

CUME #328
NMSU Department of Astronomy

Saturday, October 13th, 2007
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SOLUTION

This exam is focused upon the accompanying journal paper, "K-band transit and secondary eclipse photometry of exoplanet OGLE-TR-113b", by Snellen and Covino, MNRAS, 375, 307-312, 2007.

Answer each of the questions below, starting each new question on a new sheet of paper. Write only on one side of each sheet of paper.

Some information that you might or might not find useful:

The star (OGLE-TR-113):

Mass = 0.85 times the Sun's mass

The planet (OGLE-TR-113b):

Mass = 1.32 times Jupiter's mass

Orbital period = 1.4325 Earth days
= 1.237×10^5 seconds

Other items:

Jupiter's density : 1.3 grams per cubic centimeter

Sun's radius : 6.96×10^8 meters

Sun's mass : 2×10^{30} kg

Sun's Luminosity : 3.827×10^{26} J s⁻¹

1 Astronomical Unit : 1.5×10^{11} meters

Boltzmann's Constant : 1.38×10^{-23} J kg⁻¹ molecule⁻¹

Mass of a proton : 1.67×10^{-27} kg

Jupiter's mass : 2×10^{27} kg

Jupiter's radius : 71,000 km

Stefan-Boltzmann constant = 5.67×10^{-8} W m⁻² K⁻⁴

A priori, an earned grade of 70% (42 of 60 points) is anticipated to result in a 'PASS' grade.

1) a) Provide a clear discussion describing what limb darkening is and why it occurs. [5 points]

Limb darkening is a description of the effect that when observing a spherical object the more distant radial portions of the disk are darker (not as bright) as points nearer to the center of the disk. This limb darkening arises because the emission seen at the disk center is traversing through a shorter path length and thus photons from deeper (warmer) in the atmosphere are observed. At increasing distance from the disk center, the line-of-sight opacity is larger and the observed flux is originating from a colder (higher altitude) zone with the result of less flux being emitted and received.

b) Provide a physically-based discussion that indicates what specific reason(s) (environmental conditions) result in stellar limb darkening being much less of an issue at 'K band' than at optical wavelengths when viewing the transit of OGLE-TR-113b. [5 points]

K-band is a near-IR band (~2.2 microns). At these wavelengths, opacity due to molecular processes is beginning to dominate over atomic transitions. Within the stellar atmosphere opacity is less at 2.2 μm than at optical wavelengths so continuum flux at near-IR wavelengths is less likely (compared to optical flux) to be absorbed and reemitted at higher altitudes (lower temperatures) and thus retains its magnitude (brightness).

c) Add to the bottom-panel graph of Figure 3, **using red ink**, a curve that would represent the transit light curve under (fictitious) conditions where K-band limb darkening **IS IMPORTANT**. Provide a brief discussion indicating how you made your choices in regard to the phasing and magnitude values of your drawn curve. Keep in mind that the y-axis unit is a relative rather than an absolute measure. [2.5 points]

The curve will not decline (nor rise) as rapidly toward (away from) the peak of the transit initially (since less flux is being blocked at the now darker limb), but the curve minimum value will be less than (deeper) than) the standard curve since the then-obscured star center will have been contributing more of the total received flux from the star (since the limbs are darker) and therefore a greater percentage of the star's flux will be blocked by the planet at mid-transit.

2) a) What is planet OGLE-TRE-113b's orbital semi-major axis (OSA) length (in units of Astronomical Units)? Show your calculations. [5 points]

In its simplest form, Kepler's 3rd Law indicates: $(\text{Mass}_{\text{star}}/\text{Mass}_{\text{Sun}}) \text{Period}^2 = \text{OSA}^3$ where P is in units of Earth years and OSA in units of Astronomical Units. So:

$$(0.85/1.0) (1.4325/365.25)^2 = \text{OSA}^3$$

$$(0.85) \text{ times } 1.538 \times 10^{-5} = 1.2 \times 10^{-5} = \text{OSA}^3 \quad \text{OSA} = 0.0236 \text{ AU}$$

b) Jupiter has large moons with orbital semi-major axes as large as ~25 times Jupiter's radius. Based upon a quantitative dynamical argument, would you expect that extrasolar planet OGLE-TR-113b could have a large moon orbiting it with an orbital semimajor axis (OSA) of 25 planetary radii? A thorough discussion is fine for full credit, but can be augmented with a calculation if you like. [5 points]

Gravitational effects are the important factor here, especially the ability of the planet to hold on to the moon gravitationally ($G M_{\text{planet}} M_{\text{moon}} / r^2$) in the face of the tidal force imposed upon the planet-moon system by the central star ($2 G M_{\text{star}} M_{\text{planet}} r / a^3$). In fact, the Sun's gravitational attraction upon the Moon is twice that of the Earth upon the Moon, so it is the tidal force and not just the gravitational force that is important.

The Hill sphere describes the volume around an orbiting object (in this case the planet) within which the planet's gravity dominates over the star's tidal influence. The Hill Sphere is defined as:

$$R_H = (M_{\text{planet}} / (2 * M_{\text{star}}))^{1/3} \text{ times } a \quad [\text{the denominator factor of 2 is sometimes seen as 3}]$$

where a is the orbital semi-major axis of the planet. Since the planet's mass is $\sim 1/1000$ of the star's mass, and $1000^{-1/3}$ is $1/10$ a , or ~ 0.0023 AU ($\sim 3 \times 10^8$ meters), which is < 10 times Jupiter's (or this planet's) radius ($\sim 7 \times 10^7$ meters). Thus, a moon at $25 R_{\text{planet}}$ would be unbound.

For this situation,

$$(M_{\text{planet}} / 2 * M_{\text{star}})^{1/3} = [1.32 * M_{\text{Jup}} / (2 * 0.78 * M_{\text{Sun}})]^{1/3} =$$

$$[1.32 / (2 * 0.85 * 1000)]^{1/3} = 0.09458$$

$$\text{so, } R_H = 0.09458 * (0.0236 \text{ AU}) * (1.5 \times 10^{11} \text{ m/AU}) = 3.3 \times 10^8 \text{ meters} = 3.3 \times 10^5 \text{ km}$$

Jupiter's radius is 71,000 km, so OGLE-TR-113b's radius is $\sim 1.09 * R_{\text{Jup}} = 77,400$ km. The Hill Sphere radius, 320,000 km, is < 5 planetary radii, so the planet **WOULD NOT** be able to 'hold on' to a moon with an Orbital Semimajor Axis of 25 planetary radii.

c) Assuming that planet OGLE-TR-113b's orbit plane is parallel to the line-of-sight of an observer at Earth, what would be the maximum radial velocity measured for the star assuming the planet's orbit is circular? Show your calculations. [7.5 points]

$$\text{Orbit period} = 1.4325 \text{ days} = 1.237 \times 10^5 \text{ seconds}$$

$$\text{For the star:planet system: } M_{\text{star}} \times \text{OSA}_{\text{star}} = M_{\text{planet}} \times \text{OSA}_{\text{planet}}$$

$$\begin{aligned} \text{OSA star} &= (1.32 \text{ times Jupiter's mass}) \times (0.0236 \text{ AU}) / (0.85 \text{ times the Sun's mass}) = \\ &= (0.85) (0.0236) / (0.78 * 1000) = 3.67 \times 10^{-5} \text{ AU} \end{aligned}$$

$$\text{Star's velocity} = \text{orbit circumference} / \text{orbit period} =$$

$$[2 * \pi * 3.67 \times 10^{-5} \text{ AU} * 1.5 \times 10^{11} \text{ meters per AU}] / [123700 \text{ seconds}] =$$

$$\text{Star's velocity} \sim 280 \text{ meters per second}$$

3) Let's assume that planet OGLE-TR-113b formed at an OSA of 0.1 AU from its parent star and that its formation occurred via the process of Core Accretion. For this condition: Describe TWO attributes of this planet's formation in this environment which would result in OGLE-TR-113b's formation rate being **slower than** Jupiter's formation rate here in our Solar System at 5.2 AU. **At least one of your two reasons must invoke Safronov's formulation of protoplanet formation rate.** [7.5 points]:

Planet coagulation / accretional growth in the Safronov framework is:

$$dM/dt = \pi R_{plan}^2 \rho V_{rel} [1 + (V_{esc} / V_{rel})^2]$$

i) Since the Keplerian shear is greater at smaller OSA's, the $(V_{esc} / V_{rel})^2$ term will be ~~greater~~ ^{smaller} at smaller OSA's which will tend to minimize the growth rate.

ii) The critical core mass for gas accretion to set in is dependant upon the nebular gas temperature, so the hotter nebular disk gas temperatures nearer to the protostar will require a larger core mass (and thus greater core growth time) than at ~5 AU for gas collapse to occur.

4) a) Assuming the standard mass:luminosity relation applies for this near solar-mass star (OGLE-TR-113), determine this star's luminosity. [Note: your result will not necessarily agree with the luminosity suggested by information provide in section 4.1 of the paper.] [5 points]

If Luminosity (L) is proportional to $M^{3.5}$ (exponents between 3-4 are fine) then:

$$L(\text{relative to Sun}) = \text{Mass}(\text{relative to the Sun})^{3.5} = 0.85^{3.5} = 0.566,$$

$$\text{so the star's luminosity} = 0.566 \times 4 \times 10^{26} \text{ J s}^{-1} = 2.17 \times 10^{26} \text{ J s}^{-1}.$$

b) What Bond Albedo value, in conjunction with the stellar luminosity you calculated above, corresponds to a measured emission (think 'equilibrium') temperature of 1357 Kelvin for planet OGLE-TR-113b ? [7.5 points]

$$[(L_{star} / (4 \pi OSA_{plan}^2))] \times (1 - \text{Albedo}) \times (\pi R_{plan}^2) = 4 \pi R_{plan}^2 \sigma T_{plan}^4$$

$$[1.38 \times 10^6 \text{ W m}^{-2}] \times [1 - \text{Albedo}] = 4 \sigma T_{plan}^4$$

$$\text{Albedo} = 1 - [4 \sigma T_{plan}^4 / 1.54 \times 10^6 \text{ W m}^{-2}] = 0.44$$

c) Considering the expected physical characteristics of gas-giant planet OGLE-TR-113b (use planets in our solar system as a comparison), do you believe that the correctly calculated Bond Albedo value above represents an upper limit or lower limit of the planet's actual Bond Albedo (for the conditions specified)? Provide a clear defense of your conclusion. [5 points]

I would expect this albedo to be a **lower limit**. In our solar system both Gas Giant planets, and one-of-two of the ice giant planets, have internal heat sources which result in the planet emitting away to space more radiation than they absorb from their parent star. If the measured emission includes both absorbed-and remitted as well as internally-liberated energy, than the planet's albedo must be greater than that calculated above so that the sum of 'absorbed and re-emitted' and 'internal' energies matches the energy emitted away by the planet.

5) Let's imagine that planet OGLE-TR-113b is a tidally-locked prograde rotating planet with an obliquity of 0 (zero) degrees. For this scenario, what direction (prograde, retrograde, north, south, etc.) would 'high in the atmosphere' 45 degree latitude geostrophic winds be flowing if absorbed stellar flux 'low in the atmosphere' is the ultimate driving force for the winds? Justify your conclusion, and if you desire you can add a picture to strengthen your argument. **[5 points]**.

In this scenario, hottest temperatures at a given pressure level in the atmosphere will occur at equatorial latitudes, and the atmospheric thickness will be greatest there. Declining temperatures toward the poles will result in pressure surfaces being at lower altitudes at higher latitudes. The resulting poleward directed pressure gradient at a given height will, in balance with Coriolis forces, produce winds which flow in the same direction as the prograde planet rotation.