NMSU Department of Astronomy

Cume Exam #354

Administered by: Dr. Jim Murphy
Saturday, September 11, 2010

Total Points available: 67.5 (*a priori* passing score is 65%, or 43.9 points)

This exam is focused upon the paper, "Solar flux variability of Mars' exosphere densities and temperatures", Forbes et al., Geophysical Research Letters, Vol 35, L01201, doi:10.1029/2007GL031904, 2008.

The exam includes questions related to Mars and Earth and spacecraft orbital characteristics, questions focused upon the thermodynamics of Mars' atmosphere, and questions focused upon the martian exospheric density values discussed in the paper.

Some potentially useful 'constant' values are provided below.

Mars Characteristics of possible value:

Mars' Radius	3393 kilometers
Mars' J2 Gravitational Moment	1.96×10^{-3}
Mars' Orbital Semi-major Axis	1.52 AU
Mars' orbital eccentricity	0.093
C_p (Specific Heat)for CO_2 gas	735 J kg^{-1} K^{-1}
Gas constant (R_{CO2}) for CO_2 gas	189 J kg^{-1} K^{-1}
Mars surface gravity acceleration	3.72 m s^{-2}
Mars' bulk density	3933 kg m^{-3}
Mars' Rotation Period	24 hours 37 minutes (88775 seconds)
Mars' Axial Tilt	25.2 degrees
Mars' Bond Albedo	0.15

Some other potentially useful constant values:

Proton mass	1.67×10^{-27} kg
1 AU	1.5×10^{11} meters
Magnitude of Solar Flux at 1 AU	1370 Watts per square meter
Mass of the Sun	2×10^{30} kg

Mars vs. Earth (20 points total)

1) **Calculate** Mars' orbital period, presenting your answer in units of Earth Years. **[5.0 points]**

2) **Calculate** the Earth : Mars Synodic Period (the time interval between successive orbital opposition configurations). Present your answer in units of Earth Years. **[5.0 points]**

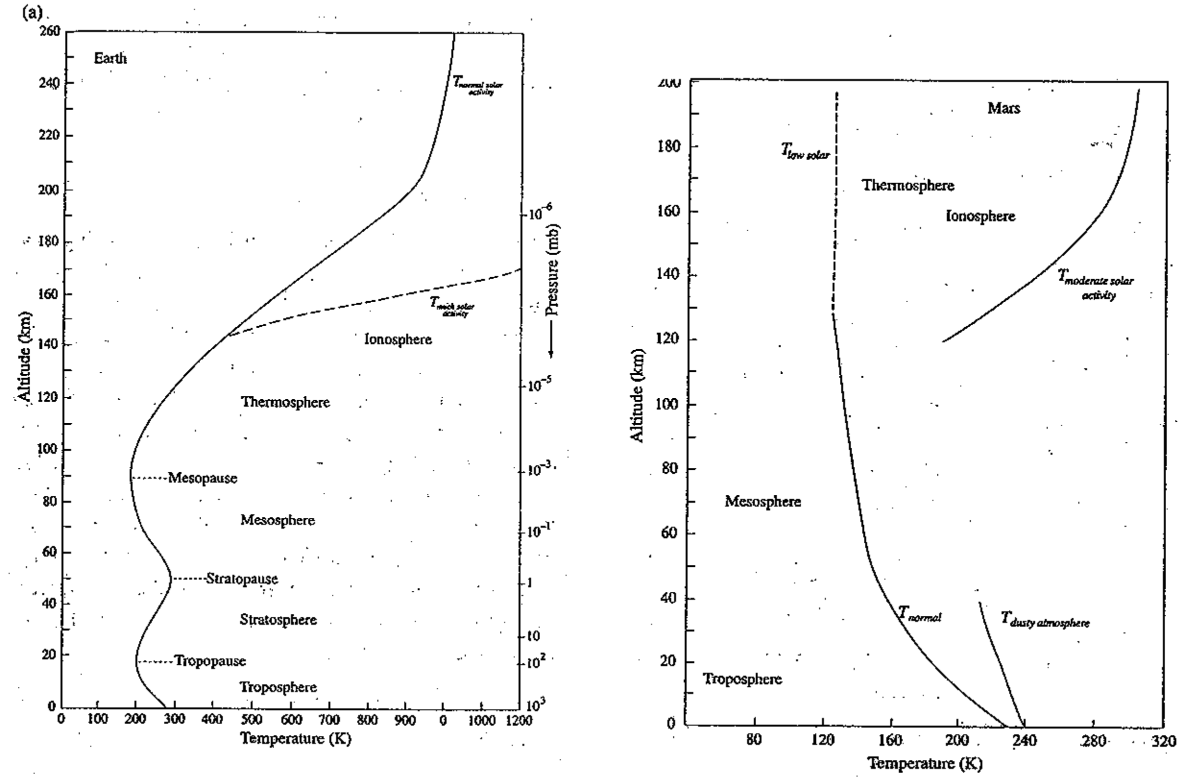
3) The Mars Global Surveyor (MGS) orbiter spacecraft arrived at Mars on September 11, 1997.

A) Assume that Mars was at its average distance from the Sun at the time that MGS arrived. Assume also that Earth's orbit is circular. **Quantitatively determine the date** on which MGS was launched toward Mars from Florida, assuming that MGS followed a minimum energy orbit (Hohmann transfer orbit) path from Earth to Mars. **[7.5 points]**

B) Draw a figure which indicates the relationship between Earth's orbit, Mars' orbit, and MGS' travel path for the stated conditions above. **[2.5 points]**

Mars' Atmosphere (27.5 points total)

Representations of the vertical temperature structures of Earth's and Mars' atmospheres are shown in the two figures below. Note that the vertical scales are not the same. The exobase (see Question 5) altitude for each of the two atmospheres occurs at altitudes above the tops of these two graphs.



4) Within Earth's and Mars' atmospheric **troposphere layers** the temperature decreases with increasing height above the surface.

A) BRIEFLY describe the physical characteristic of Mars' atmosphere that dictates that the maximum magnitude of the environmental atmospheric lapse rate *(the maximum rate at which temperature declines with height*) within Mars' troposphere will be –5 Kelvin per kilometer. [If you can provide an equation that justifies this -5 K/ km value, you should do so]. **[5.0** points]

B) Provide a BRIEF description of the physical reason why Earth's 78 % N_2 : 21% O_2 : 1% Ar composition atmosphere possesses a stratosphere layer within which *temperature increases with increasing height*, while Mars' 95% CO_2 : 3% N_2 composition atmosphere does not contain such a stratosphere layer. [**5 points**]

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5) The exosphere is the layer of the atmosphere within which upward moving atmospheric atoms/molecules can potentially directly escape to space. The lower boundary of this layer is called the 'exobase'. **Construct an appropriate equation** which illustrates that at or above the exobase an upward moving atom/molecule is unlikely to experience a collision with another atom/molecule, and thus can escape to space if it is traveling fast enough. **Describe the terms in your equation.** (*Hint: think of opacity*) [**7.5 points**]

6) Let's assume that Mars' atmosphere is isothermal, which indicates that at every altitude the temperature is the same as at every other altitude.

A) The value of the atmospheric density at Mars' surface is ~ 0.01 kg m^{-3}. Using the maximum atmospheric density value indicated in Figure 1, 40 x 10^{-18} g cm^{-3} (= 40 x 10^{-15} kg m^{-3}) at the height of 390 km above the surface, **calculate the isothermal temperature in Mars' atmosphere** that corresponds to these two density values and the 390 km atmospheric 'thickness'. **[7.5 points]**

B) Discuss the reasonableness of your calculated isothermal temperature, or **provide and justify a 'guess'** isothermal temperature if you were not able to calculate a value in part A above. **[2.5 points]**

The Data Presented in the Paper (20 points total)

7) In Paragraph 7, the authors discuss their extrapolation of the measured F10.7 cm flux (proxy for the EUV flux) at 1 AU to the EUV flux magnitude that would be received at Mars (~1.52 AU). The authors quantify this extrapolation in Figure 1 (blue and red curves). **Describe** why the Earth:Mars orbital conjunction is the <u>least certain</u> (the authors use the word 'important') orbital configuration at which to apply their extrapolation. **[7.5 points]**

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8) The martian exospheric density values discussed in this paper are determined from the MGS spacecraft directly interacting with the martian atmosphere. [MGS did not carry a gas analyzer instrument.]

A) **Think about and then describe** the nature of the spacecraft's direct interaction with Mars' atmosphere and how that interaction results in detectable changes in the spacecraft's orbit which can be related back to the atmospheric density. <u>If you can include and explain an appropriate equation as part of your answer that would be good</u>. **[7.5 points]**

B) The authors state (Paragraphs 6 & 12) that they used MGS orbit data information obtained during 48-60 orbits (4-5 days) to enable them to derive 'statistically significant' (Jim's term) individual daily exospheric density values at 390 km presented in Figure 1. If MGS' periapse had been at an altitude of only 50 km above Mars' surface (where gas density is ~8×10^{-5} kg m^{-3}), **would fewer or more** than 48-60 orbits have been needed to derive 'statistically significant' density values at or near this 50 km periapse altitude? **Explain. [5 points]**

GEOPHYSICAL RESEARCH LETTERS, VOL. 35, L01201, doi:10.1029/2007GL031904, 2008

Solar flux variability of Mars' exosphere densities and temperatures

Jeffrey M. Forbes,[1] Frank G. Lemoine,[2] Sean L. Bruinsma,[3] Michael D. Smith,[2] and Xiaoli Zhang[1]

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[1] Using densities derived from precise orbit determination of the Mars Global Surveyor (MGS) spacecraft from 1999 to mid-2005, the response of Mars' exosphere to long-term solar change is established and compared to that of Earth and Venus. At Mars, exosphere temperatures (weighted towards high-latitude Southern Hemisphere daytime conditions) change only 36–50% as much as those at Earth as solar activity increases from solar minimum to solar maximum, whereas the response at Venus is one-fifth that at Mars. General circulation models suggest that this difference may be strongly influenced by adiabatic cooling associated with the thermosphere general circulation. However, other processes such as differences in CO_2 cooling rates may also be playing a role. **Citation:** Forbes, J. M., F. G. Lemoine, S. L. Bruinsma, M. D. Smith, and X. Zhang (2008), Solar flux variability of Mars' exosphere densities and temperatures, *Geophys. Res. Lett.*, *35*, L01201, doi:10.1029/2007GL031904.

1. Introduction

[2] From a comparative planetary perspective, the responses of the upper atmospheres of the terrestrial planets (i.e., Earth, Mars and Venus) to solar variability pose interesting challenges and opportunities. The relative importance of such processes as heating efficiency, radiative cooling, thermal conduction and dynamics must be considered [e.g., *Bougher et al.*, 2000]. These planetary atmospheres are sufficiently different, yet sufficiently alike, that their comparative study allows deeper understanding and insight to be achieved for each planet.

[3] Studies have established that the thermospheres of Venus and Mars are about 10% and 30–50% as responsive as Earth, respectively, in terms of exosphere temperature changes due to solar EUV variability associated with the ~27-day rotation of the Sun [*Forbes et al.*, 2006, 2007; *Keating and Bougher*, 1987, 1992]. These differences are thought to be primarily due to the increased importance of CO_2 cooling in damping the responses at these planets, as compared to Earth and to each other.

[4] In terms of long-term solar variability, Earth has benefited from many measurements from satellite-borne and ground-based instrumentation, as well as orbital analyses of satellites, over several solar cycles. These results are embodied in empirical models such as MSISE-90 [*Hedin*, 1991], DTM94 [*Berger et al.*, 1998] and NRLMSIS-00 [*Picone et al.*, 2002]. For Mars some sense of long-term change due to solar variability has been inferred from sparse and disparate data sets such as in-situ mass spectrometers, airglow, and plasma scale heights [e.g., see *Keating and Bougher*, 1987, 1992; *Bauer and Hantsch*, 1989; *Bauer*, 1999]. Some of these data, in particular the plasma scale heights, may underestimate the exosphere temperature response since the inferred temperatures may correspond to that part of the temperature profile lower in the thermosphere (ca. 130 km) than representative of the exosphere temperature (i.e, above 180–220 km during daytime at solar maximum [*Bougher et al.*, 1999]). The widely-quoted exosphere temperature response of Mars by *Stewart* [1987] is also strongly dependent on plasma scale height data. For Venus, information on solar cycle variability of exosphere temperature is available from Magellan and Pioneer Venus Orbiter measurements (see review by *Kasprzak et al.* [1997]).

[5] The objective of the present study is to establish the response of Mars' exosphere densities and temperatures to long-term solar change, and to compare Mars' exosphere response with that of Earth and Venus. Compared to previous studies, ours is distinguished by the length, quality and quantity of data employed, and the wide range of solar conditions over which the data are collected. Our work is also relevant to the calculation of Martian volatile escape rates, which are sensitive to variations in the density and temperature of Mars' exosphere.

2. Data

[6] The Mars density data consist of daily values inferred from precise orbit determination (POD) of the Mars Global Surveyor (MGS) spacecraft from 1 February 1999 – 7 July 2005. Each density value is determined from analysis of Deep Space Network (DSN) observations over processing arcs with lengths of 4–5 days, and normalized to a constant altitude of 390 km using the DTM-Mars empirical model [*Bruinsma and Lemoine*, 2002], which is based in part on almost 2 years of MGS drag data of the type presented here. The MGS satellite is in a 93.7°-inclination 1400-0200 LT sun-synchronous 370 × 437 km frozen orbit with periapsis confined to −40° to −60° latitude. The density values presented here therefore represent averages over all longitudes, and are strongly biased towards daytime Southern Hemisphere conditions. The POD technique is similar to that utilized by *Bruinsma and Lemoine* [2002] and in recent studies devoted to solar flux changes in density due to rotation of the Sun [*Forbes et al.*, 2006, 2007]. Briefly, we employed the MGM1041c gravity model [*Lemoine*, 2003], and took into account such processes as third-body gravity perturbations due to the Sun, planets, and satellites of Mars;

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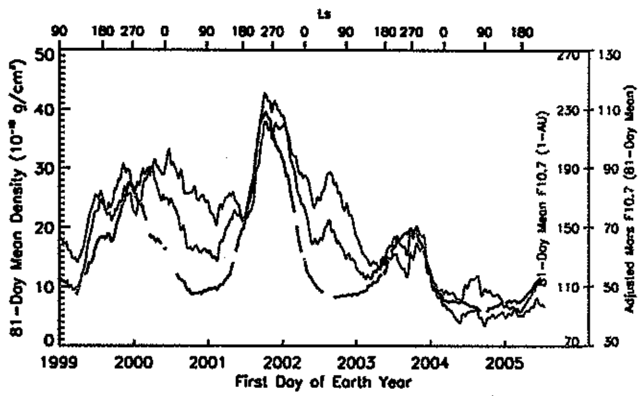

Figure 1. 81-day running mean Mars densities at 390 km (black), 10.7 cm solar flux at 1 AU (blue) and at Mars (red) vs Earth year (bottom x-axis) and L_s (top x-axis). Errors in the 81-day mean densities are estimated at 2% (RMS).

solar and planetary radiation pressure on the spacecraft; a detailed macromodel of the spacecraft; thruster firings; and changes in mass due to fuel consumption. We also used the observations to empirically infer corrections to the macromodel reflectivity parameters, the Mars k_2 Love number, and annual and semiannual variations in the gravity field due to condensation and sublimation processes in Mars atmosphere. Earth atmosphere, solid tide and ocean tidal loading corrections were also applied to the DSN tracking data. Uncertainties for the daily values are estimated at 17% (RMS), and are mainly due to tracking errors and solar radiation pressure modeling. Uncertainties in the 81-day mean values (see below) are therefore estimated at $17/\sqrt{81} \approx$ 2% (RMS). A constant bias of up to 10% is also possible due to uncertainties in the assumed drag coefficient; however, this does not affect analyses and conclusions below, which are focused on rate of change of density and derived exosphere temperature with respect to solar flux.

3. Results

[7] Since we are primarily interested in solar flux changes over much longer than a solar rotation (~27 days), we concentrate on 81-day running mean values of all parameters, which will be understood in the following unless otherwise specified. Figure 1 illustrates daily values of density normalized to a constant altitude of 390 km versus time from 1 February 1999 through 7 July 2005. Also shown are F10.7 values at 1 AU and adjusted to Mars, which are used as proxies for solar EUV fluxes. Note that the F10.7 values at 1 AU cover a wide range, from about 90 solar flux units (sfu) to 230 sfu. The values at Mars are derived from the 1 AU fluxes taking into account variations in Mars' distance from the Sun, and differences in angular distance between the positions of Mars and Earth with respect to the Sun, assuming that the EUV-emitting active regions of the Sun do not evolve during this interval. This latter assumption can be important for day-to-day solar flux variations especially near conjunction, but has relatively small effects on the 81-day mean values.

[8] The density variations in Figure 1 are characterized very well (correlation coefficient R = 0.96) by a simple linear regression formula containing a linear term in F10.7 (at Mars) and a relatively small seasonal term in Ls, the longitude of the Sun with respect to the vernal equinox of Mars. In units of 10^{-18} cm^{-3},

$$\rho_{390} = 3.72 + 0.28\overline{F}_{10.7} - 4.5\cos(L_s - 72°) \quad (1)$$

In order to compare the response of Mars' and Earth's thermospheres to changes in solar flux, it is necessary to convert to exosphere temperature (T_∞) using a model, in this case DTM-Mars. The change in the observed T_∞ with time, and the least-squares fit of the form:

$$T_\infty = 130.7 + 1.53\overline{F}_{10.7} - 13.5\cos(L_s - 85°) \quad (2)$$

are illustrated in Figure 2. The correlation coefficient is 0.99. Also shown by the lower curve in Figure 2 is the 81-day and zonal-mean dust optical depth averaged between ±30° latitude measured in Mars' atmosphere [*Smith*, 2004], an index of dust content and related heating and expansion of Mars' atmosphere. During 2001 a planet-encircling dust storm occurred. Unfortunately, the rise in dust content occurs contemporaneously with a large increase in solar flux, and the two effects on exosphere temperature cannot be distinguished. We attempted several approaches to isolate any possible effect due to dust-related heating and atmospheric expansion, but no definitive results could be obtained. For instance, when looking at daily values of T_∞, F10.7 and dust optical depth, the added variability in F10.7 and T_∞ due to rotation of the Sun [*Forbes et al.*, 2006] again precluded definitive conclusions. Adding the dust index in the least-squares fit had little effect. We also excluded the dust storm period in deriving equation (2), and attempted to correlate the difference between the fit curve and the observed temperatures, and the dust index. Equation (2) did not change much, leading us to conclude that the dust storm did not perceptibly influence exosphere temperature or density.

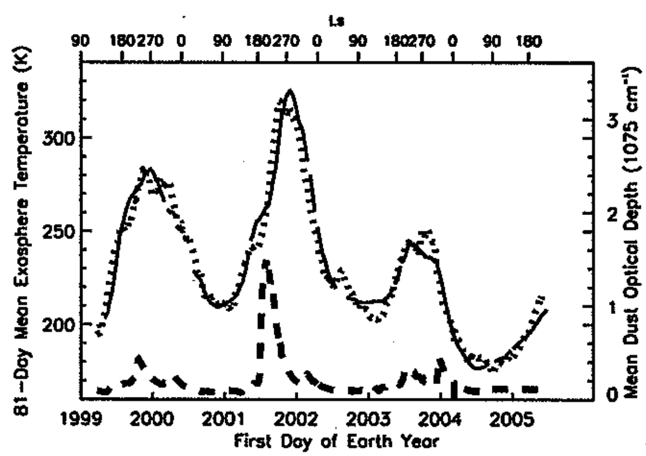

Figure 2. 81-day mean Mars exosphere temperature (solid), least-squares fit (dotted), and zonal mean optical depth averaged between ±30° latitude (dashed) vs. Earth year (bottom x-axis) and L_s (top x-axis). Errors in the 81-day mean exosphere temperatures are estimated at 2–3% (RMS).

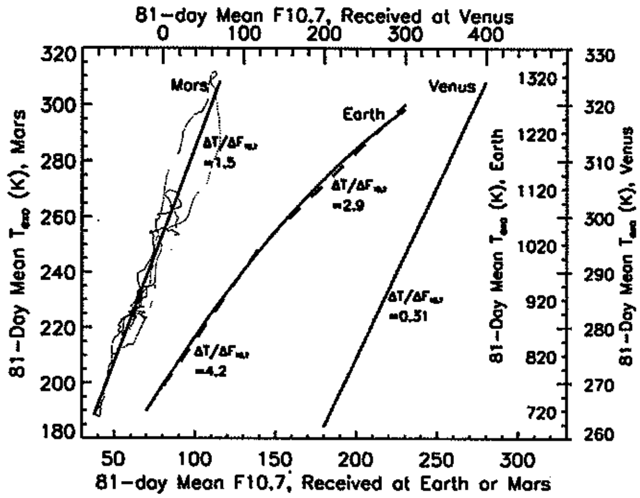

Figure 3. 81-day mean exosphere temperature at Mars, Earth and Venus versus 81-day mean 10.7 cm solar flux *received at the planet*. The Mars least-squares fit slope is $\Delta T/\Delta \overline{F}_{10.7} = 1.5$, whereas the MSISE-90 and NRLMSIS-00 models for Earth indicate a value of 4.2 at low to moderate levels of solar activity, and a value of 2.9 at higher levels of solar activity. For Venus a value of 0.31 is indicated [*Kasprzak et al.*, 1997].

[9] Figure 3 provides a comparison between the responses of the thermospheres of Earth, Mars and Venus to long-term solar variability. The line labeled "Mars" represents the solar flux term in equation (2), and the data points around it are the same as the data points in Figure 2, with the small seasonal (L_s) effect removed. The slope of the Mars line is $\Delta T/\Delta \overline{F}_{10.7} = 1.5$, which means that exosphere temperature changes by 1.5 K per solar flux unit (sfu) change received at the planet. For Venus, the value is 0.31 [*Kasprzak et al.*, 1997], about one-fifth that of Mars. The corresponding curve for Earth is also indicated, and is obtained from MSISE-90 [*Hedin*, 1991], which is a statistical representation based upon measurements from many satellites and also incoherent scatter radar data. Earth's response is not linear, and its slope varies from 2.9 at solar maximum to 4.2 at solar minimum, i.e., a factor of 2 to 3 greater than the response at Mars. The curve for NRLMSIS-00 is almost identical with that for MSISE-90.

4. Discussion and Conclusions

[10] The above value of $\Delta T/\Delta \overline{F}_{10.7} = 1.5$ for Mars is considerably less than the values of 2.7 and 4.1 derived from the multi-source (plasma scale height, airglow, mass spectrometer) data summarized by *Keating and Bougher* [1987] and *Stewart* [1987], respectively. It is also considerably greater than the value of about 0.6 inferred by *Bauer and Hantsch* [1989] and *Bauer* [1999] based strongly on plasma scale height and MGS accelerometer data, both of which correspond to about 130 km altitude. As noted previously, this latter result is not likely reflective of the exospheric temperature response at higher altitudes, and cannot be directly compared with the present results.

[11] According to MSISE-90, the response of Earth's exosphere temperature to solar flux variability is virtually independent of latitude and local time. However, general circulation model results for Mars [e.g., *Bougher et al.*, 2000, 2006] indicate that the response can differ between day and night, and also with respect to latitude. MGS is in a 370 × 437 km and 1400/0200 local time orbit with periapsis between −40° to −60° latitude throughout the time period considered here. Since most drag occurs within 1–2 scale heights of periapsis, the results presented here should be considered only applicable to 1400 LT in the Southern Hemisphere of Mars.

[12] It is important to note that the raw data for this investigation consists of density values that represent averages over 4–5 Mars days and over all longitudes. Therefore, it is not possible to comment on the possible extension into the exosphere of the large longitude variations in density known to exist in the lower thermosphere (ca. 100–140 km) due to vertically-propagating non-migrating tides [*Forbes and Hagan*, 2000; *Forbes et al.*, 2002; *Withers et al.*, 2003; *Angelats i Coll et al.*, 2004].

[13] A small exosphere temperature response to dust storm heating in the lower atmosphere is consistent with model results. For a 20-sol planet-encircling dust storm near Mars perihelion (Ls = 270), *Bougher et al.* [1997] find factors of 5–10 increases in density near 110 km. However, near the exobase (ca. 220 km) a 10–20 K *cooling* is found in the dayside Southern Hemisphere, a ~50 K warming at northern polar latitudes, and a ~20–50 K warming on the nightside at all latitudes. These temperature changes are strongly influenced by adiabatic heating and cooling due to modification of the general circulation in response to direct heating effects in the lower atmosphere, as well as the vertically-propagating semidiurnal tide. Considering the orbit of MGS with periapsis in Southern Hemisphere daytime, it is not surprising that dust storm effects are difficult to distinguish in Figure 2.

[14] Adiabatic heating and cooling may also be relevant to the interpretation of the exosphere temperature responses for both Earth and Mars in Figure 3. Several processes determine Mars' exosphere temperature and its variability [*Bougher et al.*, 1999, 2000]: (1) EUV flux, and its changes with solar cycle, solar rotation and distance from the Sun; (2) molecular thermal conduction; (3) CO_2 cooling; and (4) adiabatic heating and cooling associated with global dynamics. According to *Bougher et al.* [1999, 2000], the primary balance is between cooling by molecular heat conduction and EUV heating, with CO_2 cooling playing a tertiary role. For Venus and Earth, CO_2 cooling is more and less important, respectively, than for Mars. However, for Mars adiabatic cooling due to rising motions within the global circulation plays a progressively more important role in the heat budget as solar activity increases. At least at low to middle latitudes, for higher levels of solar EUV flux strong vertical winds (and adiabatic cooling) suppress the temperature response on the dayside of Mars. At Earth, ion drag serves to suppress any solar cycle variation of the EUV-driven circulation and along with it the adiabatic cooling effect, permitting a more robust exosphere temperature response to long-term changes in solar EUV flux [*Hagan and Oliver*, 1985]. It is not clear from the model results whether this mechanism still applies for the MGS densities and temperatures which are weighted towards middle and high latitudes. Definitive interpretation requires

closer analysis of model outputs taking into account the orbital configuration of MGS.

[15] The primary conclusion of this paper is that the response of Mars' Southern Hemisphere daytime thermosphere to long-term solar flux variability is 36–50% that of Earth and about five times that of Venus. This is in agreement with previous determinations with respect to shorter-term variations (~27 days) connected with rotation of the Sun [*Forbes et al.*, 2006, 2007]. It remains to be seen whether the latter result is attributable to the higher (yet uncertain) O/CO_2 concentration ratios in Mars' thermosphere which influence the relative efficiency of CO_2 cooling, and whether the former might be more connected with the adiabatic cooling effects discussed previously. In any case, the results presented here ought to constitute an important constraint on the heat budgets of thermosphere general circulation models that attempt to self-consistently and inter-consistently model the thermospheres of Earth and Mars [e.g., *Bougher et al.*, 2000, 2006]. Such validation is key for whole-atmosphere model frameworks which capture the global-scale dynamics of the entire Mars atmospheric system [e.g., *Bougher et al.*, 2006]. Solar cycle variations are best investigated using such models.

[16] **Acknowledgments.** J. Forbes was supported by the Glenn Murphy Professorship at the University of Colorado. F. Lemoine acknowledges the NASA Mars program for support and the MGS Radio Science Team (in particular Dick Simpson of Stanford University) for providing the DSN tracking data and other ancillary spacecraft information.

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NMSU Department of Astronomy

Cume Exam #354
SOLUTION
Administered by: Dr. Jim Murphy
Saturday, September 11, 2010

Total Points available: 67.5 (*a priori* passing score is 65%, or 43.9 points)

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Mars Characteristics of possible value:

Mars' Radius	3393 kilometers
Mars' J2 Gravitational Moment	1.96×10^{-3}
Mars' Orbital Semi-major Axis	1.52 AU
Mars' orbital eccentricity	0.093
C_p (Specific Heat)for CO_2 gas	735 J kg^{-1} K^{-1}
Gas constant (R_{CO2}) for CO_2 gas	189 J kg^{-1} K^{-1}
Mars surface gravity acceleration	3.72 m s^{-2}
Mars' bulk density	3933 kg m^{-3}
Mars' Rotation Period	24 hours 37 minutes (88775 seconds)
Mars' Axial Tilt	25.2 degrees
Mars' Bond Albedo	0.15

Some other potentially useful constant values:

Proton mass	1.67×10^{-27} kg
1 AU	1.5×10^{11} meters
Magnitude of Solar Flux at 1 AU	1370 Watts per square meter
Mass of the Sun	2×10^{30} kg

Mars vs. Earth

1) **Calculate** Mars' orbital period, presenting your answer in units of Earth Years. **[5.0 points]**

$P^2 = OSA^3$ $P^2 = 1.52^3$ $P = (3.51)^{1/2}$ $P = 1.87$ Earth years

2) **Calculate** the Earth : Mars Synodic Period (the time interval between successive orbital opposition configurations). Present your answer in units of Earth years. **[5.0 points]**

$1/P_{synodic} = 1/P_{Earth} - 1/P_{Mars}$ $1/P_{synodic} = 1/1 - 1/1.87$

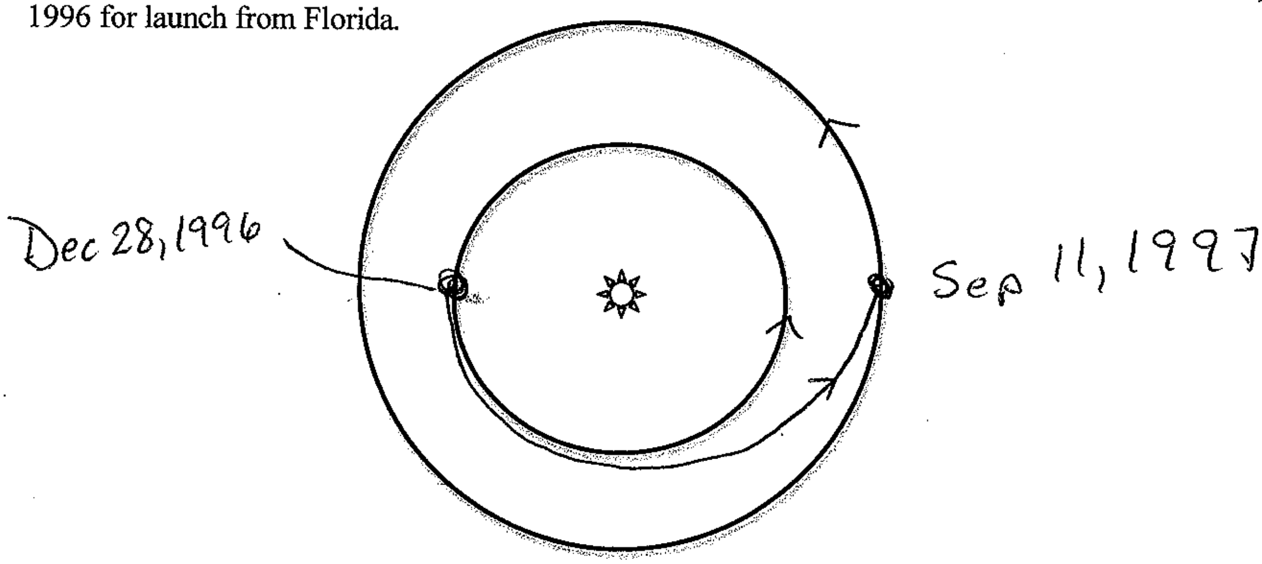
$1/P_{synodic} = 1 - 0.535$ $1/P_{synodic} = 0.465$ $P_{synodic} = 2.149$ Earth years

3) The Mars Global Surveyor (MGS) orbiter spacecraft arrived at Mars on September 11, 1997.
a) Assume that Mars was at its average distance from the Sun at the time that MGS arrived. Assume also that Earth's orbit is circular. **Quantitatively determine the date** on which MGS was launched toward Mars from Florida, assuming that MGS followed a minimum energy orbit (Hohmann transfer orbit) path from Earth to Mars. **[7.5 points]**
b) **Draw a figure** which indicates the relationship between Earth's orbit, Mars' orbit, and MGS' travel path for the stated conditions above. **[2.5 points]**

The minimum energy orbit has its apoapse at 1.52 AU (Mars' distance from the Sun) and its periapse at 1.0 AU (Earth's distance from the Sun). So, the orbit's orbital Semi-major Axis is the average of the apoapse or periapse, so MGS' OSA = (1.0 + 1.52) / 2 AU = 1.26 AU

Orbital period for an OSA of 1.26 AU : $P = (1.26^3)^{1/2} = 1.414$ Earth years
BUT
MGS need travel only one-half of the orbit path to travel from Earth to Mars, so travel time is 0.707 Earth years, which is 8.48 Earth months.

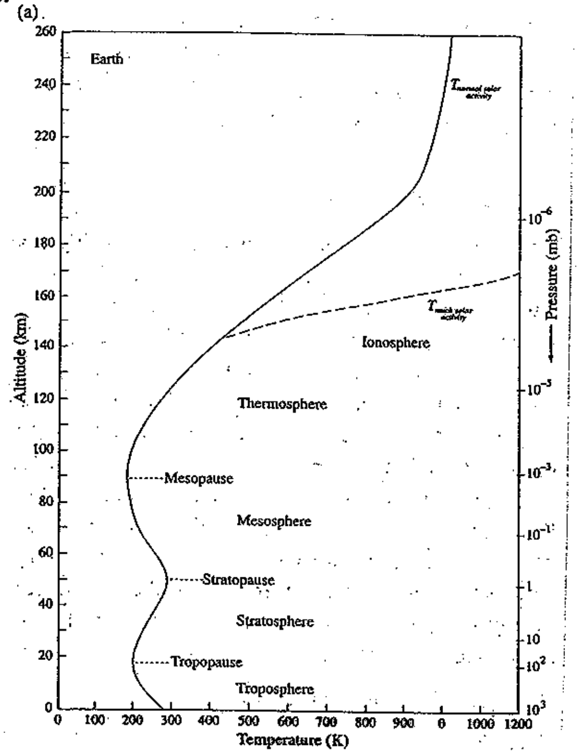
Working backward 8.48 months from September 11, 1997, one arrives at a date of ~December 28, 1996 for launch from Florida.

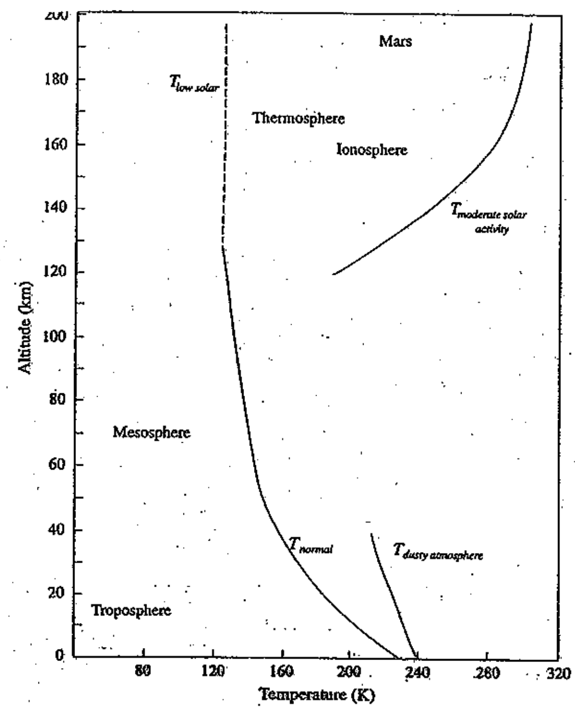


Mars' Atmosphere

Representations of the vertical temperature structures of Earth's and Mars' atmospheres are shown in the two figures below. Note that the vertical scales are not the same. The exobase (see Question 5) altitude for each of the two atmospheres occurs at altitudes above the tops of these two graphs.

(a)





4) Within Earth's and Mars' atmospheric troposphere layers the temperature decreases with increasing height above the surface.

a) **Briefly describe** the physical characteristic of Mars' atmosphere that dictates that the maximum magnitude of the environmental atmospheric lapse rate *(the maximum rate at which temperature declines with height)* within Mars' troposphere will be -5 Kelvin per kilometer. [If you can provide an equation that justifies this -5 K/ km value, you should do so]. [**5.0** points]

The maximum rate at which temperature can decline with increasing height is dictated by an atmosphere's 'adiabatic lapse rate', which is the rate of temperature change with height that a parcel of air will experience if it is raised adiabatically. This adiabatic lapse rate is quantified as the ratio of $-g/C_p$. For Mars, this amounts to ~ -5 Kelvin per kilometer.

b) **Provide a brief description** of the physical reason why Earth's 78 % N_2 : 21% O_2 : 1% Ar composition atmosphere possesses a stratosphere layer within which *temperature increases with increasing height,* while Mars' 95% CO_2 : 3% N_2 composition atmosphere does not contain such a stratosphere layer.

Earth's 20% oxygen atmosphere results in the photochemical presence of ozone at altitudes high enough for ionizing flux (short wavelength) to still be available (not yet completely absorbed as it penetrates downward through the atmosphere) but low enough in the atmosphere (large enough density values and collision frequencies) to permit significant quantities of ozone (O_3) to form. Ozone absorbs UV wavelength radiation, which produces Earth's stratospheric heating. Mars' minimal O_2 abundance results in inconsequential abundances of O_3 being present and with the absence of other photochemically active gases above the troposphere Mars does not possess a stratosphere.

5) The exosphere is the layer of the atmosphere within which upward moving atmospheric atoms/molecules can potentially directly escape to space. The lower boundary of this layer is called the 'exobase'. **Construct an appropriate equation** which illustrates that at or above the exobase an upward moving atom/molecule is unlikely to experience a collision with another atom/molecule, and thus can escape to space if it is traveling fast enough. **Describe the terms in your equation.** (*Hint: think of opacity*) **[7.5 points]**

The opacity of an atmospheric layer is: $\tau =$ the integral of $\alpha(z)\ \rho(z)\ dz$ from z1 to z2, where α is the mass extinction coefficient, ρ the gas density. ~~Opacity~~ Radiation is generally able to 'escape' to space from a layer at or above the level at which the opacity for that wavelength is <=1 when integrated downward from the top of the atmosphere.

Converting this to the notion of atmospheric atoms and molecules:
$\tau =$ integral of $\sigma\, N(z)\, dz$ where σ is the collisional cross-section of the atoms/molecules, N(z) is the number density of atoms/molecules, and dz is the height increment. We want this value of τ to be <= 1 to conclude that an upward traveling atom or molecule is unlikely to experience a collision that would prevent it from escaping.
We can simplify this to be: $\tau = \sigma\, N_{exopause}\, H_{exosphere}$ since the integral of N(Z) times dz from the exobase to the top of the atmosphere is well approximated by the product of N at the exobase times the scale height in the exosphere (assuming that the scale height in the exosphere is uniform with height). Thus, the exobase is defined as the atmospheric level at which N results in τ=1 in the above equation.

6) Let's assume that Mars' atmosphere is isothermal, which indicates that at every altitude the temperature is the same as at every other altitude.

A) The value of the atmospheric density at Mars' surface is ~ 0.01 kg m^{-3}. Using the maximum atmospheric density value indicated in Figure 1, 40 x 10^{-18} g cm^{-3} (= 40 x 10^{-15} kg m^{-3}) at the height of 390 km above the surface, **calculate the isothermal temperature in Mars' atmosphere** that corresponds to these two density values and the 390 km atmospheric 'thickness'. **[7.5 points]**

B) Discuss the reasonableness of your calculated isothermal temperature, or **provide and justify a 'guess'** isothermal temperature if you were not able to calculate a value in part a above. **[2.5 points]**

a) In an isothermal atmosphere, $\rho(z) = \rho_{sfc}\, e^{-z/H}$ where ρ (z) is density at some height z above the surface, ρ_{sfc} is the density at the surface, and H is the atmospheric scale height, $H = R_{CO2}\, T / g$, where R_{CO2} is the gas constant for CO_2 (the dominant gas in Mars' atmosphere), g is Mars' gravitational attraction (we'll assume that g is constant over the altitude range from 0-390 km.... maybe not a great assumption but OK), and T is the atmospheric isothermal temperature. So,

40×10^{-15} kg m^{-3} = 0.01 kg m^{-3} x $e^{(-390\ \mathrm{km}/H)}$

1/H (with H in units of meters) = -ln (40×10^{-15} / 0.01) / 390000 m = 6.73×10^{-5} m^{-1}

H = 1.486×10^{4} km (14.86 km)

$H = R_{CO2}\, T / g \qquad T = H\ g / R_{CO2} = 292$ Kelvin

b) this temperature is warmer than Mars' equilibrium temperature (216 K) and warmer than the temperature profiles shown in the figure presented above, but that maximum density corresponds to conditions of large EUV flux and thus hot exospheric temperatures as presented in Figure 3 of the paper, so this temperature is not too outrageous.
A guess without a calculated value should have considered the information presented in Figure 1 in the paper, which indicates that the maximum density coincided with large F10.7 cm flux values. This enhanced F10.7 cm (EUV) flux suggests tha the "T moderate solar activity" profile in the Mars figure in the exam would be appropriate, which then argues for warmer temperatures than are indicated by the primary T(z) profile in that figure.

The Data Presented in the Paper

7) In Paragraph 7, the authors discuss their extrapolation of the measured F10.7 cm flux (proxy for the EUV flux) at 1 AU to the EUV flux magnitude that would be received at Mars (~1.52 AU). The authors quantify this extrapolation in Figure 1 (blue and red curves). **Describe** why the Earth:Mars orbital conjunction is the least certain (the authors use the word 'important') orbital configuration at which to apply their extrapolation. [**7.5 points**]

The authors present within the paper an indication that the enhanced EUV flux arises from localized regions on the Sun's photosphere, and that the position of the regions which produce the enhanced EUV flux received at Earth might NOT also produce enhanced flux at Mars if Mars' orbital position is azimuthally far from Earth's position. So, when Mars and Earth are at conjunction, on opposite sides of the Sun, the EUV-generating photosphere region affecting Earth would not be 'seen' by Mars for ~13.5 days or one-half of a solar rotation. If the intensity of the emitting region was to change in that 13.5 day period then the estimate of the EUV flux being received at Mars would be inaccurate. This extrapolation would be worst when the Sun is near Solar Minimum since that is the time when EUV emitting regions are least abundant and thus azimuthal location most important.

8) The exospheric density values discussed in this paper are determined from the MGS spacecraft directly interacting with the atmosphere. [MGS did not carry a gas analyzer instrument.]
A) Think about and then describe the nature of the spacecraft's direct interaction with Mars' atmosphere and how that interaction results in detectable changes in the spacecraft's orbit which can be related back to the atmospheric density. <u>If you can include and explain an appropriate equation as part of your answer that would be good</u>. [7.5 **points**]

As MGS passed through the upper layers of Mars' atmosphere, it experienced dynamic pressure, at the magnitude of : $\frac{1}{2} \rho_{exosphere} V_{MGS}^2$ in a direction opposite to the spacecraft's orbital motion with spacecraft speed of V_{MGS}.
This episodic dynamic pressure experienced by the spacecraft would slightly slow the spacecraft's orbit speed at periapse. This will have the effect of ever so slightly reducing the apoapse distance and eccentricity of MGS' orbit; the reduced apoapse results in a reduced orbital semi major axis which slightly shortens the orbit period. By determining the changes in the spacecraft's orbit using the radio signals between Earth and MGS (Doppler shifts in MGS' signals, changed orbit period, etc.), the atmospheric density values that would produce the observed orbit changes can be calculated.

B) The authors state (Paragraph 12) that they used MGS orbit data information obtained during 48-60 orbits (4-5 days) to enable them to derive 'statistically significant' (Jim's term) individual exospheric density values at 390 km presented in Figure 1. If MGS' periapse had been at an altitude of only 50 km above Mars' surface (where gas density is $\sim 8 \times 10^{-5}$ kg m^{-3}), **would fewer or more** than 48-60 orbits have been needed to derive 'statistically significant' density values at or near this 50 km periapse altitude? **Explain. [5 points]**

At a density of $\sim 10 \times 10^{-15}$ kg m^{-3}, and an MGS speed of a few 1000 meters per second, the dynamic pressure experienced by the spacecraft would be $\sim \sim 4 \times 10^{-8}$ Pascals. When we account for the cross sectional area of the spacecraft (a few square meters), and the mass of the spacecraft (we'll assume 500 kg) and the duration of the time the spacecraft is within 1-2 atmospheric scale heights of the periapse level (this might be 30 seconds), the change in velocity of MGS during one pass through the atmosphere would be:

4×10^{-8} Pa x 3 m x 30 seconds / 500 kg = 8×10^{-9} m s^{-1} (or some small value).

This is a small signal! Accumulating the changed velocity over 50-60 orbits increases the signal by a similar factor, which starts to provide a large enough signal for density to be determined.

If the density was 10 orders of magnitude greater (at 50 km above the surface of Mars), the change in velocity calculated above would itself be 10 orders of magnitude greater, and change of 1-10 meters per second in spacecraft speed would be easily detectable... and in fact, if MGS descended down to 50 km above the surface, the dynamic heating it would experience (which scales like velocity cubed with an additional density dependence) would destroy the spacecraft, which is what happened to the Mars Climate Orbiter spacecraft when it arrived at Mars in December 1999!