 in the paper

1) Calculate GJ581d's orbital period. The paper's first paragraph provides some helpful information. [**2 points**]

1 point equation, 1 point calculation

$(M_{GJ581} / M_{Sun})\ P^2 = OSA^3$

$0.31\ P^2 = 0.22^3$

P = 0.185 Earth years = 67.7 Earth days

2) What are the inner and outer bounds (in units of AU) of the Habitable Zone annulus around star GJ581? [For the purpose of this question, assume these Habitable Zone bounds are defined by the planet's Equilibrium Temperature being 373 K or 273 K]. Assume a Bond albedo of 0.2 and unit emissivity.   [**6 points**]

(3 points for setting up, 1.5 point each for calculation).

$(1 - A)\ \pi R^2\ (L_{stellar} / 4\ \pi\ a^2) = (4\ \pi\ R^2\ \sigma\ T_{eq}^4)$

$L_{stellar} = 0.0135 \times 4 \times 10^{26}$ W = 5.4×10^{24} W

$[\, 0.8\, (L_{stellar} / 16\, \pi\, \sigma\, T^4)\,]^{1/2} = a$

For T = 273 K, a = ~0.11 AU
For T = 373 K, a = ~0.06 AU

3) Discuss at least two ways that the presence of an atmosphere can affect the inner and/or the outer boundary extent of the Habitable Zone compared to the values you calculated in 2) above. For this question, Habitable Zone is defined as stated in the paper's first paragraph. [**3 points**]

1.5 point for each valid topic discussed: clouds increase albedo, move boundary outward; greenhouse effect enhances surface temperature and moves boundary radially in; atmospheric heat transport warms cold locations so has possibility of moving boundary radially outward, etc.

i) The presence of clouds can increase the planet's Bond Albedo, which will reduce the surface temperature (other processes being ignored). This will cause a radially inward migration of both the inner and outer bound extents of the Habitable Zone

ii) The presence of a greenhouse-producing atmosphere will warm the surface temperature above the equilibrium temperature. This will cause a radially outward shift of the inner and possibly of the outer bound of the Habitable Zone

iii) other ideas can also be viable... including heat transport by ocean currents if liquid water is persistent, etc.

4) Quantify and describe how the mass-luminosity relation applicable to the information provided in Paragraph 1 in the paper compares with your expectation of the mass-luminosity relationship for a Main Sequence star and for a star of GJ581's mass. You should indicate your expectation for this Main Sequence relationship. Your quantification should include at least 3 digits. [**3 points**]

1.5 point - for establishing the numerical relationship, 1 point - for indicating main sequence value is ~3.5 but does diverge to smaller values at M < ~0.6 Msolar... so the value derived here is larger than would be expected. And in fact is much larger....

Typically, Mass:Luminosity relation for stars on the Main Sequence suggests that $L \propto M^{3.5}$

For the information provided in the first paragraph of Section 1, with M = 0.31 M_{solar} and L = 0.0135 L_{solar}, $L \propto M^{3.675}$

For much of the Main Sequence, L α $M^{3.5}$ so the paper's inferred value of 3.675 is a bit 'off', and interestingly it is 'off' in the wrong direction since at the lower-mass end of the MS the exponent is less than 3.5 since luminosity declines less rapidly at the small end of the mass distribution than it does for M >1 M_{solar}

5) a) What is the bulk density of GJ581d if its mass is the authors' 'used' value as described in paragraph 11 and its radius is the 'Radius (rocky)' value listed in Table 1. Compare your value to Earth's density. [**2 points**]

1 point-calculation, 1 point-Earth's density and comparison

$D = M / V = 8.2\ M_{Earth} / (4/3\ p\ (1.8\ R_{Earth})^3) = 7763\ kg\ m^{-3}$

This value is ~40% larger than Earth's bulk density of 5500 kg m^{-3}

b) Calculate one estimate of the central pressure of GJ581d, using an appropriate equation which provides an estimate of the central pressure of a spherical mass distribution. Describe how and why your calculated pressure value is likely to compare to the actual value. [**3 points**]

2 points – equation and calculation, 1 point – central pressure value's validity
Slab model: $P_{central} = g_{sfc}\ \rho_{bulk}\ R_{planet} = 25\ m\ s^{-2}\ \ 7763\ kg\ m^{-3}\ \ 1.8 \times 6378000\ m = 22.3\ Mbar$

The slab model applied to Earth results in an estimated central pressure of 3.5 Mbar, which is a good representation of Earth's central pressure, so the slab model's application here could be decently representative also

From integration of the hydrostatic equation, for an assumed radially uniform density,

$P_{central} = (3\ G\ M^2_{planet}) / (8\ \pi\ R^4_{planet}) = 11.4\ Mbar$

This equation applied to Earth underestimates the central pressure by a factor of 2 so a similar underestimate is likely here too. This calculation underestimates the central pressure because it does not account for greater material density existing at smaller radii which has the result of increasing the acceleration of mass above

6) Derive Equation (1) in the paper (located on page 3), and discuss the terms used in the derivation and any physical assumptions made in the derivation. Your discussion should indicate if the relaxation timescale (which is in units of time, defined as variable of interest divided by rate of change of that variable) calculated from this equation is more appropriately considered to be an upper (long) limit or a lower (short) limit for the Relaxation Timescale which applies over an extended period of time in to the future. [**4 points**]

3 points for full derivation, 1 point for discussion

$-dT / dt =$ Rate at which energy is lost divided by (Mass to be cooled times energy per unit mass per degree K)

$$dT / dt = - \sigma T_{eff}^4 / [(P_{surface} / g_{sfc}) C_p]$$

Relaxation Time is defined as : quantity of interest / rate of change of quantity

$$\text{Relaxation Time} = T / (-dT/dt) = T_{eff} / [\{ \sigma T_{eff}^4 \} / \{ (P_{surface}/g_{sfc}) C_p \}]$$

$$\tau_{relax} = g_{sfc} \; C_p \; / \; [P_{surface} \; \sigma \; T_{eff}^3]$$

An assumption that is implied is that the atmospheric column is in thermal contact throughout its depth so the entire column can be considered isothermal at T_{eff}

Since temperature will decline with time, the magnitude of τ_{relax} will increase with time, since T is being reduced and thus T^3 will decrease, so the value of τ_{relax} will increase with time as the temperature declines ; so the initial value is a lower limit for the relaxation time (indicates a maximum cooling rate)

7) I have modified the paper so that it does not clearly describe the detection technique that was employed to establish the existence of GJ581d. Quantitatively demonstrate which of the two most common forms of exoplanet detection was likely employed for GJ581d's detection. Describe how your quantitative results indicate which of the two processes is most likely to have produced the GJ581d detection. [**6 points**]

0.5 point - Identify both techniques, 2.5 points each - setting up equation and calculating a value, 1 point – indicating that Radial Velocity is the more likely technique (they should already know this because a planet mass is 'known')

Two most common techniques: Radial Velocity and Transit

Radial velocity for a 67-day 8.2 M_{Earth} planet at 0.22 AU

Stellar OSA: $a_{GJ581d} = M_{GJ81d} / M_{GJ81} \times 0.22$ AU $= 1.77 \times 10^{-5}$ AU $= 2.6 \times 10^6$ meters

Velocity $= 2 \pi a / P = 2 \pi \, 2.6 \times 10^6$ m / (67 days x 86400 s/day) = 2.8 meters per second

2.8 meter per second sensitivity is at the lower end of the capability of current (2007) technology but is achievable and this method is viable

Transit: maximum magnitude = $Area_{GJ581d} / Area_{GJ581} = (1.8 \, R_{Earth})^2 / R^2_{GJ581}$

RGJ581: $0.0135 \, L_{solar} = 4 \pi R^2_{GJ581d} \, \sigma T^4_{GJ581}$

$R_{GJ581} = 2.25 \times 10^5$ km (~30% of the solar radius)

So, transit maximum = $(1.8\ R_{Earth})^2 / (2.2 \times 10^8\ m)^2 = 2.6 \times 10^{-3}$

So the transit depth is only ~0.3% of the stellar flux and the stellar luminosity is itself only 1.35% of the solar luminosity. Since GJ581d's detection preceded KEPLER, it is unlikely that ground-based photometry was able to detect such a small signal, so Radial Velocity is a more likely (and the actual) method of detection.

8) Provide a physically-based discussion which justifies the paper's statement in paragraph 14 that the atmospheres on faster rotating planets are less efficient at directly transporting heat polewards (from equator to pole) than are atmospheres on slowly rotating planets. [**3 points**]

1.5 points – angular momentum mentioned, 1.5 points – increased zonal wind with latitude and physical complications that arise (I intend to be open minded in grading this..)

This is an issue of angular momentum conservation. For an air parcel located the equator and not moving relative to the solid surface below, its angular momentum is:

$AM_{equator}$ (per unit mass) =velocity times radius = $R^2_{planet}\ \omega_{planet}$

As the air parcel moves poleward (providing poleward heat transport), its distance from the rotation axis decreases (in the form of $R = R_{planet} \cos(\textit{latitude})$). Since the parcel must conserve angular momentum, its effective rotational frequency must increase relative to ω_{planet} as R declines. This 'additional' rotational motion is realized as zonal (east, west) flow in the same direction as the planet's rotation. The magnitude of this zonal flow increases with increasing latitude proportional to $(1/\cos(\textit{latitude}))^2$.

The meridional (north-south) gradient of this zonal flow will increase with increasing planet rotation rate. When the gradient achieves a large enough magnitude, instabilities (barotropic) will be generated. These instabilities will have the effect of limiting the latitude extent of the pole-to-equator motion and thus will result in a reduction of the heat transport to high latitudes.

9) Near the end of Paragraph 5 in the paper, N_2 and Ar are referred to as 'buffer gases'. This concept is again mentioned in Paragraph 20. Describe what the term 'buffer gas' is intended to indicate within the context of the paper, and also describe why N_2 and Ar are not directly invoked as greenhouse gases in the 3-D model. [**3 points**]

2 point – correct idea for 'buffer gas', 1 point – correct indication of lack of permanent dipoles

1 point – 'buffer gas' here indicates gases which do not directly play a role in the radiative processes which are the 'greenhouse effect', but rather gases which kinetically interact with

the greenhouse gas molecules (CO_2, H_2O, CH_4, SO_4, etc.) and result in pressure broadening of those relevant lines which enhances their greenhouse capability because the active spectral intervals are pressure broadened.

Ar and N_2 do not play the role of 'greenhouse gases' under most circumstances because they do not possess permanent dipoles