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Co./Dept.

CUME #370

Prepared and administered by Jim Murphy

Saturday, March 31, 2012

The *a priori* anticipated "PASS" grade for this exam is 70%. The Exam offers 52 total points, for which 70% amounts to 36.4 points.

Calculators are **NOT** to be used for retrieving constants or equations, only for calculations.

For any calculations you will conduct below using information from Table 1, do not worry about the error indicated for the mean values provided in the Table unless told to do so.

Write only on a single side of a sheet of paper, with no writing on the other side of the paper. For each Exam Question (1, 2, 3, 4) start with a new blank sheet of paper. When you have completed the exam, staple together IN PROPER ORDER your answer pages.

This exam focuses upon physical aspects of planetary transit observations and also upon the exoplanets described in the accompanying paper, and not upon the statistical arguments for concluding that the two Light Curves presented in Figures 1a and 1b do in fact prove the existence of those two planets. We will assume that Kepler-20e and Kepler-20f do in fact exist.

Some Possibly Useful 'Constants' Values

1 AU = 1.5×10^{11} meters

Solar Luminosity = 3.8×10^{26} W

Gravitational Constant = 6.67×10^{-11} m^3 kg^{-1} s^{-2}

Stefan-Boltzmann Constant = 5.67×10^{-8} W m^{-2} K^{-4}

Mass of a proton = 1.67×10^{-27} kg

Mass of the Sun = 2×10^{30} kg

Radius of the Sun = 6.96×10^{8} m

Earth's gravitational acceleration = 9.8 m s^{-2}

1 parsec = 206265 AU

Earth's hydrostatic surface pressure = 10^5 Pascals = 1013 millibars

Earth's radius = 6378 km

Earth's mass = 6×10^{24} kg

QUESTION 1: Total of 15 points

1 a) Using the relevant planet orbit period and stellar mass information in Table 1, **calculate** the corresponding orbital semi-major axis lengths for Kepler-20e and Kepler-20f. Your final answers should be in units of AU. Show your work. [**3 pts**]

b) Using the Kepler-20 Luminosity information provided in Table 1 and your orbital semi-major axes answer from 1a above, **calculate** the value of the planet Bond Albedo that has been assumed for either of these two planets, Kepler-20e **or** Kepler-20f. (In fact, the same value has been assumed to apply to both planets). Show your work. [**7 pts**]

c) Discuss the reasonableness of your determined Bond Albedo value from 1b. [**2 pts**]

d) Calculate the minimum required diameter of a telescope, located in orbit around Earth, needed to individually resolve planet Kepler-20f distinct from its parent star Kepler-20, at an observed wavelength of 600 nanometers (*the wavelength of greatest sensitivity within Kepler's 420-900 nanometer bandpass*). [**3 pts**]

Question 2: Total of 19.0 points

2 a) Consider the light curve of Kepler-20e presented in Figure 1a, which spans the midpoint of its transit +/- ~4 hours. **Describe** what physical processes are dictating the temporal nature of and the shape of the Light Curve during the time periods **A, B, C, D,** and **E** as I have labeled them in the Figure. Your answer should address why the Light Curve during time interval **C** is not a straight horizontal line and why there is some curvature to the Light Curve during time intervals **B** and **D.** [**5 pts**]

b) The light curves presented in the paper (Figure 1a and 1b) span only a short portion of the total orbit period for each of the two planets. **Draw** a light curve that spans the entire ~6.1 Earth Day orbit period for Kepler-20e. My primary interest is any potential observed periodic structure (variation in received total flux) in this light curve during the time period away from the ~8-hour time period displayed in Figure 1a. You should augment your drawn light curve (*be sure you label and quantify the axes!*) with a **description** of what you intend your Light Curve to display, and why the Light Curve should have that/those variations and how their magnitude(s) might compare to the Flux variation magnitude in Figure 1a (you do not need to calculate any specific values). [**5 pts**]

c) Using the information provided in Table 1, **calculate** the Synodic Period for Kepler-20e and Kepler-20f as they orbit their parent star. Your answer should be in units of Earth Days. Additionally, **draw** a "looking down from above" picture of the orbits of Kepler-20e and Kepler-20f around their parent star, and indicate the orbital azimuth positions where the first and second orbital conjunctions of Kepler-20e and Kepler-20f will occur relative to an initial conjunction location which you specify. [**5 pts**]

d) For the time interval covered by the eight observation quarters (670 Earth Days) of Kepler observations that were analyzed for this paper, **estimate** the probability that a Kepler-observed mutual transit of Kepler-20e and Kepler-20f would have occurred. Your answer should provide a quantitative probability value, and you should explain your thinking/reasoning in arriving at this value. Assume that there were no gaps in Kepler measurements (~30 minute cadence) during these eight observation quarters. [**4 pts**]

QUESTION 3: Total of 12 Points

3**a**) The text (4th page) includes a statement suggesting that Kepler-20f could possess a water vapor ("steam") atmosphere while Kepler-20e is unlikely to possess such an atmosphere, with the difference being due to atmospheric escape. Quantitatively describe why this statement has validity. You do not need to calculate numbers but you should describe how the magnitudes of the important physical terms for escape compare for the two planets. [**4 pts**]

b) If we ignore the idea of a steam atmosphere, and instead assume that Kepler-20f possesses an atmosphere identical in composition to Earth's atmosphere, **describe** how the Kepler transit Light Curve for an 'Earth Atmosphere' Kepler-20f planet might differ from the Kepler-20f light curve presented in Figure 1b? Why might an 'Earth Atmosphere" Kepler-20f light curve be deeper or shallower than that shown in Figure 1b? [**4 pts**]

c) The text (page 4) suggests that Kepler-20f's water vapor atmosphere could equal 0.05 Earth masses, and that such an atmosphere would "*protect the planet surface from further vaporization*". **Calculate** the hydrostatic surface pressure that such an atmosphere would produce on Kepler-20f (assume Kepler-20f's gravitational acceleration equals Earth's gravitational acceleration) and then **describe** how such an atmosphere could 'protect the planet's surface from further evaporation'. [**4 pts**]

QUESTION 4: Total of 6 Points

4) The caption for Figure 3 indicates that, "The non-solid lines are mass-radius relations for differentiated planets: ...".

a) **Explain** what the term "DIFFERENTIATED" indicates in this context, [**2 pts**] and then

b) **provide a brief discussion** in which you **describe** why or why not the assumption of 'differentiated' is likely to be valid for Kepler-20e and Kepler-20f based upon their inferred physical characteristics as presented in Table 1 and knowledge of planet formation processes. [**2 pts**]

c) If Kepler-20e is NOT differentiated, **explain** how this physical condition of that planet could cause its total bulk density to differ from its total bulk density in a differentiated condition. [**2 pts**]

LETTER

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Two Earth-sized planets orbiting Kepler-20

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Since the discovery of the first extrasolar giant planets around Sun-like stars[1,2], evolving observational capabilities have brought us closer to the detection of true Earth analogues. The size of an exoplanet can be determined when it periodically passes in front of (transits) its parent star, causing a decrease in starlight proportional to its radius. The smallest exoplanet hitherto discovered[3] has a radius 1.42 times that of the Earth's radius ($R_\oplus$), and hence has 2.9 times its volume. Here we report the discovery of two planets, one Earth-sized ($1.03R_\oplus$) and the other smaller than the Earth ($0.87R_\oplus$), orbiting the star Kepler-20, which is already known to host three other, larger, transiting planets[4]. The gravitational pull of the new planets on the parent star is too small to measure with current instrumentation. We apply a statistical method to show that the likelihood of the planetary interpretation of the transit signals is more than three orders of magnitude larger than that of the alternative hypothesis that the signals result from an eclipsing binary star. Theoretical considerations imply that these planets are rocky, with a composition of iron and silicate. The outer planet could have developed a thick water vapour atmosphere.

Precise photometric time series gathered by the Kepler spacecraft[5] over eight observation quarters (670 days) have revealed five periodic transit-like signals in the G8 star Kepler-20, of which three have been previously reported as arising from planetary companions[4] (Kepler-20 b, Kepler-20 c and Kepler-20 d, with radii of $1.91R_\oplus$, $3.07R_\oplus$ and $2.75R_\oplus$, and orbital periods of 3.7 days, 10.9 days and 77.6 days, respectively). The two, much smaller, signals described here recur with periods of 6.1 days (Kepler-20 e) and 19.6 days (Kepler-20 f) and exhibit flux decrements of 82 parts per million (p.p.m.) and 101 p.p.m. (Fig. 1), corresponding to planet sizes of $0.868^{+0.074}_{-0.096}R_\oplus$ (potentially smaller than the radius of Venus, $R_{Venus} = 0.95R_\oplus$) and $1.03^{+0.10}_{-0.13}R_\oplus$. The properties of the star are listed in Table 1.

A background star falling within the same photometric aperture as the target and eclipsed by another star or by a planet produces a signal that, when diluted by the light of the target, may appear similar to the observed transits in both depth and shape. The Kepler-20 e and Kepler-20 f signals have undergone careful vetting to rule out certain false positives that might manifest themselves through different depths of odd- and even-numbered transit events, or displacements in the centre of light correlated with the flux variations[6]. High-spatial-resolution imaging shows no neighbouring stars capable of causing the signals[4]. Radial-velocity measurements based on spectroscopic observations with the Keck I telescope rule out stars or brown dwarfs orbiting the primary star, but they are not sensitive enough to detect the acceleration of the star due to these putative planetary companions[4].

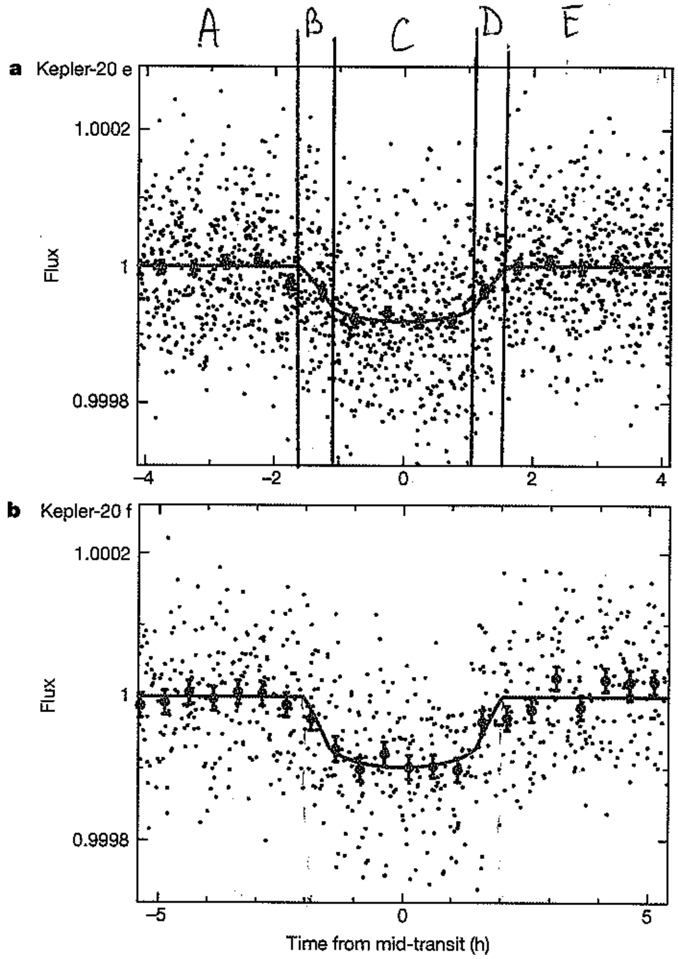

Figure 1 | Transit light curves. Kepler-20 (also designated as KOI 070, KIC 6850504 and 2MASS J19104752+4220194) is a G8V star of Kepler magnitude 12.497 and celestial coordinates right ascension $\alpha = 19$ h 10 min 47.5 s and declination $\delta = +42° 20' 19.38''$. The stellar properties are listed in Table 1. The photometric data used for this work were gathered between 13 May 2009 and 14 March 2011 (quarter 1 to quarter 8), and comprise 29,595 measurements at a cadence of 29.426 min (black dots). The Kepler photometry phase-binned in 30-min intervals (blue dots with 1σ standard error of the mean (s.e.m.) error bars) for Kepler-20 e (**a**) and Kepler-20 f (**b**) is displayed as a function of time, with the data detrended[4] and phase-folded at the period of the two transits. Transit models (red curves) smoothed to the 29.426-min cadence are overplotted. These two signals are unambiguously detected in each of the eight quarters of Kepler data, and have respective signal-to-noise ratios of 23.6 and 18.5, which cannot be due to stellar variability, data treatment or aliases from the other transit signals[4].

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Table 1 | Stellar and planetary parameters for Kepler-20

Stellar properties	Kepler-20	
Effective temperature, T_{eff}	$5,466 \pm 93$ K	
Surface gravity log[g (cm s^{-2})]	4.443 ± 0.075	
Metallicity [Fe/H]	0.02 ± 0.04	
Projected rotational velocity, $v\sin i$	0.4 ± 0.5 km s^{-1}	
Stellar mass, M_s	$(0.912 \pm 0.035)M_{\odot}$	
Stellar radius, R_s	$0.944^{+0.060}_{-0.095}R_{\odot}$	
Stellar density, ρ_s	1.51 ± 0.39 g cm^{-3}	
Luminosity, L_s	$(0.853 \pm 0.093)L_{\odot}$	
Distance, D	290 ± 30 pc	
Planetary parameters	**Kepler-20 e (KOI 070.04)**	**Kepler-20 f (KOI 070.05)**
Orbital period, P	6.098493 ± 0.000065 days	19.57706 ± 0.00052 days
Time of centre of transit, T_c	$2,454,968.9336 \pm 0.0039$ BJD	$2,454,968.219 \pm 0.011$ BJD
Eccentricity, e	<0.28	<0.32
Planet/star radius ratio, R_p/R_s	$0.0084.1^{+0.00035}_{-0.00054}$	$0.01002^{+0.00063}_{-0.00077}$
Scaled semi-major axis, a/R_s	[redacted]$^{0.21}_{-0.29}$	[redacted]$^{0.47}_{-0.63}$
Impact parameter, b	$0.630^{+0.070}_{-0.053}$	$0.727^{+0.054}_{-0.053}$
Orbital inclination, i	$87.50^{+0.33}_{-0.34}$ degrees	$88.68^{+0.14}_{-0.17}$
Planetary radius, R_p	$0.868^{+0.074}_{-0.096}R_{\oplus}$	$1.03^{+0.10}_{-0.13}R_{\oplus}$
Planetary mass, M_p	$<3.08M_{\oplus}$ (spectroscopic limit); $0.39M_{\oplus} < M_p < 1.67M_{\oplus}$ (theoretical considerations)	$<14.3M_{\oplus}$ (spectroscopic limit); $0.66M_{\oplus} < M_p < 3.04M_{\oplus}$ (theoretical considerations)
Planetary equilibrium temperature, T_{eq}	$1,040 \pm 22$ K	705 ± 16 K

$M_{\odot}$, mass of the Sun; $R_{\odot}$, radius of the Sun. The effective temperature, surface gravity, metallicity and projected rotational velocity of the star were spectroscopically determined[21] from our Keck/HIRES spectrum. With these values and the use of stellar evolution models[22], we derived the stellar mass, radius, luminosity, distance and mean density. The transit and orbital parameters (period, time of centre of transit, radius ratio, scaled semi-major axis, impact parameter and orbital inclination) for the five planets in the Kepler-20 system were derived jointly based on the Kepler photometry using a Markov-chain Monte Carlo procedure with the mean stellar density as a prior[4]. The parameters above are based on an eccentricity constraint: that the orbits do not cross each other. After calculating the above parameters, we performed a suite of N-body integrations to estimate the maximum eccentricity for each planet consistent with dynamical stability[4]. The N-body simulations provide similar constraints on the maximum eccentricity and justify the assumption of non-crossing orbits. The planetary spectroscopic mass limits are the 2σ upper limits determined from the radial velocity analysis based on the Keck radial velocity measurements. Planet interior models provide further useful constraints on mass and inferences on composition[23]. Assuming Kepler-20 e and Kepler-20 f are rocky bodies comprised of iron and silicates, and considering the uncertainty on their radii, the planet masses are constrained to be $0.39M_{\oplus} < M_p < 1.67M_{\oplus}$ for Kepler-20 e, and $0.66M_{\oplus} < M_p < 3.04M_{\oplus}$ for Kepler-20 f. The lower and upper mass bounds are set by a homogeneous silicate composition and by the densest composition from a model of planet formation with collisional mantle stripping[24]. The planet equilibrium temperatures assume an Earth-like Bond albedo of [redacted] isotropic redistribution of heat for reradiation, and a circular orbit. The errors in these quantities reflect only the uncertainty due to the stellar luminosity.

To establish the planetary nature of these signals with confidence we must establish that the planet hypothesis is much more likely than that of a false positive. For this we used the BLENDER procedure[7–9], a technique used previously to validate the three smallest known exoplanets, Kepler-9 d (ref. 8), Kepler-10 b (ref. 3), and CoRoT-7 b (ref. 10). The latter two were also independently confirmed with Doppler studies[3,11]. We used BLENDER to identify the allowed range of properties of blends that yield transit light curves matching the photometry of Kepler-20 e and Kepler-20 f. We varied as free parameters the brightness and spectral type (of the stars) or the size (for the planetary companions), the impact parameter, the eccentricity and the longitude of periastron. We simulated large numbers of these scenarios and compared the resulting light curves with the observations. We ruled out fits significantly worse (at the 3σ level, or greater) than that of a true transiting planet around the target, and we tabulated all remaining scenarios that were consistent with the Kepler light curves.

We assessed the frequency of blend scenarios through a Monte Carlo experiment in which we randomly drew 8×10^5 background main-sequence stars from a Galactic structure model[12] in a one-square-degree area around the target, and assigned them each a stellar or planetary transiting companion based on the known properties of eclipsing binaries[13] and the size distribution of planet candidates as determined from the Kepler mission itself[14]. We counted how many satisfy the constraints from BLENDER as well as observational constraints from our high-resolution imaging observations and centroid motion analysis[4], and made use of estimates of the frequencies of larger transiting planets and eclipsing binaries (see Fig. 2). In this way we estimated a blend frequency of background stars transited by larger planets of 2.1×10^{-7} and a blend frequency of background eclipsing binaries of 3.1×10^{-8}, yielding a total of 2.4×10^{-7} for Kepler-20 e. Similarly, $4.5 \times 10^{-7} + 1.26 \times 10^{-6}$ yields a total blend frequency of 1.7×10^{-6} for Kepler-20 f.

Another type of false positive consists of a planet transiting another star physically associated with the target star. To assess their frequency we simulated 10^6 such companions in randomly oriented orbits around the target, based on known distributions of periods, masses and eccentricities of binary stars[13]. We excluded those that would have been detected in our high-resolution imaging or that would have an overall colour inconsistent with the observed colour of the target, measured between the Sloan r band (12.423 ± 0.017; ref. 14) and the Warm Spitzer 4.5-μm band (10.85 ± 0.02; ref. 4). We used BLENDER to determine the range of permitted sizes for the planets as a function of stellar mass, and to each we assigned an eccentricity drawn from the known distribution for close-in exoplanets[15]. The frequency of blends of this kind is 5.0×10^{-7} for Kepler-20 e, and 3.5×10^{-6} for Kepler-20 f. Summing the contributions of background stars and physically bound stars, we find a total blend frequency of 7.4×10^{-7} for Kepler-20 e and 5.2×10^{-6} for Kepler-20 f.

We estimated the a priori chance that Kepler-20 has a planet of a similar size as implied by the signal using a 3σ criterion as in BLENDER, by calculating the fraction of Kepler objects of interest in the appropriate size range. We counted 102 planet candidates in the radius range allowed by the photometry of Kepler-20 e, and 228 for Kepler-20 f. We made the assumption that only 10% of them are planets (which is conservative in comparison to other estimates of the false positive rate that are an order of magnitude larger[16]). From numerical simulations, we determined the fraction of the 190,186 Kepler targets for which planets of the size of Kepler-20 e and Kepler-20 f could have been detected (17.4% and 16.0%, respectively), using actual noise levels. We then calculated the planet priors (the a priori chance of a planet) to be $(102 \times 10\%)/(190,186 \times 17.4\%) = 3.1 \times 10^{-4}$ for Kepler-20 e, and $(228 \times 10\%)/(190,186 \times 16.0\%) = 7.5 \times 10^{-4}$ for Kepler-20 f. These priors ignore the fact that Kepler-20 is more likely to have a transiting planet at the periods of Kepler-20 e and Kepler-20 f than a random Kepler target, because the star is already known to have three other transiting planets, and multi-planet systems tend to be coplanar[17]. When accounting for this using the procedure described for the validation of Kepler-18 d (ref. 18), we find that the flatness of the system increases the transit probability from 7.7% to 63% for Kepler-20 e, and from 3.7% to 35% for Kepler-20 f. With this co-planarity boost, the planet priors increase to 2.5×10^{-3} for Kepler-20 e and 7.1×10^{-3} for Kepler-20 f. Comparing this with the total

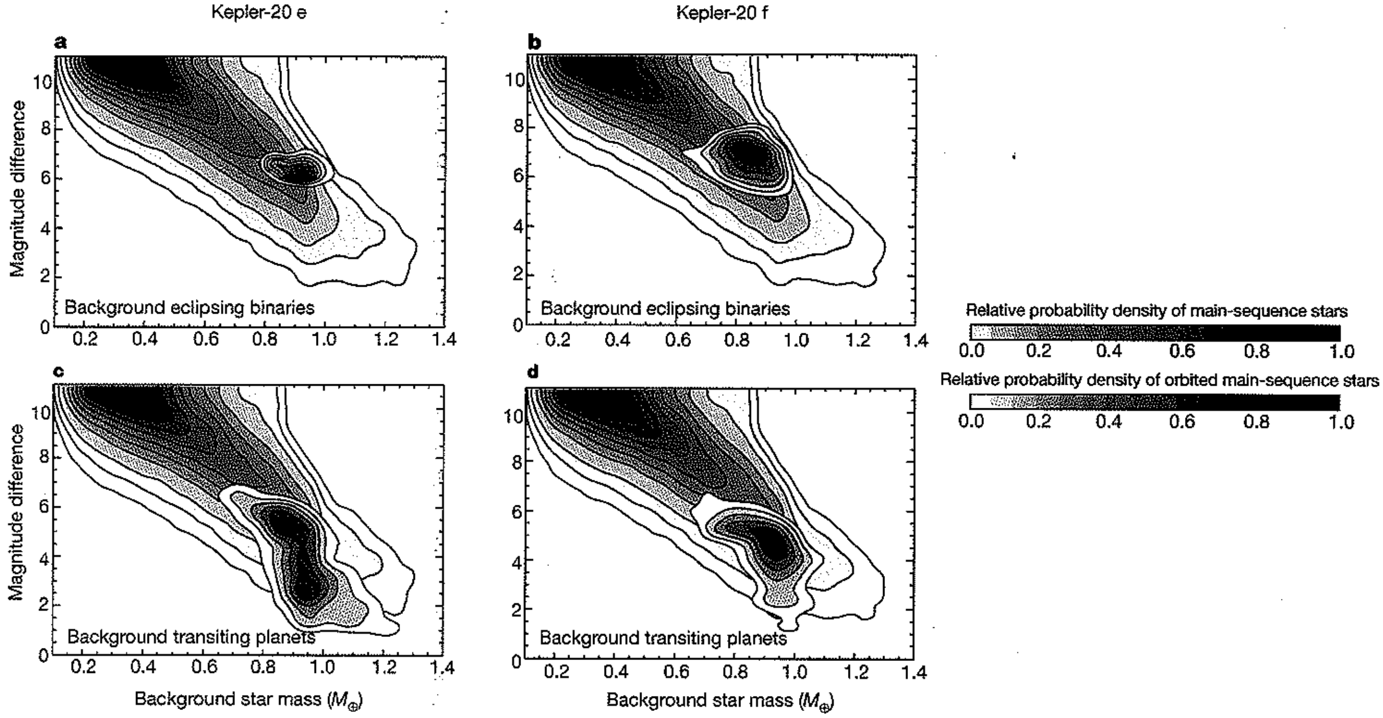

Figure 2 | Density map of stars in the background of Kepler-20. The blue-shaded contours correspond to main-sequence star counts from the Besancon model in the vicinity of Kepler-20, as a function of stellar mass and magnitude difference in the Kepler passband compared to Kepler-20. The red-shaded contours represent the fractions of those stars orbited by another smaller star (**a** and **c**) or by a planet (**b** and **d**) with sizes such that the resulting light curves mimic the transit signals for Kepler-20 e and Kepler-20 f. The displacement of the blue and red contours in magnitude and spectral type results in very small fractions of the simulated background stars being viable false positives for Kepler-20 e (1.6% when transited by a planet, and 0.1% when transited by a smaller star). We obtained similar results for Kepler-20 f (2.1% when transited by a planet, and 3.1% when transited by a smaller star). Most of these background stars have masses (spectral types) near that of the target, and are two to seven magnitudes fainter. The above fractions are further reduced because background stars able to match the signals but that are also bright enough and at large enough angular separation from the target would have been detected in our imaging observations and/or centroid motion analysis. Finally, to obtain the blend frequencies we scaled these estimates to account for the fraction of background stars expected to have transiting planets (1.29%, the ratio between the number of Kepler objects of interest and the total number of Kepler targets[25]) or stellar companions (0.79% based on the statistics of detached eclipsing binaries in the Kepler field[26]). We examined non-main-sequence stars as alternatives to either object of the blend eclipsing pair, but found that they either do not reproduce the observed transit shape well enough, or are much less common (<1%) than main-sequence blends.

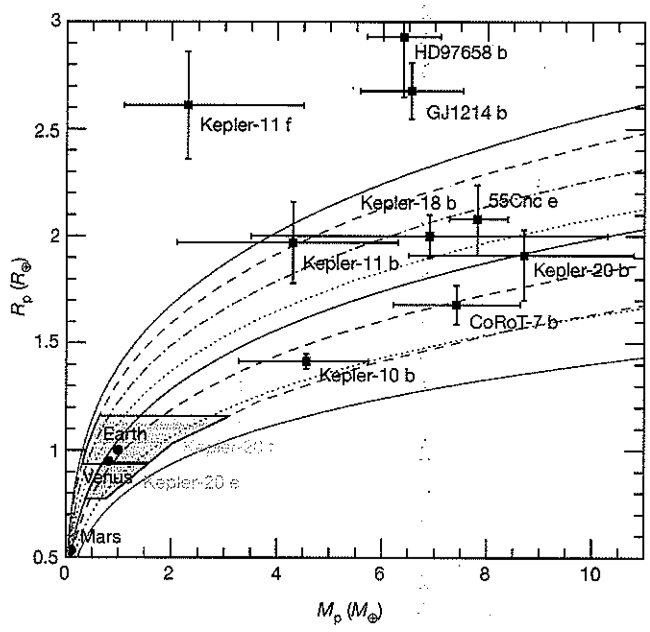

Figure 3 | Mass versus radius relation for small planets. Kepler-20 e and Kepler-20 f theoretical mass and observed radius ranges (1σ) are plotted as orange- and green-shaded areas, while the other transiting planets with dynamically determined masses are plotted in black, with 1σ error bars. The curves are theoretical constant-temperature mass–radius relations[27]. The solid lines are homogeneous compositions: water ice (solid blue), $MgSiO_3$ perovskite (solid red), and iron (magenta). The non-solid lines are mass–radius relations for differentiated planets: 75% water ice, 22% silicate shell and 3% iron core (dashed blue); Ganymede-like with 45% water ice, 48.5% silicate shell and 6.5% iron core (dot-dashed blue); 25% water ice, 52.5% silicate shell and 22.5% iron core (dotted blue); approximately Earth-like with 67.5% silicate mantle and 32.5% iron core (dashed red); and Mercury-like with 30% silicate mantle and 70% iron core (dotted red). The dashed magenta curve corresponds to the density limit from a formation model[24]. The minimum density for Kepler-20 e corresponds to a 100% silicate composition, because this highly irradiated small planet could not keep a water reservoir. The minimum density for Kepler-20 f follows the 75% water-ice composition, representative of the maximum water content of comet-like mix of primordial material in our Solar System[28].

blend frequencies, we find that the hypothesis of an Earth-size planet for Kepler-20 e is 3,400 times more likely than that of a false positive, and 1,370 times for Kepler-20 f. Both of these odds ratios are sufficiently large to validate these objects with very high confidence as Earth-size exoplanets.

With measured radii close to that of the Earth, Kepler-20 e and Kepler-20 f could have bulk compositions similar to Earth's (approximately 32% iron core, 68% silicate mantle by mass; see Fig. 3), although in the absence of a measured mass the composition cannot be determined unambiguously. We infer that the two planets almost certainly do not have a hydrogen-dominated gas layer, because this would readily be lost to atmospheric escape owing to their small sizes and high equilibrium temperatures. A planet with several per cent water content by mass surrounding a rocky interior is a possibility for Kepler-20 f, but not for Kepler-20 e. If the planets formed beyond the snowline from a comet-like mix of primordial material and then migrated closer to the star, Kepler-20 f could retain its water reservoir for several billion years in its current orbit, but the more highly irradiated Kepler-20 e would probably lose its water reservoir to extreme-ultraviolet-driven escape within a few hundred million years[19]. In this scenario, Kepler-20 f could develop a thick vapour atmosphere with a mass of $0.05M_\oplus$ that would protect the planet surface from further vaporization[20]. From the theoretical mass estimates in Table 1, we infer the semi-amplitude of the stellar radial velocity to be between 15 cm s^{-1} and 62 cm s^{-1} for Kepler-20 e and between 17 cm s^{-1} and 77 cm s^{-1} for Kepler-20 f. Such signals could potentially be detectable in the next few years, and would constrain the composition of the two planets.

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CUME #370 (Draft, as of Mar 27, 2012)
Prepared and administered by Jim Murphy
Saturday, March 31, 2012

The *a priori* anticipated "PASS" grade for this exam is 70%. The Exam offers 52 total points, for which 70% amounts to 36.4 points.
Calculators are **NOT** to be used for retrieving constants or equations, only for calculations.

For any calculations you will conduct below using information from Table 1, do not worry about the error indicated for the mean values provided in the Table unless told to do so.

Write only on a single side of a piece of paper, with no writing on the other side of the paper. For each Exam Question (1, 2, 3, etc.) start with a new blank sheet of paper. When you have completed the exam, staple together IN PROPER ORDER your answer pages.

This exam focuses upon physical aspects of planetary transit observations and also upon the exoplanets described in the accompanying paper, and not upon the statistical arguments for concluding that the two Light Curves presented in Figures 1a and 1b do in fact prove the existence of those two planets. We will assume that Kepler-20e and Kepler-20f do in fact exist.

QUESTION 1: Total of 15 points
1 a) Using the relevant planet orbit period and stellar mass information in Table 1, **calculate** the corresponding orbital semi-major axis lengths for Kepler-20e and Kepler-20f. Your final answers should be in units of AU. Show your work. **[3 pts]**

Using Newton's modified version of Kepler's 3rd Law:

$(M_{star} / M_{Sun}) \times P^2 = OSA^3$

$(M_{Kepler\text{-}20}/M_{Sun}) = 0.912$ *per Table 1 so* $0.912 \times (6.098493/365.25)^2 = OSA^3$

$OSA_{KEPLER\text{-}20e} = 0.0634\ AU$

Following the same procedure for Kepler-20f results in $OSA_{Kepler\text{-}20f} = 0.138\ AU$

b) Using the Kepler-20 Luminosity information provided in Table 1 and your orbital semi-major axes answer from 1a above, **calculate** the value of the planet Bond Albedo that has been assumed for either of these two planets, Kepler-20e or Kepler-20f. (In fact, the same value has been assumed to apply to both planets). Show your work. **[7 pts]**

For Kepler-20e:
$(1\text{-Bond Albedo}) \times (L_*/4\pi(a_{OSA}^2)) \times \pi r_p^2 = 4 \pi r_p^2 \sigma T_{eq}^4$

We can use the Stellar Luminosity value (0.853 times Solar, 0.853 x 3.8 x 10^{26} W) in Table 1 to determine that the stellar flux received at 0.0634 AU is:

Received Flux = 285208 W m^{-2}

(1-Bond Albedo) x (285208) = 4 x σ x T_{eq}^4 and with T_{eq} = 1040 K (Table 1, bottom line) we have

(1 - BondAlbedo) = 0.9303 or Kepler-20e Bond Albedo = 0.0697 (~7%)

For 20f, received stellar flux will be 60198 W m-2;
(1-BondAlbedo) = 0.9307 or Kepler-20f Bond Albedo = 0.0693 (~7%)

c) **Discuss** the reasonableness of your determined Bond Albedo value in 1b. **[2 pts]**

This calculated Bond Albedo values of ~0.09 is low when compared with terrestrial planets in our solar system (values here range from 0.12 to 0.75). The value is not physically unreasonable (it is not greater than 1.0 or less than 0.0), but it is at odds with terrestrial planet albedo values here in our solar system. Earth's albedo (~0.3) is in part the result of reflection due to clouds and ice; for 20e we would expect a smaller value since ice and clouds are unlikely, while a similar but less stringent argument can be made for 20f though its presumed 'steam' atmosphere could result in some cloud coverage (but no ice).

In fact, the paper's Table 1 caption indicates in its penultimate sentence that an assumed Earth-like Bond Albedo of 0.3 was employed to determine the Table 1-presented T_{eq} values of 1040 K and 705 K. This indicates that the author's must have used either a smaller OSA than we dynamically (and correctly) calculated in 1a above, or they used a smaller Kepler-20 luminosity value than is indicated in Table 1, or some combination. In fact, it is not possible to reproduce the Table 1 value of Luminosity for Kepler-20 using that same Table's value of stellar effective temperature and Stellar radius.

d) **Calculate** the minimum required diameter of a telescope, located in orbit around Earth, needed to individually resolve planet Kepler-20f distinct from its parent star Kepler-20, at an observed wavelength of 600 nanometers (*the wavelength of greatest sensitivity within Kepler's 420-900 nanometer bandpass*). **[3 pts]**

If Kepler-20f is located 0.138 AU from Kepler-20, the ability to resolve Kepler-20e would require

1.22 λ / Diameter = 0.138 AU/(290 x 206265 AU) so

*1.22 x (6 x 10^{-7} m) / (0.138/(290*206265)) = Diameter (meters)*

Diameter = ~317 meters

Question 2: Total of 19.0 points

2 a) Consider the light curve of Kepler-20e presented in Figure 1b, which spans the midpoint of its transit +/- ~4 hours. **Describe** what physical processes are dictating the temporal nature of and the shape of the Light Curve during the time periods **A, B, C, D,** and **E** as I have labeled them in the Figure. Your answer should address why the Light Curve during time interval **C** is not a straight horizontal line and why there is some curvature to the Light Curve during time intervals **B** and **D**. **[5 pts]**

A: Kepler-20f has not yet started to cross the star's disk and there is essentially no reflected stellar flux from the planet that is directed toward Earth, so only stellar flux is being received

B: Kepler-20e has made first contact with the star's limb and continues to cross the limb until 2nd contact at which time the planet is completely in front of the star; the curved nature of the decline arises from stellar limb darkening, which causes the Light Curve to steepen (negative slope) as time progresses and areas of the stellar disk farther from the limb are occulted by the planet; the planet's transit location/chord with respect to a chord that will pass directly in front of the star's center point also can play some role in the temporal structure of the curve here

C: this 'bottom' portion of the Light Curve is not 'flat' because of stellar limb darkening; the deepest portion of the transit occurs when the planet center crosses the star center or the central meridian of the star as seen from the telescope; as in B and C, the location of the transit chord (passing in front of the star's center, or above or below if the planet orbit plane is inclined relative to the stellar equator) also will play some role in the magnitude of the transit depth, in addition to the planet's radius relative to the stellar radius

D: the Light Curve is steepest as 3rd contact is made and the planet's trailing edge is located farthest from the exit limb; as the planet continues to exit across the stellar limb the limb darkened portion of the star is obscured so the total stellar flux 'removed' declines ...; chord location is also an issue, as mentioned in B: above

E: the planet no longer obscures any portion of the stellar disk disk and there is essentially no reflected stellar flux from the planet that is directed toward Earth, so only stellar flux is being received

b) The light curves presented in the paper (Figure 1a and 1b) span only a short portion of the total orbit period for each of the two planets. **<u>I want you to draw</u>** a Light Curve that spans the entire ~6.1 Earth Day orbit period for Kepler-20e. My primary interest is any potential observed periodic structure (variation in received total flux) in this light curve during the time period away from the 8-hour time period displayed in Figure 1a. You should augment your drawn light curve (*be sure you label and quantify the axes!*) with a **<u>description</u>** of what you intend your Light Curve to display, and why the Light Curve should have that/those variations and how their magnitude(s) might compare to the Flux variation magnitude in Figure 1a (you do not need to calculate any specific values). [**5 pts**]

See attached figure.

As the planet continues in its post-transit orbit its illuminated portion will be increasingly visible from Earth until the secondary eclipse when the planet passes behind the star. So, as the planet moves from its post-transit position to its secondary eclipse position the total received flux would increase due to the increasing addition of the planet-reflected stellar flux; since the planet is not 100% reflective (albedo < 1.0) and not all of the reflected flux is projected toward the telescope, the magnitude of this additional planet-reflected flux will be less than the magnitude of the stellar flux removed during transit, so the received total flux would be less than 1.002 on the ordinate scale; the time variation of received total flux entering in to the secondary eclipse would not exhibit and substantial limb-darkened effects; the total received flux during secondary eclipse would be equal to 1.0 (the same as during transit), and after the conclusion of the secondary eclipse the total received flux would decline *as less-and-less of the illuminated portion of the plate-reflected flux is directed toward the telescope, with no planet-reflected-flux directed toward the telescope during the transit.*

c) Using the information provided in Table 1, **calculate** the Synodic Period for Kepler-20e and Kepler-20f as they orbit their parent star. Your answer should be in units of Earth Days. Additionally, **draw** a "looking down from above" picture of the orbits of Kepler-20e and Kepler-20f orbiting around their parent star, and indicate the orbital azimuth positions where the first and second orbital conjunctions of Kepler-20e and Kepler-20f will occur relative to an initial conjunction which you specify. [**5 pts**]

$1 / P_{synodic} = 1/P_{Kepler\text{-}20e} - 1/P_{Kepler\text{-}20f}$

$1 / P_{synodic} = 1/6.098493 - 1/19.57707 = 0.1128947$ *so* $P_{synodic} = 8.8578$ *Days*

See attached figure

d) For the time interval covered by the eight observation quarters (670 Earth Days) of Kepler observations that were analyzed for this paper, **estimate** the probability that a Kepler-observed mutual transit of Kepler-20e and Kepler-20f would have occurred. Your answer should provide a quantitative probability value, and you should explain your thinking/reasoning in arriving at this value. Assume that there were no gaps in Kepler measurements (~30 minute cadence) during these eight observation quarters. [**4 pts**]

During a 670 Earth day time period, 670/8.8578 = 75.6 conjunction occurrences would happen. This indicates that one mutual transit central point would have occurred somewhere within each 4.76° angular interval of orbit azimuth. This 4.76 degree azimuth extent corresponds to 1.32% of a complete orbit.
Each Kepler-20e transit, ~4 hours in duration, spans 2.77% of Kepler-20e's 6.09 Day orbit period. For Kepler-20f, each ~5 hour transit event spans ~1% of one orbit of that planet. Each mutual transit)(assume 4 hours duration) spans ~2% of each Synodic Period.
So, since each mutual transit spans ~2% of a synodic period, and each transit of 20e spans 2.77% of its orbit period (~10 degree of orbit azimuth), and 75.6 Synodic Periods occurred, the probability is greater than 100% that a mutual transit would have been observed during Kepler's first 8 quarters of operation.

QUESTION 3: Total of 12 Points
3**a**) The text (4th page) includes a statement suggesting that Kepler-20f could possess a water vapor atmosphere while Kepler-20e is unlikely to possess such an atmosphere, with the difference being due to atmospheric escape. Quantitatively describe why this statement has validity. You do not need to calculate numbers bit you should describe how the magnitudes of the important physical terms for escape compare for the two planets. [**4 pts**]

Kepler-20e and Kepler-20f both have rather hot equilibrium temperatures. Both are likely too hot to retain liquid water (oceans, lakes) upon their surfaces if at some point in time (and/or maybe at larger OSA) they did in fact have oceans or lakes. Water vapor within the atmosphere is susceptible to molecular photochemical destruction via absorption of stellar UV flux. The UV flux at Kepler-20e will be ~4 times greater than that at Kepler-20f, so Kepler-20e's water vapor would be more susceptible to destruction. Also, Kepler-20e's hotter *temperature would result in faster atomic speeds for H (speed proportional to $T^{1/2}$), so H would be more likely to escape from Kepler-20e than from Kepler-20f AND Kepler-20e's smaller mass would result in it having a smaller escape velocity:*

$(V_{esc} = (2\ G\ M_{planet} / Radius_{planet})^{1/2})$

than that for Kepler-20f AND Kepler-20e's hotter temperature and smaller gravity would cause it to have a larger atmospheric scale height thus placing more molecules farther from the planet center (where g is reduced) so all of these processes would have the effect of permitting Kepler-20f to retain for a longer period of time than Kepler-20e a steam atmosphere.

b) If we ignore the idea of a steam atmosphere, and instead assume that Kepler-20f possesses an atmosphere identical in composition to Earth's atmosphere, **describe** how the Kepler transit Light Curve for an 'Earth Atmosphere' Kepler-20f planet might differ from the Kepler-20f light curve presented in Figure 1b? Why might an 'Earth Atmosphere" Kepler-20f light curve be deeper or shallower than that shown in Figure 1b? [**4 pts**]

The primary difference this 'Earth atmosphere' might provide is a change in the depth of the transit curve due to differing absorption of stellar flux due to the gas composition of Earth's atmosphere vs that from a 'steam' atmosphere. However, since an ~100 km 'thick' atmosphere adds only ~3% more 'cross-sectional area' to a planet of the size of Kepler-20f, the gas absorption change would need to be substantial to have an overall effect upon the total received flux and such a change is unlikely in Kepler's bandpass. The difference in gas composition would change the atmospheric scale height (larger for water vapor than for Earth's N_2/O_2 atmosphere) at the same temperature, since water vapor has a smaller molecular weight; this would change the vertical extent of the atmosphere, but as indicated already the geometrically 'thin' atmosphere is not likely to cause too much change in the overall transit depth.

c) The text (page 4) suggests that Kepler-20f's water vapor atmosphere could equal 0.05 Earth masses, and that such an atmosphere would protect the planet's surface from further evaporation. **Calculate** the hydrostatic surface pressure that such an atmosphere would produce on Kepler-20f (assume Kepler-20f's gravitational attraction equals Earth's gravitational attraction) and then **describe** how such an atmosphere could 'protect the planet's surface from further evaporation'. [**4 pts**]

Surface pressure (Pascals) = mass of gas in 1 m^2 atmospheric column times gravity

$SfcPres = g \times 0.05 \times 6 \times 10^{24}\ kg / (4\ \pi\ r_{Kepler\text{-}20f}^2) = g \times 2 \times 10^{23}\ kg / (4\ \pi\ (1.03 \times 6378000\ m)^2)$

$= 3.7 \times 10^9\ Pa$

Since water's boiling point has both a temperature <u>and</u> environment pressure dependence, this very large surface pressure value will result in a boiling temperature that is much greater than the canonical terrestrial sea level (10^5 Pa) boiling temperature of 273K. At a pressure of ~4 GPa, water's boiling temperature exceeds 700 K, so the water on Kepler-20f's surface would be evaporatively stable.

QUESTION 4: Total of 6 Points

4) The caption for Figure 3 indicates that, "The non-solid lines are mass-radius relations for differentiated planets: ...".

a) **Explain** what the term "DIFFERENTIATED" indicates in this context, [**2 pts**] and then

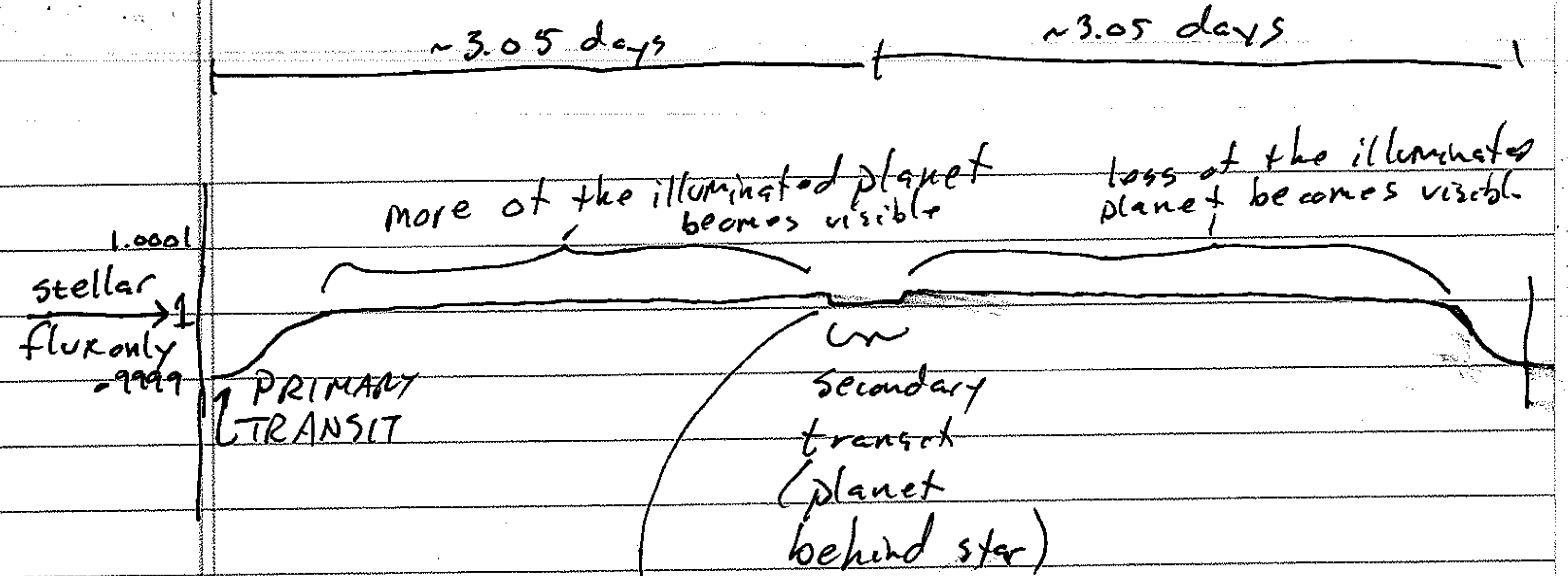
'Differentiation' indicates an internal structure consisting of the most dense material being concentrated at the planet center, with layers progressively located nearer to the surface having smaller density values than underlying layers.

b) **provide a brief discussion** in which you **describe** why or why not the assumption of 'differentiated' is likely to be valid for Kepler-20e and Kepler-20f based upon their inferred physical characteristics as presented in Table 1 and knowledge of planet formation processes. [**2 pts**]

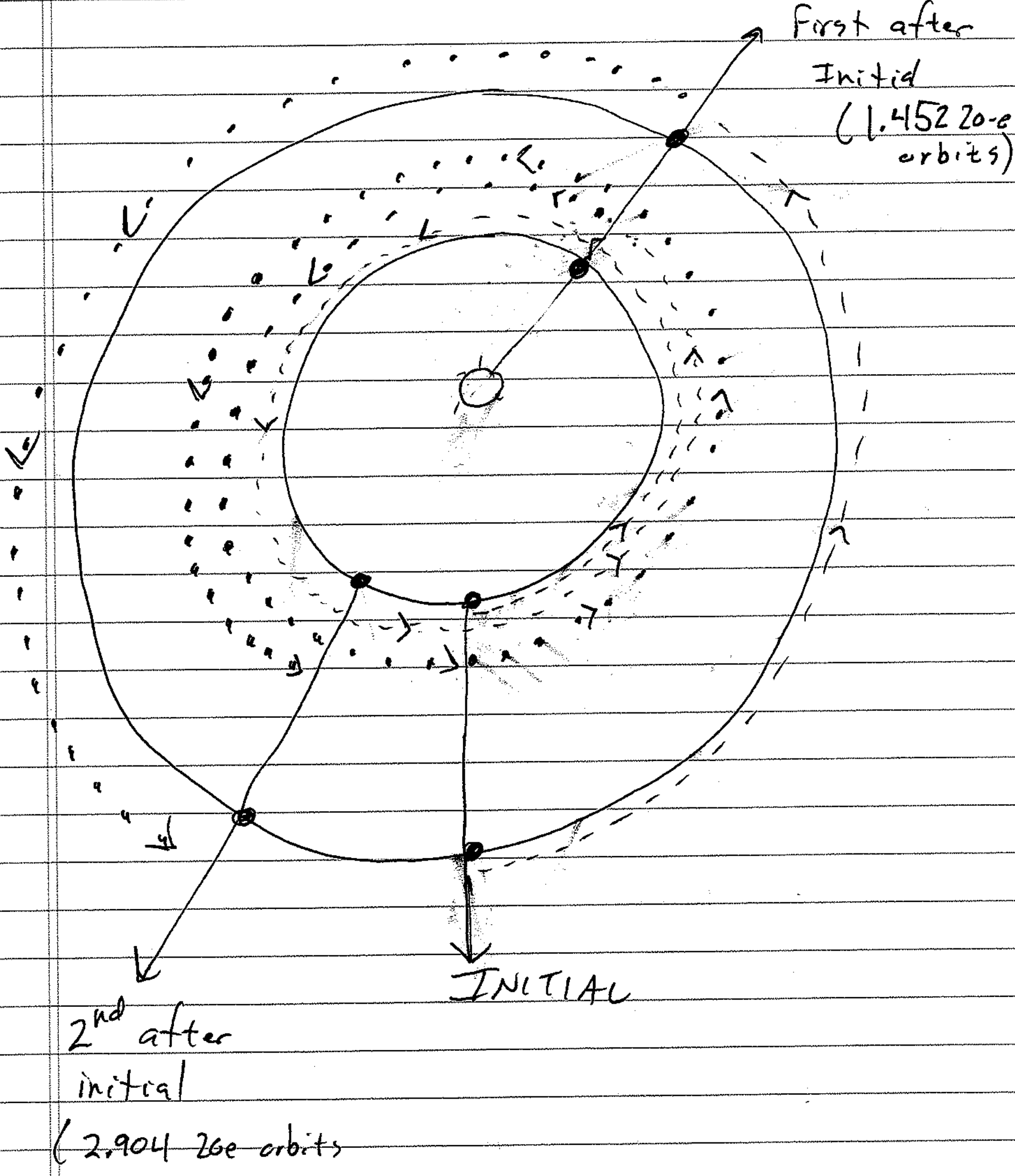
Kepler-20e and Kepler-20f are too small (not massive enough) to be gas-giant planets and thus are likely comprised of rock and metal materials. As such planets forms via accretion of mass, the incoming mass arrives at high speeds (large escape velocity values) and its impact kinetic energy is converted to thermal energy which can become buried by subsequent arriving material. This burial retains thermal energy within the planet which provides the ability for material melting. In this melt condition denser materials will sink to the center while the least dense material will 'float' to the surface. Our 4 terrestrial planets are all differentiated objects.

c) If Kepler-20e is NOT differentiated, **explain** how this physical condition of that planet could cause its total bulk density to differ from its total bulk density in a differentiated condition? [**2 pts**]

Different materials have different equations of state. If differentiation were not present than the materials at the center's of these planets would respond to the overlying pressure different than iron alone (for a rocky-iron planet) which could lead to an increase or decrease in the planet's bulk density dependent upon how those core materials respond to the core pressures and temperatures.



magnitude of secondary transit is much less than primary transit due to less than unity planet albedo, spatial distribution of planet reflected star light, planet-star distance, etc.


First after
Initid
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